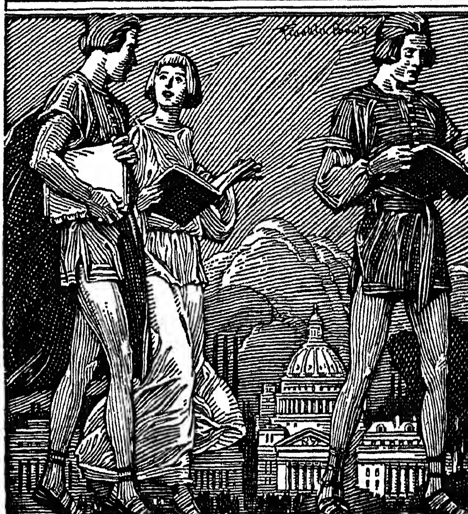

ELEMENTARY
STRUCTURAL PROBLEMS
IN STEEL AND TIMBER

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STRUCTURAL PROBLEMS
IN STEEL AND TIMBER

BY

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PREFACE TO THE SECOND EDITION

Many new problems, some of them involving the design of entire structures, or portions of structures, have been added in this edition. At the same time, opportunity has been taken to correct all known errors and to clarify the text where such appeared desirable. Certain problems concerning forms of construction or details that are becoming obsolete, and problems that experience has shown to be too advanced for elementary classes, have been omitted. Naturally, extensive changes have had to be made to conform to the new W.F. sections.

The more important additions are new problems pertaining to the following: welded connections; column bending, including the design of an eccentrically loaded corner column; torsion on a wall girder; reinforcement of a beam web for shear; a floor panel utilizing reinforced concrete joists supported on steel girders; wind stresses in a bent due to a combination of pressure and suction; deflections of a truss by analytical and graphical methods; design of an A-bent for the support of an outside travelling crane; a stepped side-column of a mill building supporting a travelling crane; a crane runway girder; moments, shears and floor-beam concentrations due to moving loads on beams, girders and trusses; and the design of a pony Warren truss highway span. A second Appendix has been added containing the derivation of certain special formulae used in the text.

C. R. YOUNG.

UNIVERSITY OF TORONTO,
February, 1935.

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CHAPTER I

TENSION MEMBERS

1. Fundamental Formulae for Tension Members.—The following formulae are applicable to tension members:

For members centrically loaded

$$f_t = \frac{P}{A} \tag{1}$$

$$A = \frac{P}{p_t} \tag{2}$$

For members subjected to direct axial tension and to cross bending arising either from eccentricity of the axial loading or from transverse loading, such bending being in the plane of a principal axis

$$f_t + f_f = \frac{P}{A} + \frac{My_e}{I} \tag{3}$$

$$A_t + A_f = \frac{P}{p_t} + \frac{My_e}{r^2 p_t} \tag{4}$$

For members subjected to combined axial tension and symmetrical bending, where the effect of deflection is considered

$$f_t + f_f = \frac{P}{A} + \frac{My_e}{I + \dfrac{Pl^2}{CE}} \tag{5}$$

$$A_t + A_f = \frac{P}{p_t} + \frac{My_e}{\left(r^2 + \dfrac{Pl^2}{CAE}\right)p_t} \tag{6}$$

* See Appendix II.

References

Merriman—Mechanics of Materials.
Johnson, Bryan and Turneaure—Modern Framed Structures, Pt. III.
Shedd—Structural Design in Steel.
Swain—Structural Engineering.

The significance of the symbols employed in the above formulae is as follows [symbols used in connection with special or empirical formulae being defined later where such formulae occur]:

A = effective area required or provided, as the expression may indicate;
A_f = area required for flexure;
A_t = area required for tension;
C = a coefficient having approximate values;

Members free to turn at both ends, $C = 10$
Members free to turn at one end and fixed at the other, uniform load,
Centre moment, $C = 13$
End moment, $C = 23$
Members free to turn at one end and fixed at the other, single central concentrated load,
Centre moment, $C = 17$
End moment, $C = 20$
Members fixed at both ends, uniform load,
Centre moment, $C = 16$
End moment, $C = 32$
Members fixed at both ends, single concentrated load,
Centre and end moments, $C = 24$

E = modulus of elasticity of the material;
f_f = existing extreme fibre stress due to flexure;
f_t = existing fibre stress due to centric axial tension;
I = moment of inertia of the cross section about its gravity axis normal to the plane of bending;
l = length of tension member in the same unit of length as is involved in A, E, f, I, M, p, r and y defined in this article;
M = bending moment;
P = external axial force;
p_t = permissible stress in tension per unit of effective net area;
r = radius of gyration;
y_e = distance from neutral axis to that extreme fibre of a member in flexure at which the bending stress is sought.

2. Non-Upset Rod Tension Members.—The tie rods in a tile arch floor are to be of mild steel, $\frac{3}{4}$ in. in diameter, threaded and not upset. If the net

thrust per lineal foot of beam is 1100 lb., find the theoretical spacing of the rods. $p_t = 16,000$ lb. per sq. in.

Net area of one rod, at root of thread, from structural handbooks, $=0.302$ sq. in.

Value of one rod $= 0.302 \times 16,000 = 4840$ lb.

Safe spacing of rods $= 4840/1100 = 4.40$ ft.

3. Upset Rod Tension Members.—A tension diagonal of the wind bracing of a tower is to carry a load of 25,000 lb. and is to be composed of one or two round or square upset threaded rods. Suggest a section. $p_t = 16,000$ lb. per sq. in.

Area required (Eq. (2), Art. 1) $= 25,000/16,000 = 1.56$ sq. in.

As standard upsets give a considerable excess in area at the root of thread over the area in the body of the bar, the latter will govern.

One $1\frac{1}{4}$-in. square bar will do. Area $= 1.563$ sq. in.

Two 1-in. round bars with a combined area of 1.570 sq. in. would also do.

The single bar is preferable by reason of less handling and erection costs.

4. Eye-Bar Tension Members.—An adjustable eye-bar tension member carrying a load of 124,000 lb. is to connect at either end to a pin, estimated to be of 4-in. diameter, and is to consist of either two or four bars, whichever number is more practicable. The thickness of the bars is not to be less than one-eighth of their width nor less than $\frac{7}{8}$ in., and their width is not to exceed eight-sevenths of the diameter of the pins. Heads are to be of the American Bridge Co. standard. $p_t = 18,000$ lb. per sq. in.

As the upset ends have a net area at the root of thread considerably in excess of the area in the body of the bar (see handbook), the latter will determine the size.

Required area in body of bars (Eq. (2), Art. 1) $= 124,000/18,000 = 6.88$ sq. in.

Maximum permissible width of bars $= 4 \times \frac{8}{7} = 4.57$ in., or using standard bars, 4 in. Minimum thickness for a bar of this width is $\frac{4}{8} = 0.5$ in., or, applying the second rule, $\frac{7}{8}$ in.; the latter governs.

Two $4 \times \frac{7}{8}$-in. bars giving 7.00 sq. in. are sufficient.

Four bars cannot be economically employed because of limiting thickness rules.

5. Net Section of Riveted Tension Members.—Experimental investigation (Section 3, Bulletin 6, 1926, School of Engineering Research, University of Toronto) has shown that the total number of rivet holes that should be deducted from the gross right sectional area of a tension member containing more than one line of holes may be considered as

$$n = 1 + x_1 + x_2 + x_3 + \quad \ldots \tag{1}$$

where x is a fraction of a rivet hole depending on the ratio of stagger s to gauge g and having approximate values for $\frac{3}{4}$-, $\frac{7}{8}$-, 1- and $1\frac{1}{8}$-in. holes as given in the diagrams of Fig. 1.

The above formula is to be applied to alternate sections, the successive terms

representing the deduction for successive holes considered in a chain across the member. The particular group of holes to be considered is that which will give the greatest total deduction, whether the holes lie on adjacent gauge lines or not. Where the ratio s/g is large, the corresponding x may be negative.

For deduction purposes a hole is considered as having a diameter $\frac{1}{8}$ in. greater than that of the rivet before driving.

6. Simplified Deduction Rules for Rivet Holes.—For determining rivet-hole deductions Professor T. R. Loudon's simplified rule (Section 10, Bulletin 9, School of Engineering Research)

$$x = 1.50 - s/g$$

may also be used. In this, x, s and g have the same significance as in Art. 5. The total number of holes to be taken out may be found by Eq. (1), Art. 5, but in no instance is a value of x to exceed unity. The results are somewhat more severe than those given by the diagrams.

The modified Cochrane rule

$$x = 1 - s^2/3.5\,gh$$

while not so easily remembered or applied as Loudon's, is more accurate for a wide range of stagger ratios s/g. In it, h = diameter of the hole.

In the specifications of the Canadian Engineering Standards Association the stipulated rule is as follows:

> "Allowance shall be made in each component of the member for as many rivet holes as it contains gauge lines, unless the distance centre to centre of rivet holes, measured on the diagonal, is at least 40% greater than the dis-distance between the gauge lines."

Another type of rule is that employed in the Specifications for Steel Highway Bridges, of the American Society of Civil Engineers (1924):

> "In sections having rivets staggered, all rows shall be deducted unless arranged so that the net section along a zigzag line, taking all distances on the diagonal at 90% of their value, is greater than the corresponding net section across the plate."

The two last rules are less convenient than the preceding ones in that the diagonal distances between holes must be calculated.

Still another type of rule that is sometimes employed is this:

> "The net section of a riveted member shall be the least area that can be obtained by deducting from the gross sectional area the area of holes cut by any plane perpendicular to the axis of the member, and parts of the areas of other holes on one side of the plane within a distance of 4 in. which are on a gauge line 1 in. or more from those of the holes cut by the plane, the area of each part being determined by the formula:
>
> $$a = A\left(1 - \frac{p}{4}\right)$$

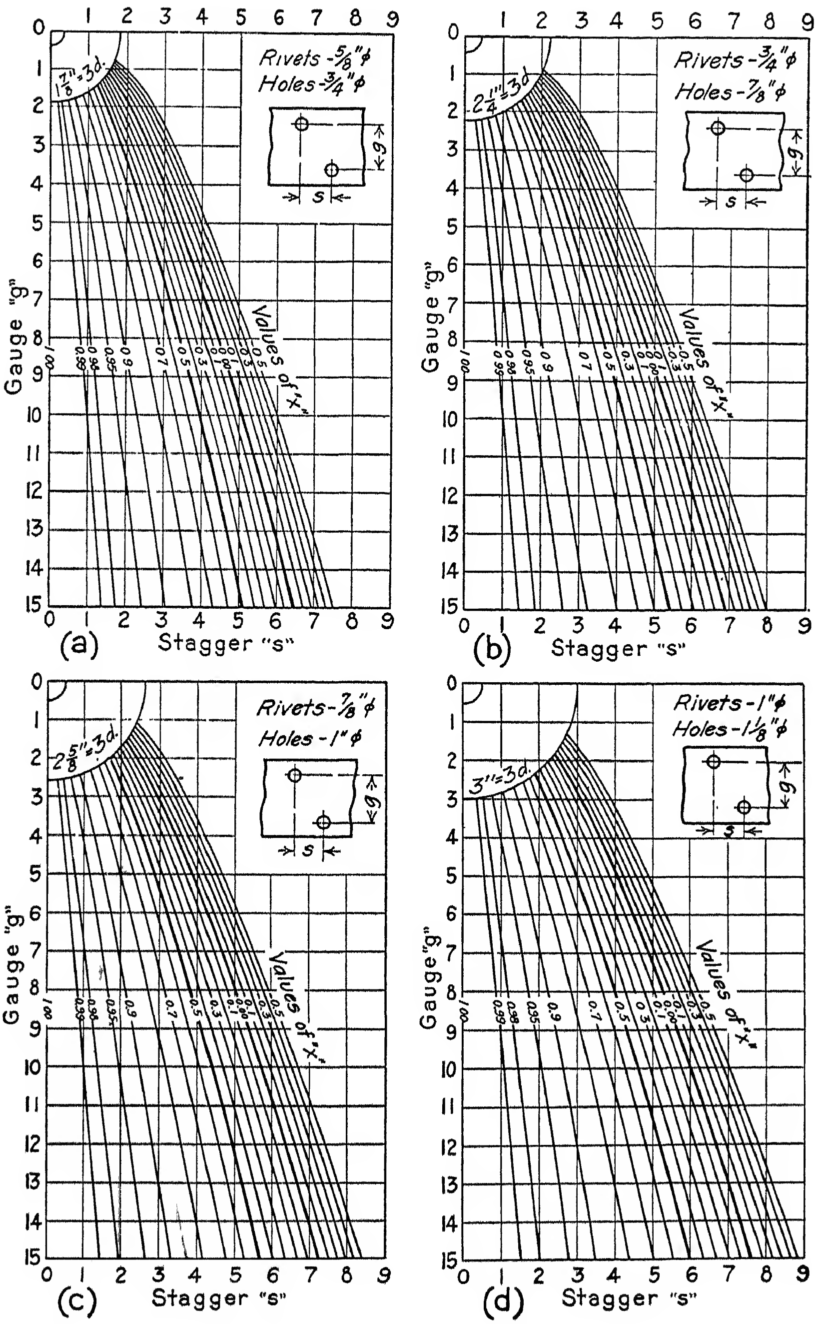

Fig. 1.—Diagrams for Deductions of Rivet Holes for Various Stagger Ratios.

in which A = area of the hole, and p = the distance, in inches, of the centre of the hole from the plane."

7. Determination of Rivet-Hole Deduction.—Find the correct number of holes to be deducted from the gross right section of the tension plate shown in Fig. 2. Rivets, $\frac{7}{8}$ in. Holes, for deduction, 1 in.

A comparison of the deductions necessary on sections *ABCD*, *ABCHI*, *ABFCD*, *ABFG*, *ABFHI*, *EFCD*, *EFG* and *EFHI* must be made. It is evident that the deductions along *ABFG*, *EFCD*, *EFG* and *EFHI* are less than along *ABCD*. These sections may, therefore, be neglected in the comparison.

Noting the relation of stagger to gauge applicable to successive rivets, and making use of the curves of Fig. 1(*c*), the numbers of holes to be deducted for the remaining sections are:

ABCD,	$1 + 1$	= 2.00 holes
ABCHI,	$1 + 1 - 0.52$	= 1.48 holes
ABFCD,	$1 + 0.47 + 0.80$	= 2.27 holes
ABFHI,	$1 + 0.47 + 0.96$	= 2.43 holes

Thus, *ABFHI* is the critical section and the deduction from the gross right section is 2.43 holes.

8. Adequacy of Riveted Plate Tension Member.—A $6 \times \frac{3}{8}$-in. flat carrying a tension of 30,000 lb. is connected to a gusset plate by seven $\frac{3}{4}$-in. rivets

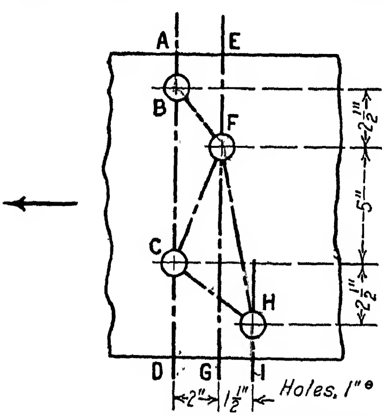

FIG. 2.—Rivet Hole Deduction.

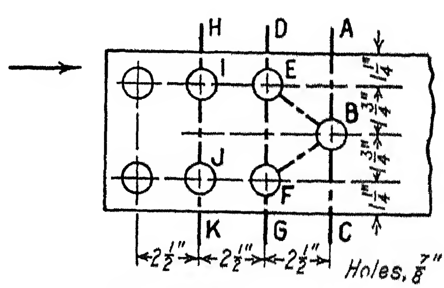

FIG. 3.—Plate Tension Member.

arranged as shown in Fig. 3. Express an opinion as to the safety of the plate in tension, apart from the security of the riveting. p_t = 16,000 lb. per sq. in. Holes, for deduction, $\frac{7}{8}$ in.

Those sections likely to be most seriously stressed are the net sections along the lines *ABC*, *ABFG*, *DEBFG* and *DEFG*, having regard, of course, to the total stress in the plate at these sections.

The deductions at the sections named are, from Fig. 1(*b*), as follows:

ABC,	$1 + 0$	$= 1.00$ hole
ABFG,	$1 - 0.29$	$= 0.71$ hole
DEBFG,	$1 - 0.29 - 0.29$	$= 0.42$ hole
DEFG,	$1 + 1$	$= 2.00$ holes

On section *ABC*, the net area is $6 \times 0.375 - 1 \times 0.875 \times 0.375 = 1.92$ sq. in.; on section *ABFG* it is $2.25 - 0.71 \times 0.33 = 2.02$ sq. in.; on section *DEBFG* it is $2.25 - 0.42 \times 0.33 = 2.11$ sq. in; and on section *DEFG* it is $2.25 - 2.0 \times 0.33 = 1.59$ sq. in.

At sections *ABC*, *ABFG* and *DEBFG* the total stress in the plate is 30,000 lb., while at section *DEFG* it is only $\frac{6}{7}$ of 30,000 = 25,710 lb., as the rivet *B* has delivered $\frac{1}{7}$ of the applied force to the gusset plate.

Along the section *ABC* the average existing stress is 30,000/1.92 = 15,620 lb. per sq. in.; along sections *ABFG* and *DEBFG* it is obviously less than this amount; along section *DEFG* it is 25,710/1.59 = 16,170 lb. per sq. in.

The plate is therefore slightly overstressed on the section *DEFG*.

HIJK is not a critical section, as the area here is the same as at *DEFG* and the total stress is only two-thirds as much.

9. Efficiency of Tension Angles with or without Lugs.—Analysis of the tests made on tension angles by Professor Frank P. McKibben (*Engineering News*, July 5, 1906, and August 22, 1907) and by J. E. Greiner (*Transactions*, A. S. C. E., Vol. 38) show, if the net areas are computed by the method of Art. 5, efficiencies of single angles connected by both legs ranging from 79.4 to 100%, with an average of 91.4%. In accordance with usual practice, where permissible stresses in tension run from 16,000 to 18,000 lb. per sq. in., the net area of such angles will be assumed as 100% efficient.

Similar analysis of single angles connected by one leg only disclosed an efficiency ranging from 74.9 to 82.8%, with an average of 78.9%. In devising an approximate formula for the efficiency of single angles connected by one leg only, it has been thought well so to frame it that the efficiencies given thereby would bear the same ratio to the test efficiences as the adopted design efficiency (100%) for angles connected by both legs bears to the average test efficiency of such angles (91.4%).

A close approximation to the fractional efficiency of the net section of a single angle connected by one leg only is given by the formula

$$e = 1.0 - 0.18\, u/c$$

where u and c are the lengths in inches of the outstanding and the connected legs respectively.

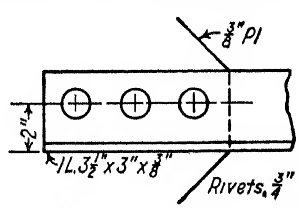

Fig. 4.—Single-Angle Tension Member.

10. Single-Angle Tension Member Connected by One Line of Rivets—Approximate Method.—Design, by the approximate method, a single-angle tension member for a load of 25,000 lb. to be connected to a $\frac{3}{8}$-in. gusset plate by a single line of rivets, as shown in Fig. 4. p_t = 16,000 lb. per sq. in. Rivets, $\frac{3}{4}$ in. Holes, for deduction, $\frac{7}{8}$ in.

For an exact design, the theory of unsymmetrical bending should be applied. Allowance for bending stress due to eccentricity in a single angle connected by one leg may be made by applying the formula of Art. 9 for the fractional efficiency of the net section

$$e = 1.0 - 0.18\,u/c$$

Required effective net area = 25,000/16,000 = 1.56 sq. in.

In general, an unequal-leg angle with the long leg connected is, on the ground of efficiency, preferable to an equal-leg one.

Assume a $3\frac{1}{2} \times 3 \times \frac{3}{8}$-in. angle with $3\frac{1}{2}$-in. leg connected. Gross area = 2.30 sq. in. Deducting one $\frac{7}{8}$-in. hole, net area = $2.30 - 0.875 \times 0.375 = 1.97$ sq. in.

Effective fraction of net section is $e = 1.0 - 0.18 \times 3/3.5 = 0.846$. Effective net section = $0.846 \times 1.97 = 1.67$ sq. in.

The assumed section is adequate.

11. Single-Angle Tension Member Connected by One Leg by Two Lines of Rivets—Approximate Method.—Design, by the approximate method, a single-angle tension member for a load of 29,000 lb., to be connected to a $\frac{3}{8}$-in. gusset plate by two lines of rivets, as shown in Fig. 5. $p_t = 16{,}000$. Permissible shearing and bearing stresses on rivets, 12,000 and 24,000 lb. per sq. in., respectively. Rivets, $\frac{3}{4}$ in. Holes, for deduction, $\frac{7}{8}$ in.

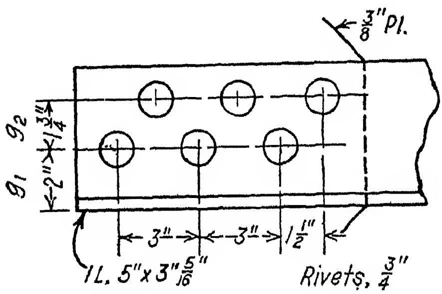

FIG. 5.—Single-Angle Tension Member with Two Rows of Rivets.

For an exact solution, the theory of unsymmetrical bending should be employed. An approximation to this result may be obtained by using the reduction formula, $e = 1.0 - 0.18\,u/c$, as for the problem of Art. 10.

Required effective net area = 29,000/16,000 = 1.81 sq. in.

An unequal-leg angle with longer leg connected is to be preferred. Assume a $5 \times 3 \times \frac{5}{16}$-in. angle with 5-in. leg connected. Gross area = 2.40 sq. in.

The arrangement of rivets at the inner end of the connection must be fixed before deduction for holes can be made. If they be arranged as shown in Fig. 5, with a stagger $s = 1\frac{1}{2}$ in. and a gauge $g_2 = 1\frac{3}{4}$ in., the deduction is found from Fig. 1(*b*) to be 1 + 0.50 = 1.50 holes.

Net area of angle = $2.40 - 1.50 \times 0.875 \times 0.3125 = 1.99$ sq. in.

Effective fraction of net section $e = 1.0 - 0.18 \times \frac{3}{5} = 0.892$. Effective net section = $0.892 \times 1.99 = 1.78$ sq. in. This is sufficiently near the requirement.

12. Maximum Stress in Double-Angle Tension Member Connected by One Leg.—A tension member consists of two $4 \times 3 \times \frac{3}{8}$-in. angles placed back to back with the long legs adjacent and separated by the thickness of the

$\frac{3}{8}$-in. end gusset plates, as shown in Fig. 6. The total load is 21,000 lb. One line of $\frac{7}{8}$-in. rivets on a $2\frac{1}{2}$-in. gauge is used in the 4-in. legs. Stitch rivets are used in the body of the member spaced 3 ft. apart. Find the maximum fibre stress.

The load will be assumed as applied at the gauge line and the member will be assumed as bending as a whole about the gravity axis of the net section parallel to the outstanding legs.

Net area of section $= 2 \times 2.48 - 2 \times 1 \times 0.375 = 4.21$ sq. in.

The position of the gravity axis of the net area, $GANA$, may be found by taking the statical moment of the net area about the backs of the outstanding legs. It is

$$(Q_{GA} - Q_H)/A_n$$

where Q_{GA} = statical moment of gross area, Q_H = statical moment of holes and A_n = net area of cross section. Inserting numerical quantities, the distance from the axis of reference becomes $(2 \times 2.48 \times 1.28 - 2 \times 1 \times 0.375 \times 2.50)/4.21 = 1.063$ in.

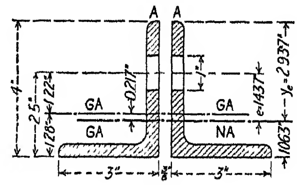

FIG. 6.—Double-Angle Tension Member without Lugs.

The shift in the neutral axis caused by the punching of holes is, therefore, $1.28 - 1.063 = 0.217$ in.

True net moment of inertia, which must be about $GANA$, is

$$I_n = I_g - I_h - A_n s^2$$

where I_n = moment of inertia of net section about the gravity axis of the net section, I_g = moment of inertia of the gross section about its own gravity axis, I_h = moment of inertia of the holes about the gravity axis of the gross section, A_n = net area of the section, and s = the distance between the gravity axes of the gross and the net sections, that is, the shift.

Inserting appropriate numerical quantities

$$I_n = 2 \times 4.0 - 2\{\tfrac{1}{12} \times 0.375 \times 1^3 + 0.375 \times 1.0 \times 1.22^2\}$$
$$-4.21 \times 0.217^2 = 6.62 \text{ in.}^4$$

Had the moment of inertia been found by the approximate formula

$$I_n = I_g - I_h$$

the results would have been only 3% too large.

The eccentricity of loading, e, is 1.437 in. and the most highly stressed fibres are at AA.

Applying Eq. (3) of Art. 1

$$f_t + f_f = \frac{21{,}000}{4.21} + \frac{21{,}000 \times 1.437 \times 2.937}{6.62}$$

$= 4990 + 13{,}400 = 18{,}390$ lb. per sq. in.

13. Single-Angle Tension Member Connected by One Line of Rivets in Each Leg.—Determine the necessary size of an unequal-leg angle connected

by one line of rivets in each leg (Fig. 7) to a $\frac{3}{8}$-in. gusset plate for a tension of 50,000 lb. Stagger of first rivets 2 in. p_t = 18,000 lb. per sq. in. Rivets, $\frac{7}{8}$ in. Holes, for reduction, 1 in. Assume full net section as effective.

Required net section = 50,000/18,000 = 2.78 sq. in.

Assume a $5 \times 3\frac{1}{2} \times \frac{7}{16}$-in. angle with a gross area of 3.53 sq. in., connected to the gusset plate directly and by means of a lug angle connected to the outstanding leg, as shown in Fig. 7.

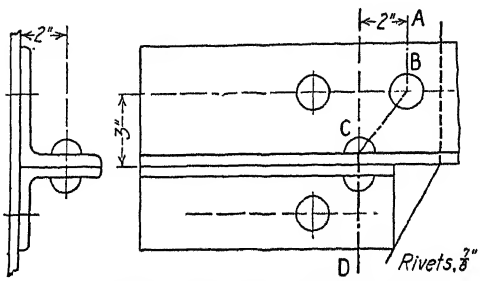

FIG. 7.—Single-Angle Tension Member with Lug.

The critical section is evidently along the line $ABCD$. The two holes cut are on gauge lines 3 + 2 − 0.4375 = 4.565 in. apart and the stagger is 2 in. From Fig. 1(c), the deduction should be 1 + 0.77 = 1.77 holes.

Net area = 3.53 − 1.77 × 1 × 0.4375 = 2.76 sq. in., which is sufficiently close to the requirement.

14. Double-Angle Tension Member of Channel Form Connected by Flanges Only, with Moment Suppressed.—Two angles arranged in channel form, with end battens so disposed as to eliminate the moment of eccentricity are to transmit a load of 43,000 lb. to outside gussets, as shown in Fig. 8. Recommend a size. Rivets, $\frac{7}{8}$ in. p_t = 16,000 lb. per sq. in.

Net area required = 43,000/16,000 = 2.69 sq. in.

Assume two $3\frac{1}{2} \times 3 \times \frac{5}{16}$-in. angles with a gross area of 3.86 sq. in.

Critical section is on the line $ABCD$. For the dimensions shown in Fig. 8,

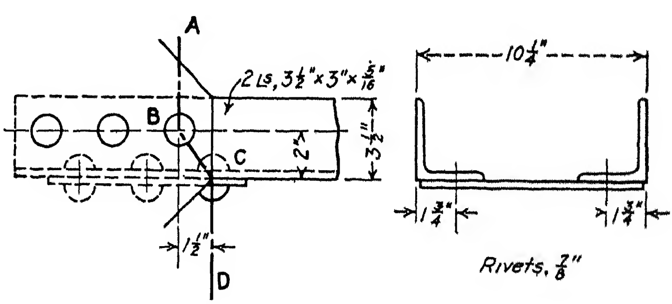

FIG. 8.—Double-Angle Tension Member with Suppressed Moment.

$s = 1\frac{1}{2}$ in. and $g = 3\frac{7}{16}$ in. Hence from Fig. 1(c), the rivet-hole deduction for the two angles is 2 × 1.81 = 3.62 holes.

Net area = 3.86 − 3.62 × 1 × 0.3125 = 2.73 sq. in.

The section is sufficient.

15. Built-Up I-Shaped Tension Member with Moment of Eccentricity Suppressed.—Find the safe capacity of the four-angle-and-plate tension member shown in Fig. 9. Rivets, $\frac{7}{8}$ in. $p_t = 16{,}000$ lb. per sq. in.

Section *ABCDEFG* is evidently the critical section. Proceeding as in Arts. 13 and 14, the deduction for one angle, using the diagram in Fig. 1(*c*), is $1 + 0.40 + 0.75 = 2.15$ holes, while the deduction for the web plate is 2 holes.

Gross area of member $= 4 \times 4.18 + 14 \times 0.375 = 21.97$ sq. in.

Net area $= 21.97 - (4 \times 2.15 \times 1 \times 0.4375) - (2 \times 1 \times 0.375) = 17.46$ sq. in.

As the web plate effectually prevents bending in the angles, the safe capacity

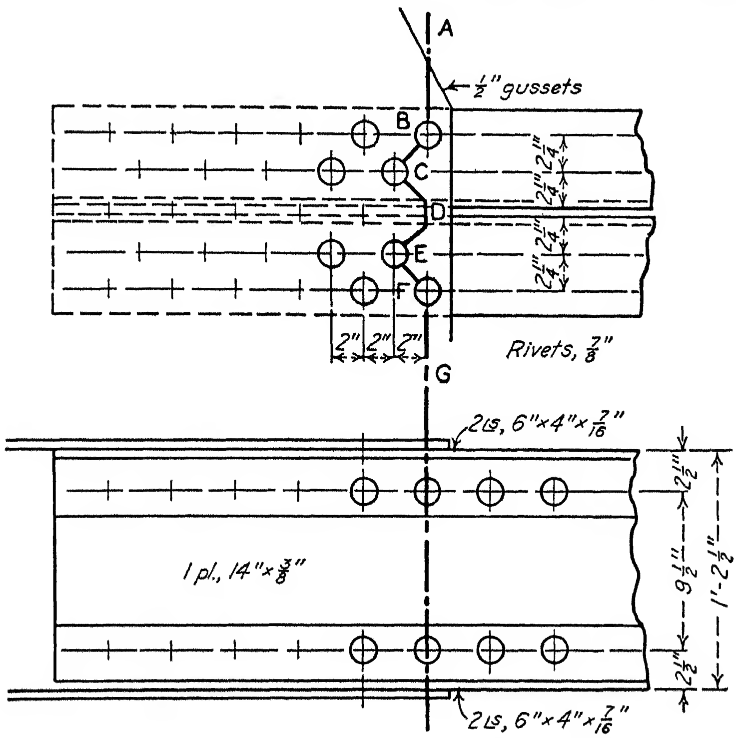

FIG. 9.—I-Shaped Tension Member with Suppressed Moment.

is based wholly on tensile value under uniformly distributed stress. It is, therefore, $17.46 \times 16{,}000 = 279{,}500$ lb.

16. Single-Channel Tension Member Connected by Web Only.—A single channel 8 in. deep, attached to $\frac{5}{16}$-in. gusset plates at each end is to transmit a tension of 27,000 lb. What weight is necessary? Rivets, $\frac{3}{4}$ in. $p_t = 16{,}000$ lb. per sq. in. Permissible shearing and bearing stresses on rivets, 12,000 and 24,000 lb. per sq. in., respectively.

Assume an 8-in., 11.5-lb. channel with general arrangement of end connections as shown in Fig. 10.

In addition to the axial tension, a bending moment about the gravity axis of the channel parallel to the web must be provided for.

Consider net sections AA and BB. According to Fig. 1(b) it is not necessary to consider a zigzag section.

Net area at section $AA = 3.36 - 1 \times 0.875 \times 0.22 = 3.17$ sq. in. Net area at section $BB = 3.36 - 2 \times 0.875 \times 0.22 = 2.98$ sq. in.

Net moment of inertia is approximately the moment of inertia of the gross area about the gravity axis of the latter parallel to the web, less the moment of inertia of the hole (or holes) about this same axis. This, for section AA is approximately $1.30 - 0.19 \times (0.47)^2 = 1.26$ in.^{4} For section BB it is approximately $1.30 - 2 \times 0.19 \times (0.47)^2 = 1.22$ in.^{4}

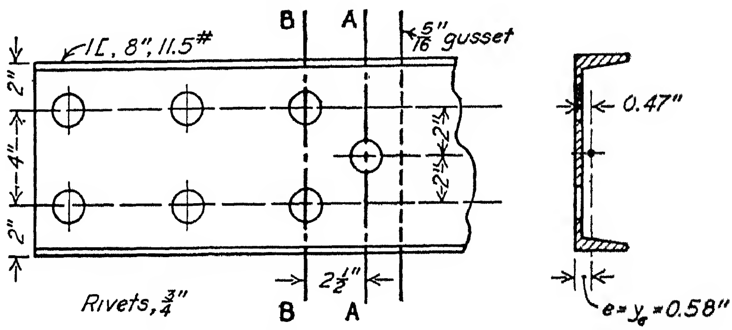

FIG. 10.—Single Channel Connected by Web Only.

Maximum stress at back of channel at section AA, assuming the 27,000-lb. force to be applied in the plane of the back of the channel, is, from Eq. (3) of Art. 1

$$f_t + f_f = \frac{27{,}000}{3.17} + \frac{(27{,}000 \times 0.58) \times 0.58}{1.26}$$
$$= 8520 + 7210 = 15{,}730 \text{ lb. per sq. in.}$$

Maximum stress at back of channel at section BB is similarly found, the axial load at this section being only $\frac{6}{7}$ of $27{,}000 = 23{,}200$ lb. if seven rivets are employed in the end connection. The combined stress is, then,

$$f_t + f_f = \frac{23{,}200}{2.98} + \frac{(23{,}200 \times 0.58) \times 0.58}{1.22}$$
$$= 7780 + 6400 = 14{,}180 \text{ lb. per sq. in.}$$

Section AA is, therefore, the critical section, and the 8-in., 11.5-lb. channel is adequate for the load.

17. Double-Channel Tension Member Subjected to Bending at End Connections.—A tension member carrying 86,000 lb. is to consist of two 10-in. battened channels with the flanges turned out. Recommend a section, if the

riveting arrangement is as shown in Fig. 11, and the battens are outside the end gusset plates. Rivets, $\frac{3}{4}$ in. p_t = 16,000 lb. per sq. in. Permissible shearing and bearing stresses in rivets 10,000 and 20,000 lb. per sq. in., respectively.

Assume two 10-in., 20-lb. channels, having a gross area of $2 \times 5.86 = 11.72$ sq. in.

Since the arrangement of the battens is such that bending of the individual channels at right angles to the plane of the web may take place at the end connections, provision must be made for both direct tension and bending.

Consider the net areas at sections *AA*, *BB*, and *CC*. According to Fig. 1(*b*), it is unnecessary to consider a zigzag section.

At section *AA*, the net area is $11.72 - 4 \times 0.875 \times 0.4375 = 10.19$ sq. in.
At section *BB*, the net area is $11.72 - 4 \times 0.875 \times 0.379 = 10.39$ sq. in.
At section *CC*, the net area is $11.72 - 6 \times 0.875 \times 0.379 = 9.73$ sq. in.

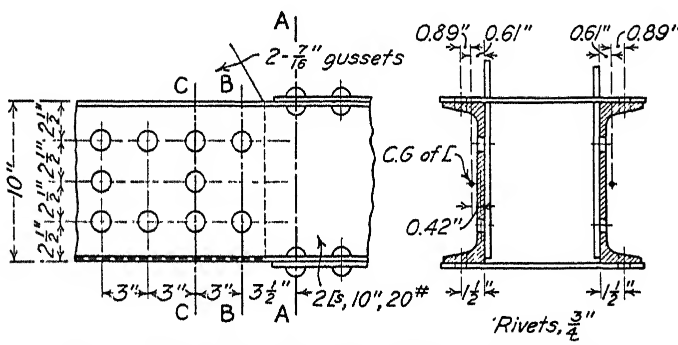

FIG. 11.—Double-Channel Tension Member with Moment.

The moment of inertia of the net section of each channel by the approximate method used in Art. 16 is found to be at section *AA*, $2.80 - 2 \times 0.875 \times 0.4375 \times (0.89)^2 = 2.19$ in.^{4}; at section *BB*, it is $2.80 - 2 \times 0.875 \times 0.379 \times (0.42)^2 = 2.68$ in.^{4}; and at section *CC* it is $2.80 - 3 \times 0.875 \times 0.379 \times (0.42)^2 = 2.62$ in.^{4}

Maximum stress at back of one channel at section *AA*, assuming that the force of 43,000 lb. is applied in the plane of the back of the channel, that is, with an arm of 0.61 in., is, from Eq. (3) of Art. 1

$$f_t + f_f = \frac{43{,}000}{5.09} + \frac{(43{,}000 \times 0.61) \times 0.61}{2.19} = 8450 + 7300$$
$$= 15{,}750 \text{ lb. per sq. in.}$$

Maximum stress at back of one channel at section *BB* is less than at section *AA*, as the net section and the net moment of inertia are each more than at section *AA* and the load is the same.

Since $43{,}000/4420 = 10$ rivets would be required for connecting each channel to its $\frac{7}{16}$-in. gusset plate, the stress in a channel at section *CC* is only $\frac{8}{10}$ of 43,000

=34,400 lb. Manifestly the combined stress at this section would for this load be less than at section AA.

The assumed section is adequate.

18. Riveted Member Subjected to Tension and Transverse Loading.—Design a double-angle, horizontal tension member, 8 ft. long, to be connected by both legs at each end (Fig. 12), and to withstand a centric axial pull of 30,000 lb. and a concentrated transverse load of 800 lb. applied 3 ft. from one end, in addition to its own weight. Consider the effect of deflection. Rivets, $\frac{3}{4}$ in. p_t = 18,000 lb. per sq. in. E = 29,000,000 lb. per sq. in.

The maximum combined stress must be calculated both at the concentrated load and at the end nearer this load. It will be assumed that the fixity at the ends is such that the moment at either of the two critical cross sections, that is at the point of concentrated transverse loading and at the end nearer this loading, is $\frac{2}{3}$ of the moment that would occur in a simply supported beam at its point of maximum moment, due to the same loading. The maximum stress due to tension and bending may be found from Eq. (5) of Art. 1, assuming such a degree of fixity at the ends that for moment at the concentrated load, C = 15, and for moment at the critical end, C = 20. See Art. 1 for values of C under different conditions.

Section at Concentrated Load.—Assume for the member, two $3\frac{1}{2} \times 3 \times \frac{5}{16}$-in. angles with the $3\frac{1}{2}$-in. legs vertical, as shown in Fig. 12.

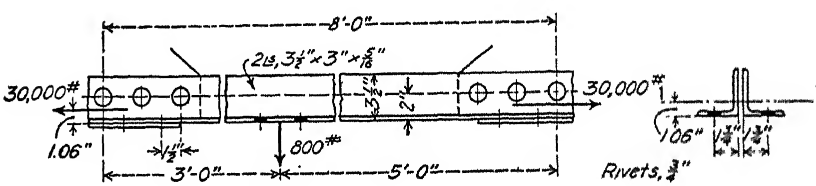

FIG. 12.—Member Subjected to Tension and Flexure.

Weight of member = 13.2 lb. per lineal ft.

Moment at point of concentrated loading, due to weight of member, allowing for restraint = $0.67\{(13.2 \times 4) \times 3 \times 12 - \frac{1}{2} \times 13.2 \times (3)^2 \times 12\}$ = 800 in.-lb.

Maximum moment due to concentrated loading = $0.67\{(800 \times \frac{5}{8}) \times 3 \times 12\}$ = 12,000 in.-lb.

Total moment at concentrated load = 800 + 12,000 = 12,800 in.-lb.

Net section at point of concentrated loading, deducting two $\frac{7}{8}$-in. holes $= 3.86 - 2 \times 0.875 \times 0.3125 = 3.31$ sq. in.

Moment of inertia of gross section of two angles about gravity axis 1 — 1 =4.6. in.^{4}

Approximate loss of I due to holes = $A_h y^2_0$, where A_h = area of holes and y_0 = distance of their centre of gravity from the neutral axis of the unpunched member. This is $(2 \times 0.875 \times 0.3125) \times (0.90)^2 = 0.44$ in.^{4}

Net I of two angles = 4.60 − 0.44 = 4.16 in.^{4}

Maximum stress on *bottom* fibres is, from Eq. (5), Art. 1

$$f_t + f_f = \frac{30{,}000}{3.31} + \frac{12{,}800 \times 1.06}{4.16 + \dfrac{30{,}000 \times (96)^2}{15 \times 29{,}000{,}000}}$$

$$= 9060 + 2820 = 11{,}880 \text{ lb. per sq. in.}$$

The assumed section is excessive at the point of concentrated loading.

Section at End Nearer Concentrated Load.—Net section at end connection, deducting 1.8 holes from each angle in accordance with the method of Art. 5, $= 3.86 - 3.60 \times 0.875 \times 0.3125 = 2.87$ sq. in.

Total moment at end same as at point of concentrated loading = 12,800 in.-lb.

Moment of inertia at end will be the gross moment of inertia, less the moment of inertia of the holes in the vertical legs. No deduction need be made for the holes in the outstanding legs, as under pure negative bending, apart from axial tension, material here would be under compression.

Reduction of $I = A_h y_o^2 = (2 \times 0.875 \times 0.3125) \times (0.94)^2 = 0.48$ in.^{4}
Net $I = 4.60 - 0.48 = 4.12$ in.^{4}

Maximum stress in *top* fibres is

$$f_t + f_f = \frac{30{,}000}{2.87} + \frac{12{,}800 \times 2.44}{4.12 + \dfrac{30{,}000 \times (96)^2}{20 \times 29{,}000{,}000}}$$

$$= 10{,}440 + 6960 = 17{,}400 \text{ lb. per sq. in.}$$

The section is, therefore, adequate.

19. Eye-Bar Subjected to Tension and Bending Due to Its Own Weight.—A single horizontal eye-bar, 20 ft. long between centres of end pins, is to resist a centric axial pull of 17,500 lb. Suggest a size. p_t, for combination of tensile stress and flexural stress due to weight = 18,000 lb. per sq. in.

Neglecting flexural stress, the area required is 17,500/18,000 = 0.97 sq. in. Assume a $2 \times \frac{1}{2}$-in. bar.

As the weight of assumed bar, neglecting heads, is 3.4 lb. per lin. ft.,
$M = \frac{1}{8} \times 3.4 \times (20)^2 \times 12 = 2040$ in.-lb.

Applying Eq. (6) of Art. 1, and letting $C = 10$, and $E = 29{,}000{,}000$ lb. per sq. in.

$$A_t + A_f = \frac{17{,}500}{18{,}000} + \frac{2040 \times 1}{\left(\frac{1}{3} + \dfrac{17{,}500 \times 240^2}{10 \times 1 \times 29{,}000{,}000}\right)18{,}000}$$

$$= 0.97 + 0.03 = 1.0 \text{ sq. in.}$$

As the section assumed provided exactly this area, the trial section is adequate.

20. Exercise Problems on the Design of Steel Tension Members.—Unless otherwise mentioned net sections of riveted tension members are computed by the method of Art. 5 and Fig. 1. See Appendix I for the answers.

(1) Determine the net section of the $14 \times \frac{3}{8}$-in. plate shown in Fig. 13, using the modified Cochrane deduction rule stated in Art. 6, that is, $x = 1 - s^2/3.5\,gh$. Where is this section located? Rivets, $\frac{3}{4}$ in.

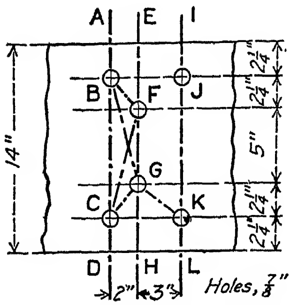

FIG. 13.—Net Section of Plate.

(2) A $6 \times \frac{1}{2}$-in. plate is put in a testing machine and pulled to destruction in tension. At about mid-length of the plate there are two holes for $\frac{7}{8}$-in. rivets punched equidistant from the edges on a 3-in. gauge and staggered $1\frac{1}{2}$ in. At what load should the plate fail if the ultimate strength of the metal is 62,000 lb. per sq. in.?

(3) An equal-leg angle connected by both legs is to transmit a pull of 35,000 lb. to a $\frac{3}{8}$-in. gusset plate. Assuming that the rivets are spaced 3 in. apart in each leg and that they are staggered perfectly, recommend a size for the angle. $p_t = 16{,}000$ lb. per sq. in. Rivets, $\frac{3}{4}$ in. Fractional deduction rule, $x = 1.3 - s/g$, but not to exceed unity.

(4) A $3\frac{1}{2} \times 3\frac{1}{2}$-in. angle connected by both legs to a gusset plate is subjected to a centric axial pull of 38,000 lb. If the rivets are spaced 4 in. in each leg and are staggered 2 in., find the required thickness of the angle. $p_t = 16{,}000$ lb. per sq. in. $\frac{7}{8}$-in. rivets.

(5) Where is the critical section in, and what is the safe capacity of, the tension member shown in Fig. 14, assuming the stress to be uniformly distributed over the cross section? Rivets, $\frac{7}{8}$ in. $p_t = 16{,}000$ lb. per sq. in.

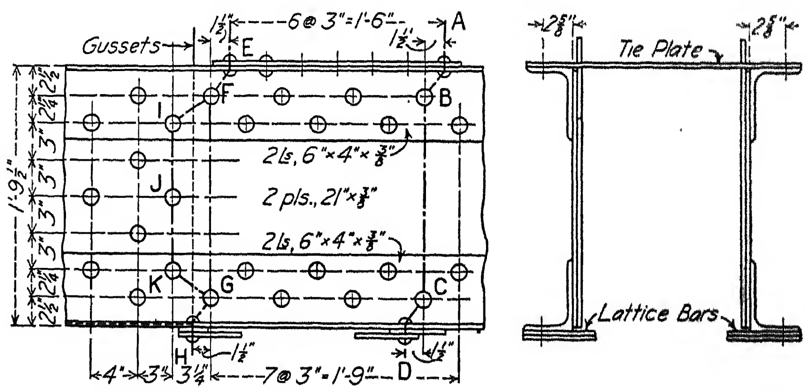

FIG. 14.—Capacity of Riveted Tension Member.

(6) A $3 \times 2 \times \frac{1}{4}$-in. angle carrying a tension of 15,000 lb. is connected by sufficient $\frac{3}{4}$-in. rivets on a $1\frac{3}{4}$-in. gauge in the 3-in. leg to transmit the stress safely to a gusset plate. Report on the safety of the angle itself. $e = 1.0 - 0.18\,u/c$. $p_t = 18{,}000$ lb. per sq. in.

(7) A $3\frac{1}{2} \times 2\frac{1}{2} \times \frac{1}{4}$-in. angle connected to a gusset plate by one line of $\frac{3}{4}$-in.

rivets in the $3\frac{1}{2}$-in. leg. What tensile force can the angle safely resist? $e = 1.0 - 0.18\ u/c$. $p_t = 16{,}000$ lb. per sq. in.

(8) An unequal-leg angle connected by the longer leg only to a gusset plate is to carry a load of 20,000 lb. Recommend a size for the angle. Rivets, $\frac{3}{4}$ in. $e = 1.0 - 0.18\ u/c$. $p_t = 16{,}000$ lb. per sq. in.

(9) A 6×4-in. angle connected by the 6-in. leg only is subjected to a pull of 42,000 lb. If the rivets ($\frac{7}{8}$ in.) are driven on two gauge lines at a staggered pitch of $1\frac{1}{2}$ in. and the gauges are 2 and $2\frac{1}{2}$ in., suggest a thickness for the angle. $e = 1.0 - 0.18\ u/c$. $p_t = 16{,}000$ lb. per sq. in.

(10) In the case of the member shown in Fig. 6, Art. 12, what is the maximum combined fibre stress due to the centric load effect and the moment in terms of the applied load P?

(11) A 1-in. square rod bent into a hook at its bottom end suspends a 500-lb. load from a ceiling. The load is applied to the hook at $1\frac{1}{2}$ in. from the inside face of the rod. What is the maximum fibre stress in the rod?

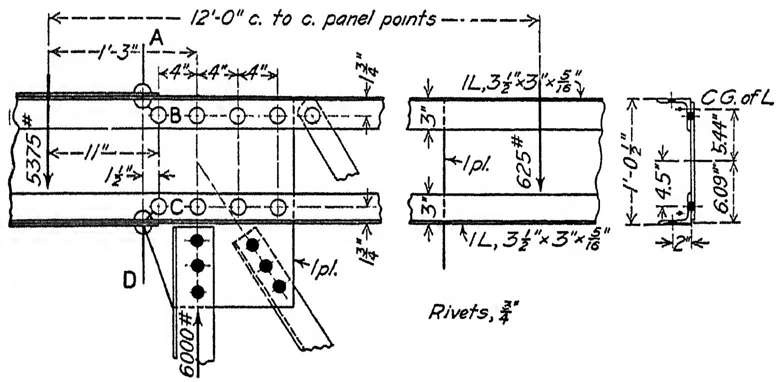

FIG. 15.—Combined Tension and Flexure.

(12) Report on the sufficiency of a 10-in., 15.3-lb. channel for transmitting a tension of 40,000 lb. to a gusset plate riveted to the back of the channel by $\frac{3}{4}$-in. rivets driven on two gauge lines, and spaced in pairs $2\frac{1}{2}$ in. apart. $p_t = 18{,}000$ lb. per sq. in.

(13) A 15,000-lb. load is attached to the lower end of a $6 \times \frac{1}{2}$-in. hanger plate by a $1\frac{1}{2}$-in. pin 2 in. from one vertical edge of the plate and 4 in. from the other. If the pin hole is 1.52 in. in diameter, find the maximum tensile stress on the plate by the ordinary flexure theory.

(14) A tension diagonal in a truss consists of two 2×10-in. eye-bars. The panel length of the truss is 25 ft. and the depth is 30 ft., centre to centre of chord pins. Determine the maximum tensile stress in the member due to an axial pull of 600,000 lb. combined with the bending due to the weight of the member. $E = 29{,}000{,}000$ lb. per sq. in. Consider ends free to turn.

(15) In one 12-ft. panel of the tension chord of a truss the axial tension is 32,000 lb. At 1 ft. 3 in. from one panel point a transverse thrust of 6000 lb. is applied, as shown in Fig. 15. If the chord consists of two $3\frac{1}{2} \times 3 \times \frac{5}{16}$-in. angles, and the details are as shown in the illustration, report on the safety of the chord. Rivets, $\frac{3}{4}$ in. $p_t = 20{,}000$ lb. per sq. in. Neglect bending in a vertical plane.

CHAPTER II

TENSION MEMBER DETAILS

21. Fundamental Formulae for Tension Member Details.—The following formulae apply generally to tension member details:

$$N = \frac{P}{v} \tag{1}$$

$$v = mAp_s \text{ or } dtp_b, \text{ whichever is the smaller} \tag{2}$$

$$T_n = \frac{Mr_n}{\Sigma r^2} \tag{3}$$

REFERENCE: Shedd—Structural Design in Steel.

The significance of the symbols employed in the above formulae is as follows:

A = area of one cross section of a rivet;
d = diameter of rivet before driving;
M = turning moment on a connection, that is, the total applied force P multiplied by the normal distance e of its line of action from the centroid of the group of resisting rivets;
m = number of cross sections of a rivet that must be sheared to bring about its failure;
N = number of rivets required to transmit force P;
P = total force to be transmitted;
p_b = permissible stress in bearing per unit of area;
p_s = permissible stress in shear per unit of area;
r = radial distance of the centre of a rivet from the centroid of the group;
r_n = radial distance of the centre of rivet number "n" from the centroid of the group;
T_n = force applied to any rivet, number "n," due to the turning effect of the moment M;
t = least thickness of material that must be crushed in order to bring about failure of a rivet in bearing;
v = least value, or safe resistance, of a rivet.

22. Minimizing Secondary Stress in a Member.—Determine the maximum combined tensile and flexural stress in the double-angle member of Art. 12 and Fig. 6, assuming that the gauge of the angles is 1.75 in. only. What extra carrying capacity will the member derive by placing the gauge line $\frac{3}{4}$ in. nearer the centroid of the member?

Proceeding as in Art. 12, the quantities necessary to the solution are found to be as follows: The net area of the member is 4.21 sq. in.; the gravity axis of the net area is 1.197 in. from the backs of the outstanding legs of the angles; the eccentricity of the connection is 0.553 in.; the net moment of inertia about the gravity axis of the net area is 7.74 in.[4]; and the distance of the extreme fibre from the gravity axis of the net section is 2.803 in.

Applying Eq. (3) of Art. 1

$$f_t + f_f = \frac{21{,}000}{4.21} + \frac{21{,}000 \times 0.553 \times 2.803}{7.74}$$
$$= 4990 + 4210 = 9200 \text{ lb. per sq. in.}$$

Since the maximum fibre stress when the gauge is 2.50 in. was found in Art. 12 to be 18,390 lb. per sq. in., the carrying capacity is increased 18,390/9200 =2.0 times by the change of gauge.

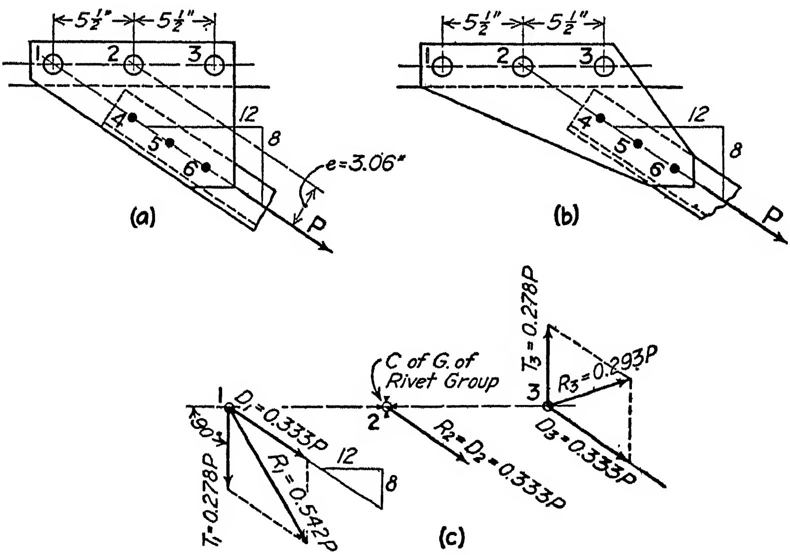

FIG. 16.—Comparison of Gusset-Plate Attachments.

In general, the nearer the centroid of the group of rivets attaching a member to a gusset is to the gravity axis of the member, the less will be the secondary stress, as far as such is due to eccentricity of connection of the member under consideration.

23. Efficiency of Gusset-Plate Attachment.—Find the safe capacity of an end connection for a tension member, if the arrangement of rivets be, first, as in Fig. 16(*a*), and second, as in Fig. 16(*b*). Rivets, $\frac{7}{8}$ in. p_s = 12,000 lb. per sq. in. for shop rivets and 10,000 lb. per sq. in. for field rivets p_b = 24,000 lb. per sq. in. for shop rivets and 20,000 lb. per sq. in. for field rivets.

Case A.—The group of rivets, 1, 2, 3 must withstand a direct force P and a turning moment $M = Pe = 3.06\ P$ in.-lb.

Direct force on any rivet, number "n" is $D_n = P/3$ lb. This acts parallel to the force P.

Turning force on any rivet, number "n," is, from Eq. (3), Art. 21

$$T_n = \frac{Mr_n}{\Sigma r^2}$$

Now, as rivet 2 is the centroid of the group 1, 2, 3, hence $\Sigma r^2 = 2 \times (5.5)^2 = 60.5$ in.² Hence turning force on rivets 1 and 3 is $T_1 = T_3 = 3.06\,P \times 5.5/60.5 = 0.278\ P$ lb., and on rivet 2 it is zero. These turning forces act normally to the radii from the centroid of the rivet group to rivets 1 and 3, that is to the line $1 - 3$.

Combining the direct and turning forces by graphical means, as in Fig. 16(c), it is seen that the resultant force on rivet 1, that is 0.542 P, is the maximum resultant.

Now, as the maximum safe load on shop rivet 1 is its single shearing value, that is $0.601 \times 12{,}000 = 7210$ lb., 0.542 P must not exceed 7210 lb. Solving for P, it is found to be 13,300 lb.

The safe capacity of the group of field rivets 4, 5, 6 is three times the single shearing value of a $\frac{7}{8}$-in. field rivet, or $3 \times 0.601 \times 10{,}000 = 18{,}030$ lb.

The capacity is, therefore, only 13,300 lb.

Case B.—As there is no moment on the rivet group 1, 2, 3, its safe capacity is three times the single shearing value of a $\frac{7}{8}$-in. shop rivet, or $3 \times 7210 = 21{,}630$ lb.

The safe capacity of the group 4, 5, 6 is, as already found, 18,030 lb.

A comparison of the capacities of connections a and b shows that a is only 73.7% as strong as b. The value of eliminating moment in connections is thus apparent.

24. Connection of Single-Angle Tension Member by One Leg Containing Two Lines of Rivets.—Design the connection for the angle of Art. 11, Fig. 5.

Safe resistance of a $\frac{3}{4}$-in. rivet in the situation is its single shearing value $= 0.442 \times 12{,}000 = 5300$ lb.

Number of rivets required, according to Eq. (1), Art. 21, is $29{,}000/5300 = 6$. These may be accommodated in two gauge lines as shown in Fig. 5, with a staggered pitch of $1\frac{1}{2}$ in.

The rivet-hole deduction might be reduced to one hole, as may be found from Fig. 1, by making the stagger for the first pair of rivets $2\frac{1}{8}$-in.

25. Connection of Double-Angle Tension Member by One Leg.—Design the connection for the member of Art. 12, Fig. 6. Rivets, $\frac{7}{8}$ in. $p_s = 10{,}000$ lb. per sq. in. $p_b = 20{,}000$ lb. per sq. in.

Safe resistance of a $\frac{7}{8}$-in. rivet in this situation is its bearing value on the $\frac{3}{8}$-in. gusset plate $= 0.875 \times 0.375 \times 20{,}000 = 6560$ lb.

Number of rivets required, Eq. (1), Art. 21 = 21,000/6560 = 3.2, that is 4.

These rivets are driven in one gauge line and may be spaced a minimum distance of three diameters, centre to centre.

26. Connection for a Single-Channel Tension Member Connected by Web Only.—Design the end connection for the single-channel tension member of Art. 16, Fig. 10.

Safe resistance of one $\frac{3}{4}$-in. rivet in this situation is its bearing on the 0.22-in. web of the channel = $0.75 \times 0.22 \times 24{,}000 = 3960$ lb.

Number of rivets required for tension of 27,000 lb., Eq. (1), Art. 21 = 27,000/3960 = 7.

The rivets are best arranged as in Fig. 10, that is with a single rivet at an apex of the group at the inner end of the connection. With a stagger of $2\frac{1}{2}$ in. and a gauge of 2 in., a zigzag section becomes stronger than a right one, as is seen by consulting Fig. 1(*b*).

27. Connection for Double-Channel Tension Member of Box Type.—Design the end connections for the double-channel tension member of Art. 17, Fig. 11.

Safe resistance of one $\frac{3}{4}$-in. rivet in this case is its single shearing value $= 0.442 \times 10{,}000 = 4420$ lb.

Number of rivets required for a tension of 86,000 lb., Eq. (1), Art. 21 = 86,000/4420 = 20 rivets, that is 10 in. each channel.

To keep the net area of the channels as great as possible place only two rivets in the first transverse now at the inner end of the connection. The recommended arrangement of the rivets is as shown in Fig. 11.

28. Effect of Improving Rivet Value in a Connection.—Find the effect on the end connection of the built-up I-shaped member of Art. 15, Fig. 9, brought about by adding plates overlapping the end gussets and the member itself on the outside, thus raising the value of the included rivets from single shear to bearing on a $\frac{1}{2}$-in. plate. Rivets, $\frac{7}{8}$ in. $p_s = 10{,}000$ lb. per sq. in. $p_b = 20{,}000$ lb.

Safe resistance of a rivet in single shear $= 0.6010 \times 10{,}000 = 6010$ lb.

Safe resistance in bearing on $\frac{1}{2}$-in. gusset plate $= 0.875 \times 0.50 \times 20{,}000 = 8750$ lb.

Number of rivets required in order to develop safe capacity of member, if connection is as in Fig. 9, that is, if the single shearing value of the rivets is their least value, is 279,500/6010 = 47. The smallest practicable number is the multiple of 4 nearest 47 on the high side, or 48.

All, or only a part, of the rivets through the gusset may be raised in value, according to the length of the outside plates employed, as indicated in Fig. 17(*a*) and Fig. 17(*b*). In either case the minimum practicable width of the added plates is 12 in.

All Rivets Improved in Value.—For the case of Fig. 17(*a*), the total number of rivets required, Eq. (1), Art. 21, is 279,500/8750 = 32. These may conveniently be arranged as shown.

The total amount of stress transmitted by one of these plates to the gusset

will equal the number of rivets passing through it and the gusset multiplied by the difference between the bearing value of a $\frac{7}{8}$-in. rivet or a $\frac{1}{2}$-in. gusset and the single shearing value, or $16/(8750 - 6010) = 43{,}850$ lb.

At the critical section *ABCDEF*, the deduction, according to Fig. 1(*c*), is $(1.0 + 0.4 + 1.0 + 0.4) = 2.8$ holes. Assuming a $\frac{3}{8}$-in. plate, the net area is, therefore $(12 - 2.8 \times 1)0.375 = 3.45$ sq. in., and the safe capacity $= 3.45 \times 16{,}000 = 55{,}200$ lb. This is considerably above the requirement, but a thinner plate is not permissible in such heavy construction.

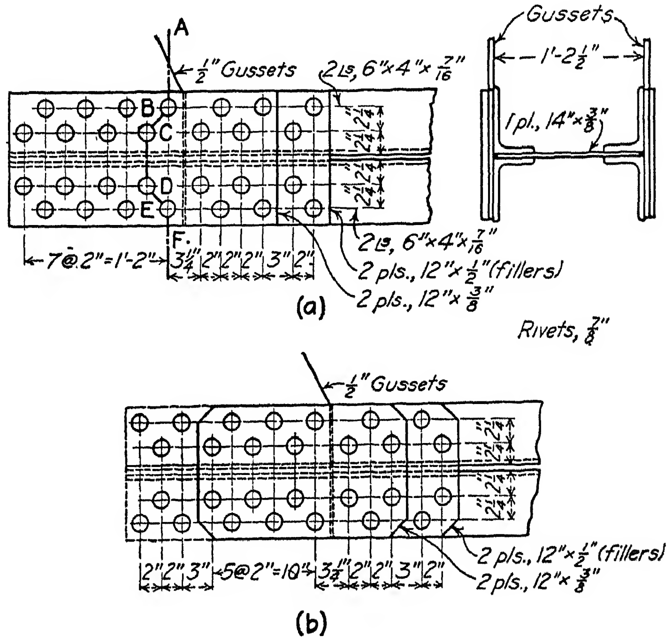

FIG. 17.—Improvement of Rivet Value in a Connection.

To the right of the gusset, through one of these plates, it is necessary to have sufficient rivets acting in single shear to transmit 43,850 lb. This number is $43{,}850/6010 = 8$, as shown in Fig. 17(*a*).

Each filler should be attached to the member by sufficient rivets beyond the end of the outer plate to ensure that the filler takes by virtue of the relation of its thickness to the thickness of the flange material of the member its proper share of the stress going to the outer plate. For one filler, in this case, the number should be $8 \times 0.5/0.9375 = 4.3$. Four will be used.

As compared with the detail of Fig. 9, eight more rivets are needed, as well as the additional material in the outer plates and fillers. To offset this, the gusset plates may be made smaller, and if they be thick this advantage is often deemed sufficient to employ a detail such as shown in Fig. 17(*a*).

Part of Rivets Improved in Value.—If the joint is arranged as shown in

Fig. 17(*b*), the capacity of the riveting through the gusset plate is $(24 \times 8750) + (12 \times 6010) = 282{,}000$ lb., or slightly more than the capacity of the member.

One outside plate must resist $12(8750 - 6010) = 33{,}900$ lb. As previously determined, the capacity of the minimum plate allowable—a $12 \times \frac{3}{8}$-in. plate—is 55,200 lb.

To the right of the gusset plate $33{,}900/6010 = 6$ rivets are needed.

Each filler should have outside the end of the outer plate $6 \times 0.5/0.9375 = 3.2$, say 4 rivets.

The same number of rivets is required in this detail as in that of Fig. 17(*a*), but the added plate material is less. It does not reduce the size of gussets so much as the design of Fig. 17(*a*).

29. Connection for Single-Angle Tension Member with Lug.—Design an end connection for both legs of a $4 \times 3 \times \frac{3}{8}$-in. tension angle with the 4-in. leg riveted directly to a $\frac{3}{8}$-in. gusset and the 3-in. leg connected thereto by a lug angle having the outer end flush with the outer end of the member. The detail is to be of maximum efficiency and is to be for the safe capacity of the member. Rivets, $\frac{7}{8}$ in. $p_t = 16{,}000$ lb. per sq. in. $p_s = 12{,}000$ lb. per sq. in. $p_b = 24{,}000$ lb. per sq. in. Consider the full net section as effective.

Number of Rivets.—Assuming that the detail is such as to require the deduction of only one rivet hole, the net area of a $4 \times 3 \times \frac{3}{8}$-in. angle $= 2.48 - 1 \times 0.375 = 2.10$ sq. in. At 16,000 lb. per sq. in., the capacity $= 2.10 \times 16{,}000 = 33{,}600$ lb.

Safe resistance of one rivet is its single shearing value $= 0.601 \times 12{,}000 = 7220$ lb.

Number of rivets required, Eq. (1), Art. $21 = 33{,}600/7220 = 4.7$. Use 5 rivets.

Distribution of Rivets.—To ensure equalized stress over the section of the angle, the five rivets should be divided between the two legs of the main angle in proportion to their gross areas. The 4-in. leg should, therefore, contain $5 \times (4 \times 0.375)/\{(4 + 2.625) \times 0.375\} = 3.0$ rivets. Three rivets will therefore be used to connect the 4-in. leg directly to the gusset and two rivets to connect the 3-in. outstanding leg to the lug angle.

In order to keep the gusset plate as small as practicable and the lug angle short, 3-in. spacing, the normal minimum for $\frac{7}{8}$-in. rivets, will be used. Two rivets, spaced 3 in. are first located in the 4-in. leg of the main angle, with the outer one $1\frac{1}{2}$ in. from the end—the normal minimum distance of a $\frac{7}{8}$-in. rivet from a sheared edge. Two rivets are then placed directly opposite these through the lug and the gusset. The necessary two rivets in the outstanding leg of the lug angle to develop the stress borne by the rivets in the other leg are spaced 3 in. apart and staggered $1\frac{1}{2}$ in. with those in the other leg, to facilitate driving.

For a connection of maximum efficiency, the remaining, or first, rivet in the 4-in. leg must be located at such a distance forward of the first rivet in the outstanding leg that only one hole need be deducted. The distance apart of the gauge lines, if the angle were developed, assuming a gauge of $2\frac{1}{2}$ in. in the 4-in. leg and $1\frac{3}{4}$ in. in the 3-in. leg, would be $2.50 + 1.75 - 0.375 = 3.875$ in. The

necessary stagger of these rivets, in order that only one hole need be deducted, is found from Fig. 1(c) to be $3\frac{3}{8}$ in. The final spacing is therefore as shown in Fig. 18, there being $4\frac{7}{8}$ in. between the first two rivets in the directly connected leg of the main angle.

Proceeding as in Art. 8, it may be shown that the member is stronger along section *DEFG* than on section *ABC*.

Lug Angle.—Since the lug angle must transmit to the gusset the proportion of the total stress that should be borne by the outstanding leg of the main angle, the outstanding leg of the lug angle should be of the same section as the outstanding leg of the main angle, and is preferably of the same width and thickness. The leg in contact with the gusset should be made as narrow as the driving of the rivets in it at the required stagger will permit. In the present case, this is 3 in. The length of the lug is determined by the rivet spacing adopted and by the minimum end distances permissible.

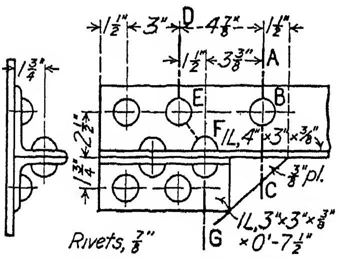

FIG. 18.—Connection of Single Angle for Maximum Efficiency.

Frequently, the maximum efficiency obtained by making the distance between the first and second rivets greater than that between the other rivets is sacrificed in order to reduce the size of the gusset plate or shorten the main angle.

30. Fillet Welded Connection for Flat Bar Tension Member.—A 6 × $\frac{3}{8}$-in. steel bar carrying a total tension of 45,000 lb. is to be welded to a $\frac{3}{8}$-in. gusset plate by parallel (longitudinal) shear $\frac{5}{16}$-in. fillet welds and a similar normal (transverse) shear fillet weld across the end of the bar. Design the connection.

Permissible shear in pounds per lineal inch of single fillet welds will be assumed as follows: Parallel (longitudinal) shear, $p_s = 1/6\ b\ (3300 - 50\ l)$, but not over $500\ b$; normal (transverse) shear, $p_s = 500\ b$. In these formulae, b = side of the largest isosceles triangle contained in the cross section of the fillet expressed in sixteenths of an inch and l = actual length in inches of the longitudinal fillet under consideration.

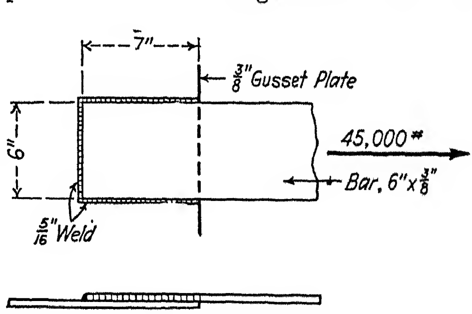

FIG. 19.—Fillet Welded Connection for Flat Bar.

When calculating the effective length, deduct $\frac{1}{2}$ in. from the actual length of each weld to allow for craters.

Assume, as indicated in Fig. 19, that the bar laps 7 in. on the gusset plate. This will give a total of 14 in. of parallel fillet and 6 in. of normal fillet, or effective lengths of 13 in. and 5.5 in. respectively.

For each parallel fillet the permissible shear per lineal inch is $p_s = \frac{5}{6}(3300 - 50 \times 7) = 2458$ lb., while for the normal fillet the permissible shear is $p_s = 500 \times 5 = 2500$ lb.

The provided capacity of the connection is, then, having regard to the effective lengths of the welds:

Parallel welds, 13 in. at 2458 lb. per lin. in.	=31,950 lb.
Normal weld, 5.5 in. at 2500 lb. per lin. in.	=13,750 lb.
Total provided capacity	=45,700 lb.

This slightly exceeds the total stress in the bar.

31. Fillet and Slot Welded Connection for Tension Members of I-Section. —A 12-in., 65-lb. W.F. carrying a total tension of 335,000 lb. is to be connected to and between two $\frac{5}{8}$-in. gusset plates which are to be welded to the I-section flanges by a combination of fillet and slot welds. Parallel (longitudinal) shear $\frac{3}{8}$-in. triangular single fillet welds are to be used at the edges of the flanges of the I-section, and, in addition, two parallel shear $\frac{3}{4} \times \frac{3}{8}$-in. slot welds will be made to each flange of the section, beginning at the inner edge of the gusset

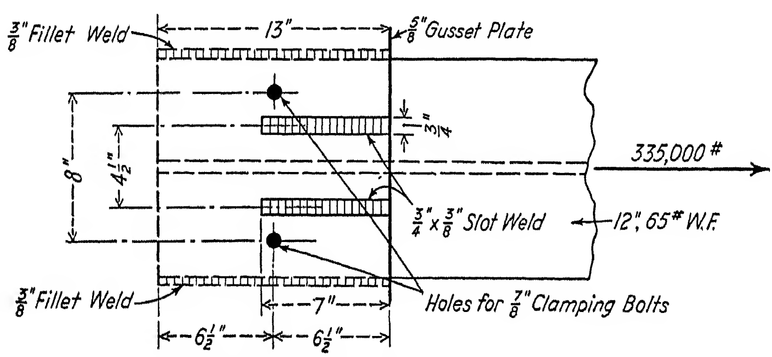

FIG. 20.—Fillet and Slot Welded Connection for Tension Member of I-Section.

plates. On account of assumed inaccessibility, there will be no weld at the end of the section. Design the connection.

Permissible shear in pounds per lineal inch for parallel shear fillet welds $p_s = 1/6\ b\ (3300 - 50\ l)$, with a maximum of $500\ b$. In this, b and l have the same significance as in Art. 30.

Permissible shear in pounds per lineal inch for slot welds, $p_s = \frac{t}{16}(12{,}440 - 190\ l)$, with a maximum of $706\ t$. In this, t is the throat width of the weld expressed in sixteenths of an inch and l is its actual length in inches. The depth b of the weld must not be less than one-half of t.

Allow 10% excess capacity to compensate for lessened effectiveness of welds due to craters.

As indicated in Fig. 20, assume that the gussets extend out 13 in. from the end of the member. This will make possible $4 \times 13 = 52$ in. of parallel shear fillet welding.

According to the data, the connection should be designed for a capacity of $1.10 \times 335{,}000 = 368{,}500$ lb.

Permissible shear per lineal inch of the parallel shear fillet welds is $p_s = \frac{8}{8}$ $(3300 - 50 \times 13) = 2650$ lb., and the safe resistance of these welds is $52 \times 2650 = 137{,}800$ lb.

There must then be developed by the parallel shear slot welds a total resistance of $368{,}500 - 137{,}800 = 230{,}700$ lb.

Permissible shear per lineal inch of $\frac{3}{4} \times \frac{3}{8}$-in. slot weld, assuming each of the four welds to be 7 in. long, is $p_s = \frac{12}{16}(12{,}440 - 190 \times 7) = 8330$ lb., and the safe resistance of the four welds is $28 \times 8330 = 233{,}240$ lb. These welds will start at the edge of the gusset, as shown in Fig. 20.

The holes for the $\frac{7}{8}$-in. bolts used for holding the member in place between the gussets during the welding operation will be filled by rivet (plug) welds, but no credit for the value of these will be given.

32. Fillet Welded Connection of Single Angle.—A single $3\frac{1}{2} \times 2\frac{1}{2} \times \frac{1}{4}$-in. angle is to be welded to a gusset by the $3\frac{1}{2}$-in. leg, with $\frac{1}{4}$-in. parallel shear fillet

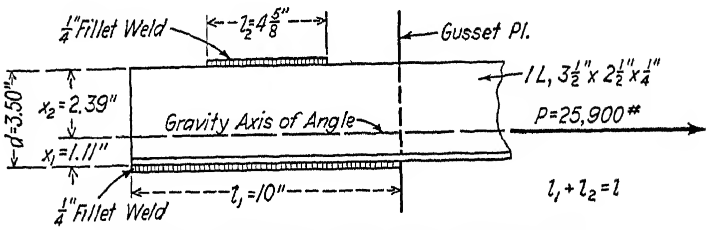

FIG. 21. Fillet Welded Connection of Single Angle.

welds at the two edges of the leg. Design a connection to develop the capacity of the angle.

Permissible tensile stress, $p_t = 18{,}000$ lb. per sq. in.; permissible shear in pounds per lineal inch for parallel shear fillet welds, $p_s = \frac{b}{8}(3300 - 50\,l)$, with a maximum of $500\,b$.

Add $\frac{1}{2}$ in. to the required length of each fillet to compensate for the effect of craters.

As the area of the angle is 1.44 sq. in., the connection must be designed for $P = 1.44 \times 18{,}000 = 25{,}900$ lb.

In order to obviate secondary stress in the angle, the lengths and resistances of the two fillet welds should be such that the resultant resistance coincides with the gravity axis of the angle. Referring to Fig. 21, it is thus evident, by taking moments about one edge or the other, that

$$l_1 = \frac{x_2}{d} \cdot l$$

$$l_2 = \frac{x_1}{d} \cdot l$$

As the permissible longitudinal shear on the fillets will depend upon their

length, it will be necessary to estimate approximately the respective lengths of the fillets. If there were no reduction for lengths, p_s would be 500 b or 2000 lb. per lin. in., and a total length of 25,900/2000 = 12.95 in. would be required. The two fillets would then have as their effective lengths

$$l_1 = \frac{2.39}{3.50} \times 12.95 = 8.84 \text{ in.}$$

and

$$l_2 = \frac{1.11}{3.50} \times 12.95 = 4.10 \text{ in.}$$

Since the formula for permissible shear obviously requires no reduction for lengths of 6 in. and less, the effective fillet at the toe of the angle will be 4.10 in. long.

The fillet at the heel at a certain reduced stress per lineal inch must have as much safe resistance as 8.84 in. at 2000 lb. per lin. in., or 17,680 lb. Assume this fillet as having an actual (not net) length of 10 in. Then $p_s = \frac{4}{6}(3300 - 50 \times 10) = 1867$ lb. per lin. in. From this $l_1 = 17{,}680/1867 = 9.5$ in.

Adding $\frac{1}{2}$ in. to the length of each fillet will give $l_1 = 10$ in. and $l_2 = 4\frac{5}{8}$ in.

Frequently the parallel shear welds are not directly opposite each other, but are staggered, that is, one is set forward or back with respect to the other. Although tests show some falling off in strength due to staggering, the permissible stress specified above is sufficiently conservative to allow for this feature.

33. Transfer of Dead-Load Stress to Welded Reinforcement of a Tension Member.—A tension member of 16 sq. in. is subjected to a dead load stress of 14,000 lb. per sq. in. It is desired, by welding a heated plate of suitable section and of length $l = 180$ in. thereto, to reduce the dead load stress to $f_t = 9000$ lb. per sq. in. What area of plate should be used, and to what increase of temperature should the plate be subjected after the first end is welded to the original member and before the other end is welded and the intermediate welding is done, in order that the stress in the reinforced member may be 9000 lb. per sq. in.?

Coefficient of expansion of steel per degree F. is $\omega = 0.0000065$; modulus of elasticity of steel is $E = 29{,}000{,}000$ lb. per sq. in.

Dead load tension carried by member $= P = 16 \times 14{,}000 = 224{,}000$ lb.

Required area of reinforced member to reduce stress to 9000 lb. per sq. in. $= 224{,}000/9000 = 24.88$ sq. in.

Area of reinforcing plate $= 24.88 - 16.0 = 8.88$ sq. in.

Under the desired tensile stress of 9000 lb. per sq. in., the reinforcing plate would elongate

$$\Delta l = \frac{f_t l}{E} \tag{1}$$

In order to produce this length change before the final attachment of the plate, a temperature rise in the plate must be produced amounting to

$$t = \frac{\Delta l}{l\omega} \tag{2}$$

Combining (1) and (2),

$$t = \frac{f_t}{E\omega} = \frac{9000}{29{,}000{,}000 \times 0.0000065} = 47.8 \text{ deg. F.}$$

This method has been successfully employed in the strengthening of old structures in service. If the reinforcing material is welded without preheating, the dead load stress carried by the original member would not be lessened. If a reduction in it is desired without heating of the reinforcement, the member should be shored up during the welding process. When load then comes on, it will be distributed in proportion to area.

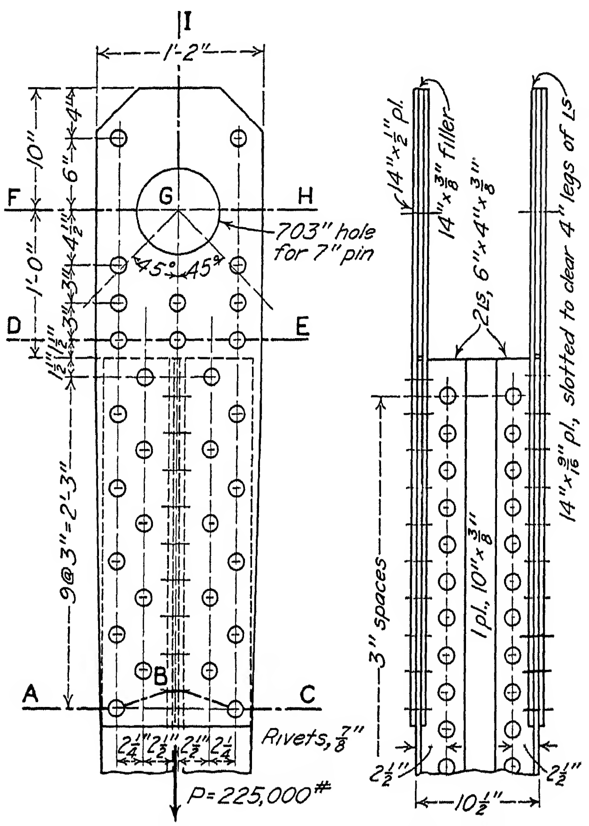

FIG. 22.—Pin Connection for Built-Up Tension Member.

34. Pin Connection for Built-Up Tension Member of H-Form.—A built-up tension member consisting of four 6 × 4 × ⅜-in. angles and a 10 × ⅜-in. plate arranged in H-form, as shown in Fig. 22, is called upon to carry a centric axial tension of 225,000 lb. It is to be connected to a 7-in. pin with its web parallel thereto by necessary pin plates. Assume that the joint clearances require that the main material be cut off 12 in. short of the pin, and that the pin plates do not extend more than 10 in. beyond the pin. Design the end connection.

Permissible tensile stress p_t = 16,000 lb. per sq. in. Rivets, ⅞ in. Permissible shearing stress on pins and shop rivets, p_s = 12,000 lb. per sq. in. Permissible bearing stress on pins and shop rivets, p_b = 24,000 lb. per sq. in.

The details are to be designed for 125% of the capacity of the member; the net section through the pin hole, at the back thereof, or any transverse section

through the pin plates and fillers, must, therefore, exceed the net section of the body of the member by at least 25%. Pin plates shall be as wide as the dimensions of the member will allow. Only the rivets in front of the two lines drawn from the centre of the pin towards the body of the member and inclined at 45 deg. to the axis of the member shall be considered as effective in distributing pin pressure to the full cross section of the member.

Effective Area of Member.—To carry a load of 225,000 lb., a net area of 225,000/16,000 = 14.05 sq. in. is required. The riveting arrangement at the inner end of the pin plates must be such that at least this net area is realized, and, besides, the net area of the member must be determined in order to arrive at the strength of the details. If, as shown in Fig. 22, a staggered pitch of 3 in. is adopted in the 6-in. leg of each angle and the rivets in the 4-in. legs are spaced 3 in. apart and staggered with the rivets in the 6-in. legs, the net area will be as large as is practicable. The section *ABC* is evidently the critical one.

Calculating deductions by the method of Art. 5, for any angle, at section *ABC*, $s = 1.5$, and for the gauges shown, $g = 6.875$ in. From Fig. 1(*c*) it is seen that the deduction for one angle is 1.95 holes, or 7.80 holes for the four angles. The loss of area for the angles is $7.80 \times 1 \times 0.375 = 2.93$ sq. in., and for the plate $2 \times 1 \times 0.375 = 0.75$ sq. in., making a total of 3.68 sq. in. Since the gross area is $4 \times 3.61 + 10 \times 0.375 = 18.19$ sq. in., the net area is $18.19 - 3.68 = 14.61$ sq. in., which is in excess of the required net area.

Pin Plate Sections.—The section of the pin plates must be chosen so as to give an area (1) through the hole on the section *FGH*, (2) along the rivet line *DE*, and (3), along the section *GI* back of the hole, 25% in excess of the net area in the body of the member, that is an area of $14.51 \times 1.25 = 18.15$ sq. in. Besides, the bearing area on the pin must be sufficient to transmit 125% of the capacity of the member.

To transmit the load to the pin, two pin plates on the outstanding legs of each pair of angles will be employed. One plate will be on the backs of the angles, as shown in Fig. 22, and the other on the inside, slotted to clear the 4-in. legs of the angles. Between each pair of pin plates a $\frac{3}{8}$-in. filler. equal in thickness to the angles, is employed.

If the gross width of the plates be taken as 14 in., and the net width at the pin hole as $14 - 7.03 = 6.97$ in., the combined thickness of the two plates and the filler on one side of the member should be $18.15/(2 \times 6.97) = 1.30$ in. As the filler is $\frac{3}{8}$ in. thick, each of the pin plates should be, so far as section *FGH* is concerned, $(1.30 - 0.38)/2 = 0.46$, or say $\frac{1}{2}$ in.

At the section *DE*, the fillers may not be counted as effective for tension area. The provided area is $(14 \times 2.0) - 3 \times 1 \times 2.0 = 22.0$ sq. in. and hence the section is ample.

For the section *GI* back of the pin, an area 125% of 14.51, or 18.15 sq. in., is also required. Since the combined thickness of plates for the two sides is 2.75 in. the area provided for a length of 10 in. back of the pin is $(10 - 7.03/2) \times 2.75 = 17.85$ sq. in. This is insufficient. Increase the inner (slotted) plates to $\frac{9}{16}$ in.

Required bearing area on the pin $= 14.51 \times 1.25 \times 16{,}000/24{,}000 = 12.10$

sq. in. Provided area = $2 \times 7.0 \times 1.4375 = 20.1$ sq. in., or much more than required.

In this problem it is assumed that the 7-in. pin itself is sufficient.

Riveting.—As the longitudinal distortion of pin plates and fillers back of the pin is the same, no stress-equalizing rivets are required there. Two are shown, however, to hold the plates tightly together.

From the pin a filler receives a stress proportional to its thickness. The stress for which the plates on one-half the member must be connected is $\frac{1}{2} \times 18.15 \times 16{,}000 = 145{,}200$ lb. A filler should, therefore, resist $145{,}200 \times 0.375/1.4375 = 37{,}900$ lb. As the rivets are in bearing on the $\frac{3}{8}$-in. filler, their value is 7880 lb. and $37{,}900/7880 = 5$ rivets only are required in each half of the detail. To secure the plates well together, 8 rivets are shown in Fig. 22. Only 6 of these lie within the two 45-deg. lines from the pin centre, and hence only 6 are effective.

To attach the pin plates on one side of the member to the outstanding legs of the angles, $145{,}200/7880 = 19$ rivets are required. The nearest practicable number is 20. These are arranged on a staggered pitch of 3 in.

For the sake of clearances, certain rivet heads on the outside of the pin plates, depending on the joint arrangement, would need to be flattened.

35. Exercise Problems on Tension Member Details.—Unless otherwise indicated, the method of making deductions for rivet holes is that of Art. 5 and Fig. 1.

(1) A $5 \times \frac{1}{4}$-in. plate carrying a total tension of 16,500 lb. is to be connected to a $\frac{3}{8}$-in. gusset plate by $\frac{3}{4}$-in. rivets. How many should be employed? $p_s = 9000$ lb. per sq. in. $p_b = 18{,}000$ lb. per sq. in.

(2) A $3 \times \frac{5}{16}$-in. flat carrying a tension of 9000 lb. is riveted to a $\frac{3}{8}$-in. gusset plate. If the rivets are $\frac{3}{4}$ in. and p_s and p_b are 8000 and 16,000 lb. per sq. in. respectively, find the number of rivets required in the connection.

(3) A $\frac{3}{4}$-in. bar carrying a stress of 50,000 lb. is connected to two $\frac{1}{2}$-in. plates by $\frac{7}{8}$-in. rivets, the bar being placed between the two plates. Find the number of rivets required to transmit the stress. p_s and p_b are 11,000 and 22,000 lb. per sq. in. respectively.

(4) A truss member consisting of two angles $3\frac{1}{2} \times 3 \times \frac{5}{16}$ in., with the $3\frac{1}{2}$-in. legs adjacent, connects to a $\frac{3}{8}$-in. gusset plate which passes between the two angles. Assuming $\frac{3}{4}$-in. rivets and p_s and p_b as 10,000 and 20,000 lb. per sq. in., respectively, find how many rivets would be required in the end connection to transmit a total tension of 30,000 lb.

(5) A truss member consisting of four angles $5 \times 3 \times \frac{3}{8}$ in. laced together to form an I-section is connected to, and between, two $\frac{1}{2}$-in. gusset plates at either end. How many $\frac{7}{8}$-in. field rivets would be necessary at each end in order to provide for a tension in the member of 120,000 lb. $p_s = 9000$ lb. per sq. in. $p_b = 18{,}000$ lb. per sq. in.

(6) A $5 \times \frac{1}{4}$-in. flat is connected to a $\frac{1}{4}$-in. gusset plate by three rivets arranged in isosceles triangular formation, the two rivets forming the base of the triangle being nearest the end of the member and driven on gauge lines $2\frac{1}{2}$ in. apart. The staggered pitch of rivets is $2\frac{1}{2}$ in. What is the maximum safe load on the member? Rivets, $\frac{3}{4}$ in. $p_t = 16{,}000$ lb. per sq. in. $p_s = 12{,}000$ lb. per sq. in. $p_b = 24{,}000$ lb. per sq. in.

(7) By what percentage would the safe capacity of the tension angle of Fig. 18 be lessened if the $1\frac{1}{2}$ in. stagger between the rivets in the two legs were maintained throughout the connection?

(8) A $10 \times \frac{7}{16}$-in. hanger plate carrying a total tension of 63,500 lb. is connected to a $\frac{7}{16}$-in. gusset by $\frac{3}{4}$-in. rivets. Design a connection of maximum efficiency, having regard to desirable compactness. $p_t = 16{,}000$, $p_s = 12{,}000$, and $p_b = 24{,}000$ lb. per sq. in.

(9) A member is connected to another by two rivets which are spaced 3 in. apart on centres and are located on a line at right angles to the line of pull. If the force to be resisted is 6000 lb. and its line of action passes through one of the rivets, find the total stress on each of the rivets.

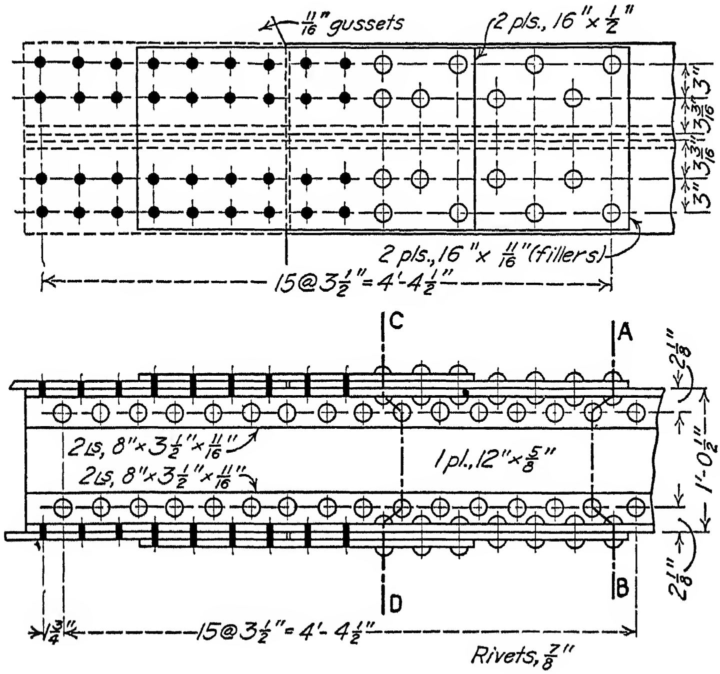

FIG. 23.—Capacity of Riveted Tension Member.

(10) A tension lateral making an angle of 45 deg. with a chord is connected thereto by means of a detail similar to that shown in Fig. 16(*a*). The distance between rivets 1–2 and 2–3 is 5 in. Find the total stress on each of the rivets 1, 2 and 3 due to a pull of 12,000 lb. in the lateral.

(11) A tension member consisting of two $6 \times 4 \times \frac{3}{8}$-in. angles, with the 4-in. legs outstanding is to be connected to a $\frac{7}{16}$-in. gusset plate by $\frac{7}{8}$-in. rivets. It is desirable that the distance from the end of the angles to the edge of the gusset plate where the member crosses it be not more than 14 in. What should be the rivet arrangement to give the maximum efficiency for the member? What load can be carried safely? Rivets, $\frac{7}{8}$ in. $p_t = 16{,}000$ lb. per sq. in. $p_s = 12{,}000$ lb. per sq. in. $p_b = 24{,}000$ lb. per sq. in.

(12) A steel flat 4 in. wide is to be used as a hanger for the suspension of a 35,000-lb. load. It must not be over 2 ft. 9 in. long and the lower end must be forked by the addition of fillers and outside plates, all 4 in. wide, so as to give a clear space 6 in. high and $1\frac{7}{16}$ in. between outside plates. The loads are applied by field rivets to the outside of the lower end of these fork plates and the reaction is applied to the upper end of the main plate by field rivets. Design the hanger. Rivets, $\frac{7}{8}$ in. p_t = 18,000 lb. per sq. in. p_s = 12,000 and 10,000 lb. per sq. in. for shop and field rivets respectively. p_b = 24,000 and 20,000 lb. per sq. in. also for shop and field rivets.

(13) The detail of Fig. 23 is supposed to transmit a tension of 497,000 lb. Analyze it and find out (*a*) the location of the critical net section, the area at this point, and what it should be to resist the existing stress, (*b*) the strength of the attachment to the gussets, (*c*) the required and provided net areas in the two outside $\frac{1}{2}$-in. plates, (*d*) the number of rivets (two rows being field) required in the $\frac{1}{2}$-in. plates outside the gussets, and (*e*) the number of shop rivets required outside the $\frac{1}{2}$-in. plates in the fillers. Rivets, $\frac{7}{8}$ in. p_t = 16,000 lb. per sq. in. p_s = 12,000 and 10,000 lb. per sq. in. for shop and field rivets respectively. p_b = 24,000 and 20,000 lb. per sq. in. for shop and field rivets respectively.

CHAPTER III

COMPRESSION MEMBERS

36. Fundamental Formulae for Compression Members.—The following formulae are applicable to compression members:

For members centrically loaded.

$$f_c = \frac{P}{A} \tag{1}$$

$$A = \frac{P}{p_c} \tag{2}$$

For members subjected to direct axial compression and to cross bending arising either from eccentricity of the axial loading or from transverse loading, such bending being in the plane of a principal axis

$$f_c + f_f = \frac{P}{A} + \frac{My_e}{I} \tag{3}$$

$$A_c + A_f = \frac{P}{p_c} + \frac{My_e}{r^2 p_c} \tag{4}$$

$$P_e = P\left(1 + \frac{ey_e}{r^2}\right) \tag{5}$$

For members subjected to combined axial compression and symmetrical bending, where the effect of deflection is considered

$$f_c + f_f = \frac{P}{A} + \frac{My_e}{I - \dfrac{Pl^2}{CE}} \tag{6}$$

$$A_c + A_f = \frac{P}{p_c} + \frac{My_e}{\left(r^2 - \dfrac{Pl^2}{CAE}\right)p_c} \qquad * \text{(7)}$$

Values of C may be taken as in Art. 1.

References

Hool and Kinne—Structural Members and Connections.
Johnson, Bryan and Turneaure—Modern Framed Structures, Pt. III.
Merriman—Mechanics of Materials.

See Appendix II.

Shedd—Structural Design in Steel.
Spofford—Theory of Structures.
Swain—Structural Engineering.

In the above formulae the symbols not already defined in Art. 1 have significances as follows:

A_c = area required for compression;
e = eccentricity of application of an axial load;
f_c = existing fibre stress due to centric axial compression;
P_c = equivalent centric axial load which will produce the same maximum fibre stress as an axial load P acting with an eccentricity of e;
p_c = permissible stress in compression per unit of area.

37. Short H-Column Supporting Centric Load.—Select an H-section 4 ft. 8 in. long to support a total centric axial compression of 51,000 lb.; $p_c = 19{,}000 - 100\ l/r$, with a maximum of 13,000 lb. per sq. in.

According to the tables of safe loads of H-columns, a 4-in., 13.8-lb. H will probably be adequate.

Least radius of gyration = 0.95 in. Slenderness ratio $l/r = 56/0.95 = 59$. $p_c = 19{,}000 - 100 \times 59 = 13{,}100$ lb. per sq. in., but 13,000 lb. per sq. in. must not be exceeded.

Area required = 51,000/13,000 = 3.92 sq. in. Area provided = 3.99 sq. in. Section is adequate.

38. Medium-Length H-Column Supporting Centric Load.—Find the necessary size of an H-section column one story high to carry a total centric axial load of 55 tons, if the story height be 12 ft. and $p_c = 16{,}000 - 70\ l/r$.

From the tables of capacity of columns in *Steel Construction*, based on the formula $p_c = 18{,}000/(1 + l^2/18{,}000\ r^2)$ it appears that an 8-in., 31-lb. W.F. would be sufficient.

Radius of gyration normal to the plane of the web = 2.01 in. Hence $l/r = 144/2.01 = 71.6$. $p_c = 16{,}000 - 70 \times 71.6 = 10{,}990$ lb. per sq. in.

Area required = 110,000/10,990 = 10.01 sq. in. Area of 8-in., 31-lb. W.F. is 9.12 sq. in. Section is insufficient, and hence we must use an 8-in., 35-lb. W.F., with an area of 10.30 sq. in.

39. Medium-Length Round Cast-Iron Column Supporting Centric Load.—Determine to the nearest $\frac{1}{8}$ in. the thickness of a hollow round cast-iron column 15 ft. high and of 10 in. outside diameter to carry a total centric axial load of 230,000 lb. $p_c = 9000 - 125\ l/d$. (New York Building Code formula modified.)

Permissible compressive stress $p_c = 9000 - 125 \times 180/10 = 6750$ lb. per sq. in.

Area required = 230,000/6750 = 34.10 sq. in. From the properties of hollow round sections as given in structural handbooks, a column with outside diameter of 10 in. and a shell thickness of $1\frac{1}{4}$ in. has an area of 34.36 sq. in., which is adequate.

40. Effect of Two-Way and Four-Way Lateral Support.—A column consists in two successive 12-ft. stories of a 6-in., 25.0-lb. Carnegie H. In the lower of the two stories there exists a total centric axial load of 72,000 lb. At the bottom of the first of these stories, and at the top of the second, 4-way lateral support exists, but at the floor level between, lateral support is available only in a direction normal to the column web, as indicated in Fig. 24. Express an opinion as to the safety of the column. C.E.S.A. formula for building columns (1924), $p_c = 14{,}000 - \frac{1}{3}(l/r)^2$.

The sufficiency of the column should be investigated about the two principal axes of the column section.

Normal to the web, buckling can take place only over a height of 12 ft. Slenderness ratio in this direction $= 144/1.43 = 100.7$. $p_c = 14{,}000 - \frac{1}{3}(100.7)^2 = 10{,}620$ lb. per sq. in. Existing stress in the lower of the two stories is $f_c = P/A = 72{,}000/7.35 = 9790$ lb. per sq. in., which is safe.

In the plane of the web, buckling can take place over two stories in height, that is, over 24 ft. Slenderness ratio in this direction $= 288/2.53 = 113.8$. $p_c = 14{,}000 - \frac{1}{3}(113.8)^2 = 9680$ lb. per sq. in. Existing stress, as already found $= 9790$ lb. per sq. in. The column is, therefore, unsafe in the plane of the web.

If there were 4-way support at the intermediate floor, the column would be much stronger in the plane of the web than in a plane at right angles thereto.

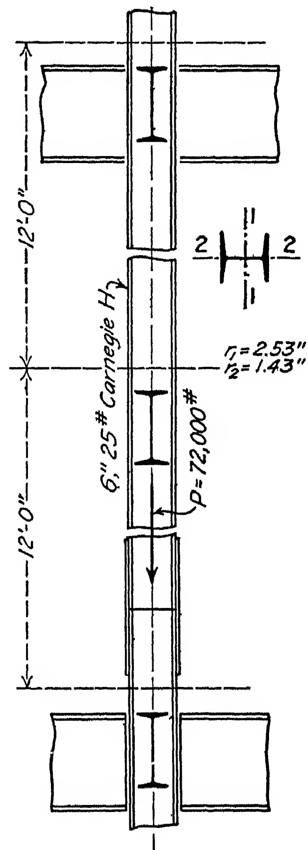

FIG. 24.—Effective Length of Column.

41. Long Tubular Column Centrically Loaded.—A column 19 ft. long between lateral supports is to consist of a section of galvanized standard-weight steel pipe, and is to carry a total centric axial load of 7000 lb. Suggest a size if $p_c = 13{,}000 - 50\ l/r$.

Assume a nominal 3-in. standard pipe which has an external diameter of 3.50 in. and an internal diameter of 3.068 in. Area = 2.23 sq. in. Radius of gyration, from handbook = 1.16 in.

Slenderness ratio $= 19 \times 12/1.16 = 196.5$. $p_c = 13{,}000 - 50 \times 196.5 = 3175$ lb. per sq. in.

Area required $= 7000/3175 = 2.21$ sq. in. Area provided = 2.23 sq. in., which is sufficient.

42. Medium-Length, Double-Angle Strut.—Find the total safe capacity of a strut 8 ft. 4 in. long between centres of end connections, if it be composed

of two $3 \times 2\frac{1}{2} \times \frac{5}{16}$-in. angles stitch riveted together, with the 3-in. legs adjacent and separated by a space of $\frac{1}{4}$ in. The 3-in. legs only are connected, the gauge being $1\frac{3}{4}$ in. as shown in Fig. 25. $p_c = 16{,}000/(1 + l^2/9000,\ r^2)$, ends being assumed as free to turn. Neglect the effect of deflection.

The maximum existing stress which, according to Eq. (3) of Art. 36, is

$$f_c + f_f = \frac{P}{A} + \frac{My_c}{I}$$

will be found in terms of P, and by equating this to the permissible stress the safe capacity can be found.

Area of member, no deduction of rivet holes being necessary, is 3.24 sq. in.

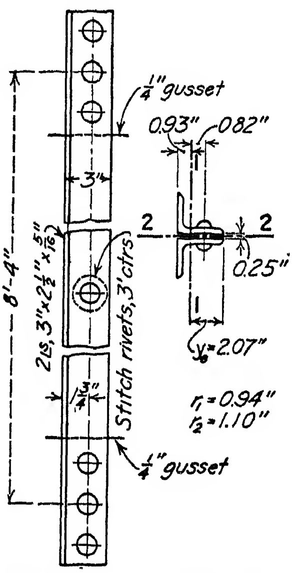

FIG. 25.—Double-Angle Strut.

Eccentricity of loading = 1.75 − 0.93 = 0.82 in. Moment = 0.82 P in.-lb. Distance from neutral axis to most highly compressed fibre is y_c = 3.00 − 0.93 = 2.07 in. Moment of inertia about axis 1 − 1 at right angles to the plane of bending = 2 × 1.40 = 2.80 in.⁴

Hence, maximum existing stress

$$f_c + f_f = \frac{P}{3.24} + \frac{0.82\,P \times 2.07}{2.80} = 0.914\,P$$

Least radius of gyration is in the plane of bending and is 0.94 in. Slenderness ratio, l/r = 100/0.94 = 106.4. Hence $p_c = 16{,}000/(1 + 106.4^2/9000)$ = 7120 lb. per sq. in.

Now since 0.914 P = 7120, therefore P = 7780 lb.

If the load were wholly centric the equation would be 0.309 P = 7120, and P would be 23,000 lb. Hence because of the eccentricity of the connection the capacity is only 33.8% of what it would be if the loading were centric.

The assumption of perfectly hinged ends is, of course, unduly severe, since the end connections actually afford considerable resistance to end rotation.

43. Effect of Restraining Action of End Connections of Strut.—Determine the safe capacity of the strut of Art. 42 if it be assumed that only one-third of the apparent moment of eccentricity is absorbed by the strut, while two-thirds is absorbed by the other members that are connected to the same gusset plates. Assume the other data as in Art. 42.

Eccentricity moment to be absorbed by the strut under consideration is

$$\tfrac{1}{3} \times 0.82\,P = 0.273\,P$$

Maximum existing stress is

$$f_c + f_f = \frac{P}{3.24} + \frac{0.273\,P \times 2.07}{2.80} = 0.511\,P$$

The permissible flexural stress being 7120 lb. per sq. in., then $0.511\ P = 7120$, and $P = 13{,}920$ lb.

The safe capacity is thus $13{,}920/23{,}000 = 0.606 = 60.6\%$ of that for a perfectly hinged centrically loaded strut. This is more nearly in accordance with the demonstrated capacity of struts in riveted frames than the result of the previous article.

44. Eccentricity Required to Realize a Desired Ratio of Flexural to Direct Stress.—If the actual moment on the strut of Art. 42 be only one-third of the apparent moment, what eccentricity of connection should there be in order to make the flexural stress one-half of the direct stress?

Equating the flexural stress to one-half of the direct stress, we have, if e be the apparent eccentricity,

$$\frac{Pey_e}{3I} = \frac{P}{2A}$$

whence, since

$$I = Ar^2,$$

$$e = \frac{3r^2}{2y_e}$$

As r in the plane of bending $= 0.94$ in. and $y_e = 2.07$ in., $e = 3 \times (0.94)^2/2 \times 2.07 = 0.64$ in.

45. Orientation of H-Column for Best Meeting of Eccentricity Moment.—A 14.25×14.58-in. 103-lb. H-column passing through a floor is to receive a

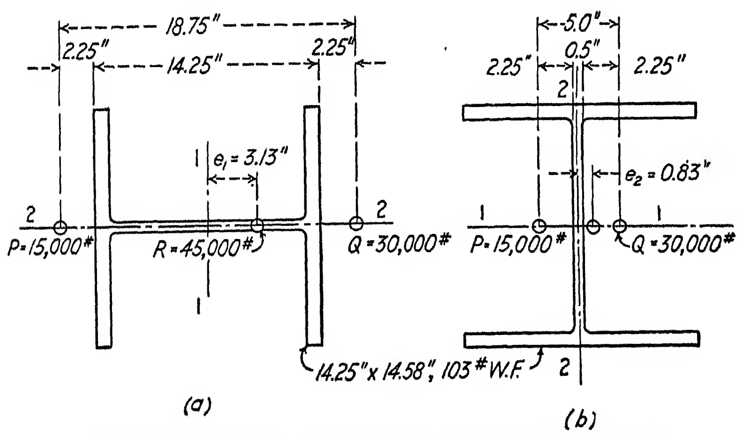

FIG. 26.—Orientation of H-Column for Eccentricity Moment.

30,000-lb. load on one face and a 15,000-lb. load on the opposite face, in each case applied 2.25 in. from the face of the column as shown in Fig. 26. So far as *these loads alone* are concerned, which way should the column be oriented?

If the two loads are applied to the flanges, as in Fig. 26(a), they act at a dis-

tance of 18.75 in. apart. Their resultant acts at $18.75/3 = 6.25$ in. from the 30,000-lb. load, or with an eccentricity of $e_1 = \frac{1}{2} \times 18.75 - 6.25 = 3.13$ in.

If the loads are applied to the web, as in Fig. 26(*b*), they act with an eccentricity of $e_2 = \frac{1}{2} \times 5.0 - 1.67 = 0.83$ in.

So far as the effect of the two prescribed loads is concerned, the two orientations shown will be equally effective in resisting *centric* axial load, but for moment that orientation will be best for which the ratio of section modulus to eccentricity in the plane of bending is greatest. For the (*a*) orientation this ratio is

$$\frac{S_1}{e_1} = \frac{163.6}{3.13} = 52.3$$

while for the (*b*) orientation it is

$$\frac{S_2}{e_2} = \frac{57.6}{0.83} = 69.5$$

It thus appears that for the loading condition specified, arrangement (*b*) is the better. This arises from the fact that each of the two loads, and consequently their resultant, is applied nearer the column axis in (*b*) than in (*a*).

Because of the greater resistance to wind moment, and because heavy loads should be applied as directly as possible to the larger masses of metal and to broad, clear, flat surfaces, girders are connected to the flanges rather than to the web of a column section.

46. Stress in a Stay Member Supporting a Centrically Loaded Hinged Column Buckled at Mid-Height.—A W.F. section used as a column 25 ft. long is discovered after erection to have a centre deviation normal to the web of 0.5 in. from the straight line joining its ends. A horizontal stay member is connected to the column in such a manner as to meet directly and prevent any further buckling. If, subsequent to the introduction of the stay member, a centric axial load of 100,000 lb. is applied to the column, what stress will be produced in the stay member? Assume the bend of the column to be parabolic.

The necessary transverse force to prevent further buckling under a centric axial load P_2 applied after the stay member is brought into service is approximately

$$Q = \frac{5\,P_2 d}{l}$$

where d = the central deflection of the column and l = the total length of the column (not merely the distance from one end to the point of attachment of the stay member).

Introducing the values of P_2, d and l,

$$Q = \frac{5 \times 100{,}000 \times 0.5}{300} = 833 \text{ lb.}$$

It is obvious from the expression for Q that, for a perfectly straight column under centric axial load, the smallest possible lateral force, that is the weakest possible stay member, would, in theory, prevent buckling at mid-height.

47. Effect of Deflection in Reducing Capacity of a Strut.—Find the safe capacity of the strut of Art. 42 if the effect of deflection be considered. E = 29,000,000 lb. per sq. in. Ends considered as free to turn.

Maximum existing stress, according to Eq. (6) of Art. 36, is

$$f_c + f_f = \frac{P}{A} + \frac{My_c}{I - \dfrac{Pl^2}{10\,E}}$$

Equating this to the permissible stress

$$\frac{P}{3.24} + \frac{0.82\,P \times 2.07}{2.80 - \dfrac{P \times (100)^2}{10 \times 29{,}000{,}000}} = 7120$$

Solving, P = 7300 lb., or 480 lb. less than for the case of Art. 42 in which the deflection was neglected. For struts of medium length the effect of deflection is, therefore, not important.

48. Four-Angle Latticed Column Eccentrically Loaded.—The top story of a 4-angle latticed column supports a load of 45,000 lb. and one of 27,000 lb. applied as shown in Fig. 27. Assume that the weight of the column is included in these loads. Neglecting the effect of deflection, suggest a size. A.R.E.A. formula, $p_c = 15{,}000 - 50\,l/r$.

Assume a column consisting of four $4 \times 3 \times \frac{5}{16}$-in. angles arranged as shown in Fig. 27.

Area of section $= 4 \times 2.09 = 8.36$ sq. in.

Effective length of column = 150 in.

Moment of inertia about axis 1 — 1 $= 4\{1.70 + 2.09(4.49)^2\} = 175.6$ in.^{4}

Radius of gyration about axis 1 — 1 $= (I_1/A)^{1/2} = (175.6/8.36)^{1/2} = 4.57$ in.

Moment of inertia about axis 2 — 2 $= 4\{3.40 + 2.09(1.51)^2\} = 32.68$ in.^{4}

Radius of gyration about axis 2 — 2 $= (I_2/A)^{1/2} = (32.68/8.36)^{1/2} = 1.98$ in.

Permissible compressive stress, based upon the tendency of the column to buckle transversely to the plane of the lacing, $p_c = 15{,}000 - 50 \times 150/1.98 = 11{,}210$ lb. per sq. in.

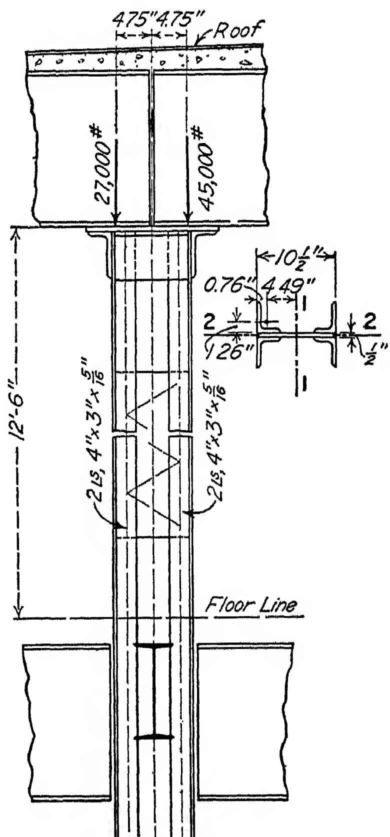

Fig. 27.—Eccentrically Loaded Latticed Column.

Distance of line of action of resultant

from 45,000-lb. load = (27,000 × 9.5 + 45,000 × 0)/72,000 = 3.56 in. Eccentricity of resultant load = 4.75 − 3.56 = 1.19 in.

Moment of eccentricity of loading = 72,000 × 1.19 = 85,700 in.-lb.

Area of column required, from Eq. (4) of Art. 36

$$A_c + A_f = \frac{72,000}{11,210} + \frac{85,700 \times 5.25}{(4.57)^2 \times 11,210}$$

$$= 6.42 + 1.92 = 8.34 \text{ sq. in.}$$

The assumed section is sufficient.

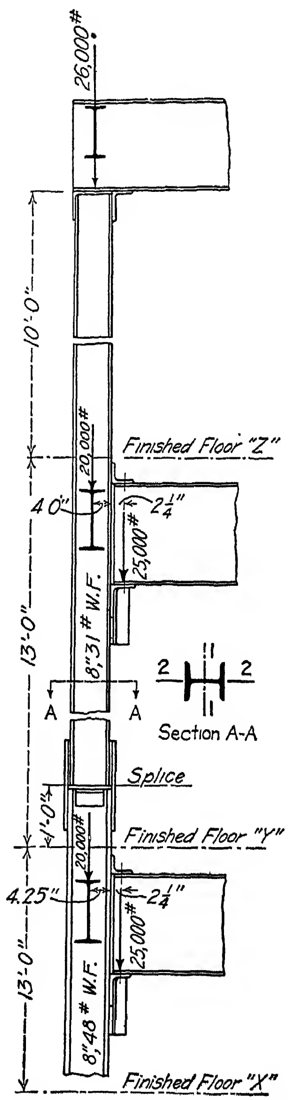

FIG. 28.—Three-Story Eccentrically Loaded H-Column.

49. Three-Story Eccentrically Loaded H-Column.—Design the top three stories of a continuous H-column with web in the plane of eccentricity, if the loading and dimensions are as in Fig. 28 and the same section is used for the top story as for the one immediately below. $p_c = 16,000 - 55\ l/r$, with a maximum of 13,000 lb. per sq. in. E = 29,000,000 lb. per sq. in.

Assume (1) that the effect of deflection in producing moment is neglected, and (2) that it is included. In the latter case, assume the column as fixed at one end and free to turn at the other. Assume the effective eccentricity as the apparent eccentricity divided by 2, and regard the eccentricity that originates in a given story as dissipated at the floor level below, provided that there is complete lateral support in the plane of eccentricity at this latter floor level.

Top Story.—As the column is to be made continuous for two top stories, the section found for the second story down from the roof will be amply strong for the top story.

Story YZ.—Load on the second story down from the roof, that is YZ = 26,000 lb. from roof + 20,000 lb. centric load at floor Z + 25,000 lb. eccentric load at floor Z + weight of column for $1\frac{1}{2}$ stories, say 700 lb., or a total of 71,700 lb.

Moment of eccentricity of 25,000-lb. load, assuming an 8-in., 31-lb. W.F., with depth 8 in. = $\frac{1}{2}\ Pe = \frac{1}{2} \times 25,000 \times 6.25$ = 78,125 in.-lb.

Unsupported length is approximately the distance between finished floors, that is 13 ft. = 13 × 12 = 156 in. For an 8-in., 31-lb. W.F., area = 9.12 sq. in.;

radius of gyration about the axis $1-1$ (Fig. 28) = 3.47 in.; radius of gyration about the axis $2-2$ = 2.01 in. Hence l/r in plane of web = 156/3.47 = 44.9 and at right angles to plane of web, it is 156/2.01 = 77.6.

Maximum permissible stress, considering strength in plane of web must, therefore, since $p_c = 16{,}000 - 55\,l/r = 13{,}530$ lb. per sq. in., be limited to 13,000 lb. per sq. in. to conform to the specification. At right angles to the plane of the web, $p_c = 16{,}000 - 55 \times 77.6 = 11{,}730$ lb. per sq. in., which governs.

Approximate Design, Story YZ.—As the column is evidently weakest at right angles to the plane of the web, it must be proportioned so that the sum of the axial compressive and flexural stresses will not exceed 11,730 lb. per sq. in. To ensure this, the area required, Eq. (4), Art. 36, is

$$A_c + A_f = \frac{71{,}700}{11{,}730} + \frac{78{,}125 \times 4.0}{(3.47)^2 \times 11{,}730} = 6.12 + 2.21 = 8.33 \text{ sq. in.}$$

As the area provided is 9.12 sq. in., the section is adequate.

Allowance for Deflection. Story YZ— The design will be checked for the effect of the total axial load acting on the deflection in the column arising from eccentricity of loading. The total area required, if one end be assumed as fixed and the other as free to turn, with the value of C consequently approximating 15 (Art. 1), is from Eq. (7) of Art. 36.

$$A_c + A_f = \frac{71{,}700}{11{,}730} + \frac{78{,}125 \times 4.0}{\left(3.47^2 - \dfrac{71{,}700 \times 156^2}{15 \times 9.12 \times 29{,}000{,}000}\right)11{,}730}$$
$$= 6.12 + 2.30 = 8.42 \text{ sq. in.}$$

As this is also in excess of the area required, the section is adequate.

Story XY.—Since the column is well supported in the plane of eccentricity at floor Y, the eccentricity of the 25,000-lb. load at floor Z may be considered as wholly dissipated below floor Y. The loading originating above floor Y becomes here, therefore, a centric load of 71,700 lb. If the column from mid-height of story YZ to mid-height of story XY be assumed to weigh 650 lb., the total load on this tier is 71,700 lb. centric from above floor Y + 20,000 lb. centric at floor Y + 25,000 lb. eccentric at floor Y + 650 lb. centric column weight = 117,350 lb.

Moment of eccentricity of 25,000-lb. load at floor Y, if section is $8\frac{1}{4}$ in. deep $= \frac{1}{2} \times 25{,}000 \times 6.38 = 79{,}800$ in.-lb.

Assume an 8-in., 40-lb. W.F.; area = 11.76 sq. in.; radius of gyration about the axis $1-1$ = 3.53 in.; radius of gyration about the axis $2-2$ = 2.04 in. Hence l/r at right angles to the plane of the web = 156/2.04 = 76.5, and the governing permissible stress = $p_c = 16{,}000 - 55 \times 76.5 = 11{,}790$ lb. per sq. in.

Approximate Design, Story XY.—Area required for centric axial loading and moment of eccentricity is

$$A_c + A_f = \frac{117{,}350}{11{,}790} + \frac{79{,}800 \times 4.13}{(3.53)^2 \times 11{,}790} = 9.98 + 2.25 = 12.23 \text{ sq. in.}$$

The section assumed is not sufficient. Adopt an 8-in., 48-lb. W.F. which has an area of 14.11 sq. in.

Allowance for Deflection. Story XY.—The total area required, allowing for the effect of deflection, and assuming the 48-lb. W.F., is found, proceeding as for story YZ, to be

$$A_c + A_f = \frac{117{,}350}{11{,}870} + \frac{\frac{1}{2} \times 25{,}000 \times 6.50 \times 4.25}{\left(3.61^2 - \dfrac{117{,}350 \times 156^2}{15 \times 14.11 \times 29{,}000{,}000}\right)11{,}870}$$

$$= 9.90 + 2.32 = 12.22 \text{ sq. in.}$$

As the area of an 8-in., 48-lb. W.F. is 14.11 sq. in., this section is adequate.

50. Centric Axial Load Equivalent to an Eccentric Axial Load.—In the case of the latticed column of Art. 48, find what centric axial load would be equivalent to the actual eccentric loading.

Total resultant load = 72,000 lb., applied 1.19 in. off centre.
Distance from neutral axis to extreme fibre, y_e = 5.25 in.
Radius of gyration in the direction of the eccentricity = 4.57 in.
Equivalent centric axial load, from Eq. (5), Art. 36, is

$$P_e = 72{,}000\left\{1 + \frac{1.19 \times 5.25}{(4.57)^2}\right\} = 72{,}000 \times 1.299 = 93{,}500 \text{ lb.}$$

51. Design of a Crane Runway Column.—The side columns of a building support both the roof and a crane runway in the manner shown in Fig. 29.

The load imposed by the roof truss due to dead weight and snow is 12,000 lb. This will be assumed to include such roof load as is transmitted directly to the column by the eave purlin and the weight of the column. Consider it and a downward wind force of 2400 lb. as applied at the face of the inner flange of the column. A horizontal wind force of 3600 lb. will be assumed as applied at the top of the column.

The maximum reaction from the runway is 30,000 lb. and is applied to the bracket 10 in. from the face of the column. The horizontal and the upward vertical components of the wind reaction at the leeward column base are respectively 3600 and 2400 lb., the columns being assumed as free to turn at the base, and at the top as well.

Assuming the dimensions shown in Fig. 29, recommend a single rolled section for the leeward column. $p_c = 20{,}000 - 80\,l/r$, with a maximum of 20,000 for a

combination of dead load, snow load, crane load and wind load stresses. Neglect the effect of deflection.

As the permissible working stress will be determined by the tendency of the column to buckle transversely to the web, a beam with a comparatively wide flange is desirable. Hence assume tentatively an 18-in., 47-lb. W.F., with an

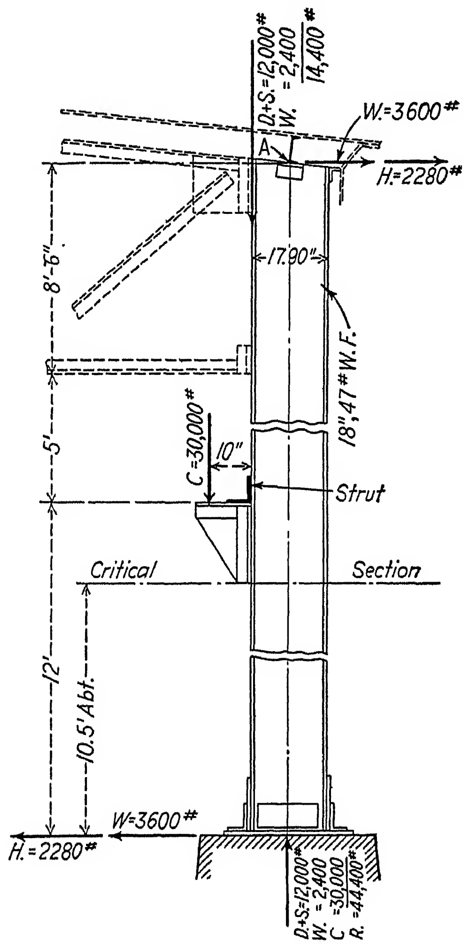

FIG. 29.—Design of a Leeward Crane Runway Column.

area of 13.81 sq. in., a section modulus of 82.3 in the plane of bending, and a radius of gyration of 1.56 at right angles to the web.

Above the crane runway bracket the total axial load = 12,000 lb. dead and snow load + 2400 lb. wind load = 14,400 lb. Below the crane runway bracket it is 14,400 + 30,000 = 44,400 lb.

In addition to the horizontal reaction at the base of the column due to wind, there will be one at the top and the bottom of the column due to the eccentricity

of the axial loading. Taking moments about the point A on the axis of the column and at its top,

$$H = \frac{(14{,}400 \times 8.95) + (30{,}000 \times 18.95)}{25.5 \times 12} = 2280 \text{ lb.}$$

This force acts to the right at the top and to the left at the bottom of the column.

Two cross sections require consideration: (1) at the top of the bracket, and (2) at the bottom of the bracket.

At the top of the bracket the moment about a point on the axis of the column, considering forces above the bracket, is $(2280 + 3600) \times 13.5 \times 12 - 14{,}400 \times 8.95 = 826{,}000$ in.-lb.

Maximum fibre stress at the top of the bracket is, from Eq. (3) of Art. 36,

$$f_c + f_f = \frac{14{,}400}{13.81} + \frac{826{,}000}{82.3} = 1040 + 10{,}010$$
$$= 11{,}050 \text{ lb. per sq. in.}$$

If a strut be run along the inside of the columns at the level of the top of the crane bracket, the column will be supported on *one* flange at this point. Although the compression flange is not directly stayed, it derives considerable restraint from the support on the tension flange, and the permissible stress would be at least as great as the existing stress, 11,050 lb. per sq. in.

At the bottom of the bracket, considering forces below this level, and remembering the assumption of no moment at the base, the moment is $(2280 + 3600) \times 10.5 \times 12 = 742{,}000$ in.-lb.

Maximum fibre stress at the bottom of the bracket is

$$f_c + f_f = \frac{44{,}400}{13.81} + \frac{742{,}000}{82.3}$$
$$= 3210 + 9020 = 12{,}230 \text{ lb. per sq. in.}$$

The section at the bottom of the bracket is only 1.5 ft. away from a point of lateral support, and hence an assumption of a 12-ft. unsupported length is very much on the severe side. According to this, however, l/r transverse to the plane of the web $= 114/1.56 = 92.4$, and $p_c = 20{,}000 - 80 \times 92.4 = 12{,}610$ lb. per sq. in.

The column is thus amply safe.

52. Design of Eccentrically Loaded Corner Column.—Design the two stories of the eccentrically loaded H-section corner column shown in Fig. 30, assuming the orientation and dimensions there indicated, and the loads delivered to the column at each of the floors X, Y and Z as equal and having the magnitude and point of application shown in section A–A.

Permissible combined stress, $p_c = 18{,}000 \Big/ \left(1 + \dfrac{l^2}{18{,}000\, r^2}\right)$, with a maximum of 15,000 lb. per sq. in.

Assume that the effect of deflection in producing moment is neglected, that

the effective eccentricity is the apparent eccentricity divided by 2, and that the eccentricity which originates in a given story is dissipated at the floor level below, if there be complete lateral support in the plane of eccentricity at this latter floor level.

Story YZ.—The column in this story carries (1) a centric axial load of 75,000 lb. from the story above; (2) a load of 25,000 lb. applied in the central plane of the column web and 10 in. from the column axis, and (3) a load of 18,000 lb. applied 5 in. to one side of the central plane of the web and at a distance from the axis of the column amounting to half the depth of the column section plus 2.25 in.

Total axial load, P, is 75,000 + 25,000 + 18,000 = 118,000 lb.

Assuming an H-column section 10 in. deep, the apparent eccentricity of the 18,000-lb. load about the axis 1–1 is 5 + 2.25 = 7.25 in.

The total apparent moment about the 1–1 axis is then 25,000 × 10 − 18,000 × 7.25 = 119,500 in.-lb., and the actual moment is one-half of this, or M_1 = 59,750 in.-lb.

About the axis 2–2, the apparent moment is 18,000 × 5 = 90,000 in.-lb. and the actual moment is M_2 = 45,000 in.-lb.

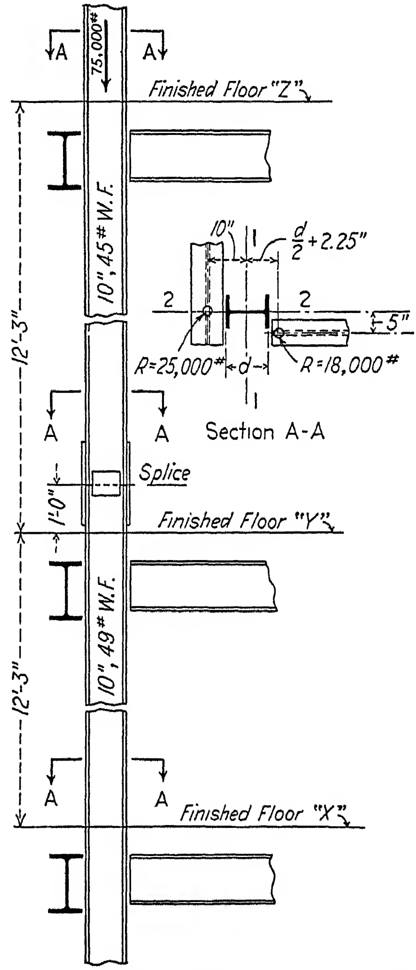

FIG. 30.—Eccentrically Loaded Corner Column.

Assume a 10-in. 45-lb. W.F. section, for which A = 13.24 sq. in., S_1 = 49.1, S_2 = 13.3 and r_2 = 2.00 in. The maximum existing stress, extending Eq. (3) of Art. 36 to cover bending parallel to the two principal axes, and using section moduli rather than moments of inertia, is

$$f_c + f_{f1} + f_{f2} = \frac{P}{A} + \frac{M_1}{S_1} + \frac{M_2}{S_2}$$

$$= \frac{118{,}000}{13.24} + \frac{59{,}750}{49.1} + \frac{45{,}000}{13.3}$$

$$= 8920 + 1220 + 3380$$

$$= 13{,}520 \text{ lb. per sq. in.}$$

Since the permissible stress for the maximum slenderness ratio, which is $147/2.00 = 73.5$, is, from the table in "Steel Construction," 13,850 lb. per sq. in., the section is adequate.

The actual depth of the section is 10.12 in., but the excess over the assumed 10 in. affects the computed moment but slightly.

Story XY.—According to the assumptions of the design, the axial load of 118,000 lb. in story YZ becomes wholly centric below floor Y. At this floor, however, an eccentric load of 25,000 lb. and one of 18,000 lb. are applied, as shown in Fig. 30. In story XY there is then a total axial load of $P = 161{,}000$ lb. part of which is eccentric.

If the section be a 10-in. H, the moment will be, as for story YZ, $M_1 = 59{,}750$ and $M_2 = 45{,}000$ in.-lb.

Assume a 10-in., 49-lb. W.F., for which $A = 14.40$ sq. in., $S_1 = 54.6$, $S_2 = 18.6$ and $r_2 = 2.54$ in.

$$f_c + f_{f1} + f_{f2} = \frac{161{,}000}{14.40} + \frac{59{,}750}{54.6} + \frac{45{,}000}{18.6}$$
$$= 11{,}180 + 1095 + 2420$$
$$= 14{,}695 \text{ lb. per sq. in.}$$

The slenderness ratio being less than 60, the permissible stress is 15,000 lb. per sq. in.

53. Design of Double-Angle Member Subjected to Axial Compression and Cross Bending.—A double-angle compression member of a truss, 10 ft. long between panel points and inclined at 45 deg. to the horizontal, carries a thrust of 22,000 lb. The angles are stitch riveted together at intervals of about 2 ft., as shown in Fig. 31, and are connected to the end gussets by rivets in the adjacent legs only.

Recommend a size for the angles, making allowance for the cross bending due to the weight of the angles, and for the effect of deflection. $p_c = 20{,}000 - 80\ l/r$, with a maximum of 17,000 lb. per sq. in. Consider the strut as partially fixed at the ends, so that C in Eq. (6) of Art. 36 is 15. $E = 29{,}000{,}000$ lb. per sq. in.

The strut will be assumed to bend as a whole in the plane of the gussets. For an exact analysis, the theory of unsymmetrical bending should be applied, each angle being considered as partially stayed by the other.

Assume two $4 \times 3 \times \frac{5}{16}$-in. angles with 4-in. legs back to back, separated by the thickness of gussets, or $\frac{3}{8}$ in. Gross area (no deduction being required for rivet holes) = 4.18 sq. in. Moment of inertia about axis $1 - 1 = 6.8$ in.^{4}

Apparent moment is due (1) to eccentricity of force P, which, for the special gauge of 2 in. is $2.00 - 1.26 = 0.74$ in., and (2) to weight of angles, or 14.4 lb. per lin. ft.

Eccentric moment $= Pe = 22{,}000 \times 0.74 = 16{,}300$ in.-lb.

Component of weight of member normal to its length $= 14.4 \cos 45° = 10.2$ lb. per lin. ft. Moment at centre of strut, making allowance for partial fixity at the ends, is approximately, $\frac{1}{10} \times 10.2 \times (10)^2 \times 12 = 1230$ in.-lb.

Since the eccentric moment tends to bend the member upward and the weight

moment tends to bend it downward, the latter must be taken from the former, and the net apparent moment = 16,300 − 1230 = 15,070 in.-lb.

Neglecting the component of the weight of the member parallel to its length, the maximum stress on the lower fibres at the centre is, from Eq. (6) of Art. 36

$$f_c + f_f = \frac{22{,}000}{4.18} + \frac{15{,}070 \times 2.74}{6.8 - \dfrac{22{,}000 \times (120)^2}{15 \times 29{,}000{,}000}}$$

$$= 5270 + 6800 = 12{,}070 \text{ lb. per sq. in.}$$

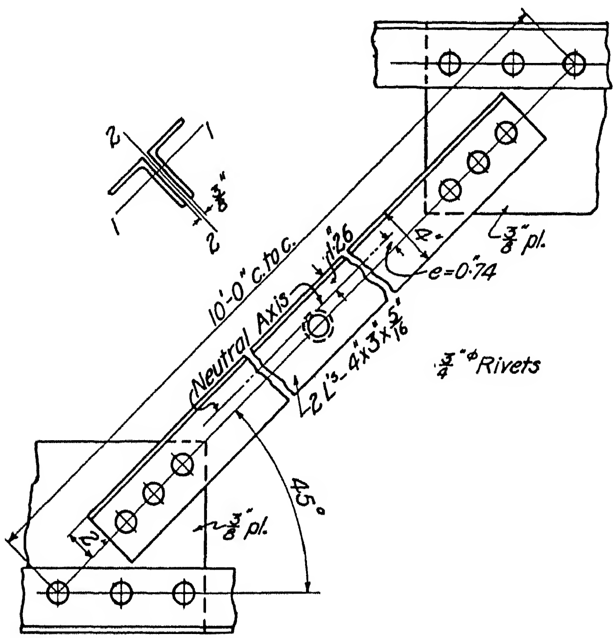

FIG. 31.—Member Subjected to Compression and Cross Bending.

Greatest allowable stress on strut, for the maximum value of l/r (in plane of gussets) of 120/1.27 or 94.5, is p_c = 20,000 − 80 × 94.5 = 12,440 lb. per sq. in. The section is adequate.

54. Exercise Problems on the Design of Compression Members.—The following exercise problems are based on the principles employed in the problems of this chapter. See Appendix I for the answers.

(1) A single 6 × 6 × ½-in. angle, 10 ft. long, is used as a column in a building. Estimate its safe carrying capacity, assuming the load as centrically applied. $p_c = 13{,}000 - 0.25(l/r)^2$.

(2) A column 15 ft. high between lateral supports consists of four 5 × 3½ × ⅜-in. angles and a 12 × ⅜-in. web plate arranged in an I-section 12½ in. deep. What centric axial load will it safely carry? $p_c = 16{,}250/(1 + l^2/11{,}000r^2)$.

(3) Report on the safety of a steel column 18 ft. long subjected to a centric axial load of 300,000 lb., if the column consists of two 10-in., 20-lb. channels and two $12 \times \frac{1}{2}$-in. flange plates, the flanges of the channels being turned out. $p_c = 14,000 - \frac{1}{3}(l/r)^2$.

(4) A 4-in., 13.8-lb. H-column, 8 ft. long, carries a centrically applied load of 35,000 lb. Express an opinion as to the safety of the column. $p_c = 19,000 - 100\ l/r$ with a maximum of 13,000 lb. per sq. in.

(5) A steel column 13 ft. long is required to carry a centric axial load of 57,000 lb. Suggest a section. $p_c = 14,000 - 0.5\ (l/r)^2$.

(6) A steel column in a 2-story building receives from the roof a load of 20,000 lb. and from the first floor 85,000 lb., the loads in each case being centrically applied. If the column is supported laterally in all directions at the roof and at the first floor levels, suggest a section for the column in the first story. $p_c = 15,000 - 50\ l/r$. Story heights, 12 ft.

(7) Express an opinion as to the safety of an 8-in. 18.4-lb. I-beam 8 ft. long used as a column and carrying a load of 35,000 lb. applied $1\frac{1}{2}$ in. off centre in the central plane of the web. $p_c = 15,000 - 50\ l/r$.

(8) A steel column consisting of two 10-in., 15.3-lb. channels and two $12 \times \frac{3}{8}$-in. plates on the flanges carries a total axial load of 200,000 lb. 1 in. off centre in the direction of the channel webs. Express an opinion as to the safety of the column if the permissible stress on the extreme fibre is $p_c = 16,000 - 70\ l/r$ and the story height is 14 ft.

(9) A hollow circular cast-iron column, 12 ft. long, with 8 in. outside diameter and 1 in. thickness, is subjected to an axial load applied $\frac{1}{2}$ in. off centre. What is the maximum permissible amount of this load? Use formula, $p_c = 10,000/(1+ l^2/800\ d^2)$.

(10) A 6-in., 12.5-lb. I-beam is used as a vertical strut supporting a load of 20,000 lb. applied 1 in. off centre in the plane of the web. If the length of the strut is 6 ft., report on the safety thereof, taking account of the effect of deflection. $p_c = 15,000 - 50\ l/r$. $E = 30,000,000$. Ends free to turn.

(11) Determine the maximum fibre stress on the 4-angle laced top story column of Art. 48, taking into account the effect of deflection. $C = 10$ in. Eq. (6) of Art. 36. $E = 29,000,000$ lb. per sq. in.

(12) A steel column supporting a centric axial load of 70,000 lb. is 22 ft. long and is supported laterally *in one plane* at the mid-point. Recommend a section. $p_c = 14,000 - \frac{1}{3}(l/r)^2$.

(13) Select a suitable section for a steel column 13 ft. long to carry an axial load of 70,000 lb. applied $\frac{1}{2}$ in. off centre. Indicate clearly the recommended orientation of the column. Neglect the effect of lateral deflection. $p_c = 13,000 - 0.25(l/r)^2$.

CHAPTER IV

COMPRESSION MEMBER DETAILS

55. Steel Plate Column Base.—A 14-in., 273-lb. W.F. section is to deliver a centric axial compression of 600 tons to a concrete pedestal through a solid steel slab, or plate, base. Design the base, using the approximate method recommended by the American Institute of Steel Construction in its handbook "Steel Construction" (1934).

Permissible flexural stress on steel plate, $p_f = 18{,}000$ lb. per sq. in.; permissible bearing on concrete pedestal, $p_c = 600$ lb. per sq. in. Neglect the uplift effect.

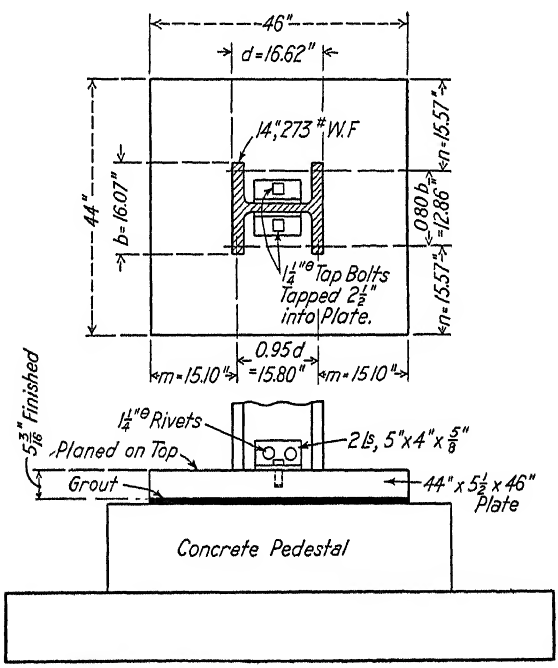

FIG. 32.—Steel Plate Column Base.

The required area of the plate = 1,200,000/600 = 2000 sq. in.

Use a 44 × 46-in. plate with the long dimension parallel to the depth of the H-section, as shown in Fig. 32.

Since the depth of the column section is 16.62 in., the dimension m in Fig. 32

is $(46.0 - 0.95 \times 16.62)/2 = 15.10$ in. Similarly, since the flange width of the section is 16.07 in., the dimension n is $(44.0 - 0.80 \times 16.07)/2 = 15.57$ in.

According to the method employed in the handbook mentioned, the net thickness of the plate should be at least

$$t = \left(\frac{um^2}{6000}\right)^{\frac{1}{2}} \tag{1}$$

or

$$t = \left(\frac{un^2}{6000}\right)^{\frac{1}{2}} \tag{2}$$

whichever is the greater, u being the existing upward pressure per unit of area.

In the present case Eq. (2) governs. The value of u being 1,200,000/(44 × 46) = 593 lb. per sq. in.

$$t = \left\{\frac{593 \times (15.57)^2}{6000}\right\}^{\frac{1}{2}} = 4.89 \text{ in.}$$

An allowance of $\frac{5}{16}$ in. must be made for planing the top of a plate of this thickness, although the bottom may be left rough for the grouted bearing. The rough plate should thus be at least 5.20 in. thick. The nearest available thickness is $5\frac{1}{2}$ in., which will be used.

To hold the steel column section in place on the plate during erection, a pair of $5 \times 4 \times \frac{5}{8}$-in. angles is riveted to the column web and connected to the plate by two $1\frac{1}{4}$-in. studs tapped $2\frac{1}{2}$ in. into it. Connection is made to the column web rather than to the flanges to lessen the amount of drilling.

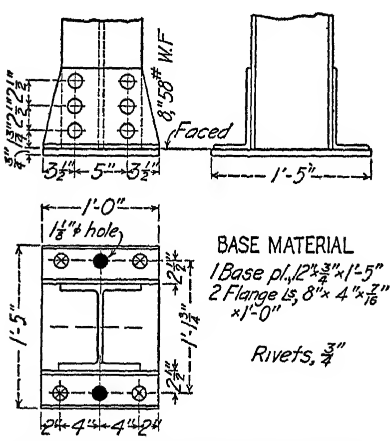

Fig. 33.—Riveted Base for H-Column.

56. Design of Riveted Steel Base for H-Column.—Design a riveted steel plate and angle base of the type shown in Fig. 33 for an 8-in., 58-lb. H-column, the total load being a 95,000-lb. centric, axial load. Rivets, $\frac{3}{4}$ in. Anchor bolt holes $\frac{1}{8}$ in. larger than anchor bolts. p_f = 20,000 lb. per sq. in. p_s = 12,000 lb. per sq. in. p_b = 24,000 lb. per sq. in. p_b, on concrete pedestal = 500 lb. per sq. in. Consider 40% of the total axial load as carried directly to the masonry by bearing of the faced end of the column shaft.

Base Plate.—Required area of base plate = total load divided by permissible pressure on masonry = 95,000/500 = 190 sq. in.

Assuming base angles with 4-in. horizontal legs, the length of base plate parallel to the column web would need to be 17 in. to accommodate an 8.75×

8.22-in. column section. Adopting a stock width, the plate will be 12 × 16 in., giving 204 sq. in. of area.

The thickness of the base plate should be sufficiently great to ensure that no undue flexural stresses arise in it and that the upward deflection will not be excessive at any point. The strip of plate between the column web and the 17-in. edges should be investigated for stress and deflection, as also the part of the plate and its base angle reinforcement extending outwards from each flange.

For the first-named area of plate, an approximate empirical determination of the required thickness may be made by considering a strip 1 in. wide as a cantilever beam projecting outwards from the web at right angles to the 16-in. edge.

The intensity of upward pressure on such a cantilever strip may be assumed in accordance with the approximate semi-rational formulae

$$w' = \left(0.25\frac{l}{b} - 0.10\right)w, \text{ for } \frac{l}{b} \leqq 2 \tag{1}$$

$$w' = 0.20\frac{l}{b}w, \text{ for } \frac{l}{b} \geqq 2 \tag{2}$$

where w' = virtual uniformly distributed upward pressure on the cantilever strip;

l = mean length of the unsupported area of plate under consideration, measured at right angles to the cantilever strip;

b = cantilever span;

w = average existing upward pressure on the base plate, per unit of area.

In the present case, l = 7.0 in., approximately, and b = 5.74 in., and, therefore, l/b = 7.0/5.74 = 1.22.

Assumed upward loading on cantilever, from Eq. (1) above = (0.25 × 1.22 −0.10) × 95,000/192 = 102 lb. per sq. in.

Maximum moment on 1-in. strip for projection of 5.74 in. clear of the web $= \frac{1}{2} \times 102 \times (5.74)^2 = 1680$ in.-lb.

Required section modulus = 1680/20,000 = 0.083 in.^{3}

A $\frac{3}{4}$-in. base plate giving a section modulus of $\frac{1}{6} \times 1 \times (0.75)^2 = 0.094$ in.^{3} will be adequate.

The upward deflection will not become objectionable for such a cantilever strip, restrained as it is by the support at right angles to it, if the thickness of the base plate is at least $\frac{1}{10}$ of the cantilever span. In this case it is 0.75/5.74 =0.131.

On the flange side of the column, considering the combined plate and base angle outside the fillet of the latter, there is a cantilever of about 3.25-in. span loaded uniformly by an upward pressure of 95,000/204 = 466 lb. per sq. in.

Moment on a 1-in. strip $= \frac{1}{2} \times 466 \times (3.25)^2 = 2460$ in.-lb.

Required section modulus = 2460/20,000 = 0.123 in.^{3}

The section modulus of the combined $\frac{3}{4}$-in. plate and $\frac{7}{16}$-in. angle, is $\frac{1}{6} \times 1 \times (1.188)^2 = 0.235$ in.^{3}, which is ample.

The deflection of this simple cantilever will not be objectionable if the combined thickness is at least $\frac{1}{4}$ of the projection. It is 1/3.25.

Base Angles.—Since only 40% of the total axial load is assumed to be transferred to the masonry by the faced end bearing of the column shaft, the remaining 60%, or $95{,}000 \times 0.60 = 57{,}000$ lb., must be delivered to the base plate by the base angles. To make this possible, enough rivets must be placed through the column shaft to develop 57,000 lb.

The least value of a rivet through a flange is its single shear value $= 0.442 \times 12{,}000 = 5300$ lb. The required number of rivets is, therefore, $57{,}000/5300 = 10.8$. Twelve is the least practicable number.

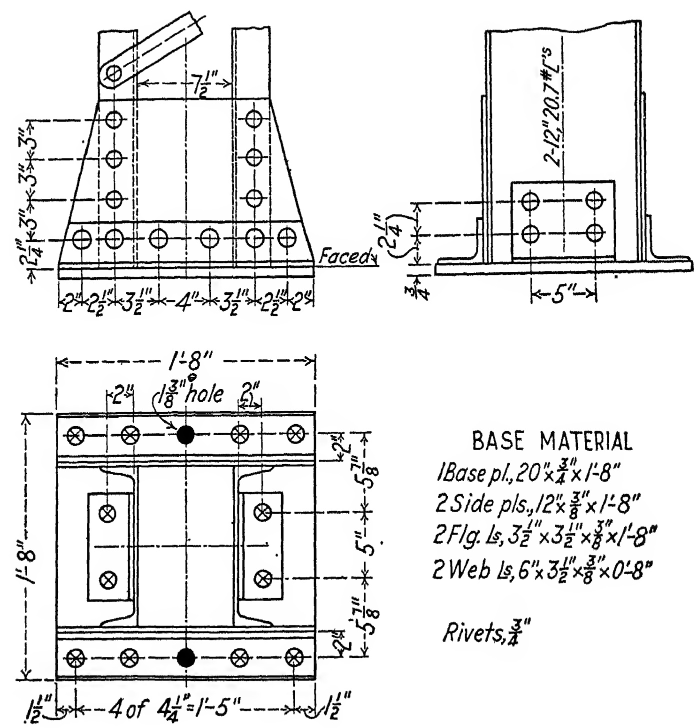

Fig. 34.—Riveted Base for Channel Column.

The $8 \times 4 \times \frac{7}{16}$-in. base angles are run full width of the base. No base angles are required on the web for a column of this weight.

All vertical rivets through base angles must be countersunk on the under side. Sufficient rivets should be employed to develop, in conjunction with the friction of the parts, a horizontal shearing resistance at the plane of contact of the base angles and base plate consistent with the joint flexural action of these two elements. Anchor bolt holes of $1\frac{1}{8}$ in. diameter are provided for 1-in. anchors.

57. Design of Riveted Steel Base for a Double-Channel Column.—Design a riveted steel plate and angle base of the type shown in Fig. 34 for a column consisting of two 12-in., 20.7-lb. channels with flanges turned out, the total load being a centric axial load of 155,000 lb. Side plates and base angles on both the flanges and the web are to be used. Rivets, $\frac{3}{4}$ in. Anchor bolt holes $\frac{1}{8}$ in. larger than the anchor bolts. p_f = 18,000 lb. per sq. in. p_s = 10,000 lb. per sq. in. p_b = 20,000 lb. per sq. in. p_b, on concrete pedestal, = 400 lb. per sq. in. Consider 35% of the total axial load as carried to the pedestal by bearing of the faced end of the column shaft.

Base Plate.—Required area of base plate = 155,000/400 = 388 sq. in.

To provide for $\frac{3}{8}$-in. side plates on the column flanges and $3\frac{1}{2}$-in. horizontal legs for the flange angles, the length of base plate parallel to the channel webs would need to be $12 + (2 \times \frac{3}{8}) + (2 \times 3\frac{1}{2}) = 19\frac{3}{4}$, or say 20 in. A plate 20 × 20 in. will be adopted, giving an area of 400 sq. in.

As in the problem of Art. 56, the cantilevered base plate should be investigated for flexure and deflection, as also should the area inside the column section.

For a 1-in. strip projecting outward from the edge of one of the base angles, the span is 2.47 in. The mean length, l, of the unsupported area of plate in this 2.47-in. zone is 12 in. Hence $l/b = 12.0/2.47 = 4.86$. According to Eq. (2) of Art. 56, the cantilever strip should be designed for a uniformly distributed upward pressure of

$$w' = 0.20 \times 4.86 \times 155{,}000/400 = 377 \text{ lb. per sq. in.}$$

Maximum moment for 2.47-in. projection $= \frac{1}{2} \times 377 \times (2.47)^2 = 1150$ in.-lb.

Required section modulus = 1150/18,000 = 0.064 in.^{3}

A $\frac{5}{8}$-in. base plate, giving a section modulus of 0.065 in.^{3} per inch of width would be adequate. For a thickness in excess of $\frac{1}{10}$ of the cantilever span (see Art. 56), the deflection would not be excessive. This thickness will be adopted tentatively.

Consider next a similar strip extending back to the fillet of a web base angle, a distance of 5.1 in. The mean length of the zone having this unsupported width is about 11.5 in. Hence $l/b = 11.5/5.1 = 2.26$. According to Eq. (2) of Art. 56, this cantilever strip should be designed for an upward pressure of

$$w' = 0.20 \times 2.26 \times 155{,}000/400 = 175 \text{ lb. per sq. in.}$$

Maximum moment $= \frac{1}{2} \times 175 \times (5.1)^2 = 2280$ in.-lb.

Required section modulus = 2280/18,000 = 0.127 in.^{3}

Counting the combined thickness of the assumed plate and the base angle, or say 1 in., the section modulus provided per inch of width is 0.167 in.^{3}, which is ample. The projection is within the rule for deflection.

On the flange side of the column, the projection from the fillet of the base angle is 2.75 in. and the upward pressure is 155,000/400 = 388 lb. per sq. in. On a 1-in. strip, the moment $= \frac{1}{2} \times 388 \times (2.75)^2 = 1470$ in.-lb.

Required section modulus = 1470/18,000 = 0.082. Section modulus of combined thickness of 1 in. = 0.167 in.^{3}, which is adequate.

The combined thickness is 1/2.75 of the projection, and, therefore, ample for deflection.

Assume that the ratio of upward pressure borne by the base plate in the direction of the short dimension of the area within the column section to the total nominal unit pressure on the plate is given by the formula $r = l^4/(l^4 + b^4)$, where l = long side and b = short side of the inner rectangle. Since l = 12 in. and b = 7.5 in., r = 0.87. Hence strips in the short direction should be figured for an upward pressure of $0.87 \times 388 = 338$ lb. per sq. in.

Assuming perfect restraint under the column webs, the moment on a 1-in. strip running in the direction of the $7\frac{1}{2}$-in. dimension $= \frac{1}{12} \times 338 \times (7.5)^2 = 1580$ in.-lb.

Required section modulus = 1580/18,000 = 0.088 in.^{3}

The $\frac{5}{8}$-in. plate tentatively adopted will not be sufficient. For a $\frac{3}{4}$-in. plate, the section modulus provided $= \frac{1}{6} \times 1 \times (0.75)^2 = 0.094$ in.^{3}

The deflection will not be excessive if the unsupported length of plate in the interior area is not in excess of 16 times the thickness of the base plate. In this case it is exactly 16 times that thickness.

Side Plates and Base Angles.—The side plates and base angles must deliver to the base plate 65% of the total column load, or $0.65 \times 155{,}000 = 101{,}000$ lb. Assume 16 rivets through the column flanges and 8 rivets through the column webs, as shown in Fig. 34. The safe resistance of each of the former rivets is its single shearing value $= 0.442 \times 10{,}000 = 4420$ lb., and of each of the latter its bearing value on the 0.28-in. web of column channel, or $0.28 \times 0.75 \times 20{,}000 = 4200$ lb. The total resistance of the two groups is $(16 \times 4420) + (8 \times 4200) = 70{,}700 + 33{,}600 = 104{,}300$ lb., which is slightly in excess of requirements.

The side plates will be made $\frac{3}{8}$ in. thick, will extend across the base, as shown in Fig. 34, and will be 12 in. deep to allow for three rows of rivets above the base angles riveted to them.

The base angles on the column flanges have 12 rivets driven through them into the side plates, or enough to develop $12 \times 4420 = 53{,}000$ lb., leaving 48,000 lb. to be applied to the base plate by the faced side plates and the base angles on the web. The 8 rivets in the latter will develop of themselves 33,600 lb.

Two $1\frac{1}{4}$-in. anchor bolts will be used, and the holes will be $1\frac{3}{8}$ in. diameter.

58. Design of Riveted Steel Base for a Column Subjected to Overturning.—Design a riveted steel plate and angle base of the type shown in Fig. 35 for an 18-in., 50-lb. W.F. column to withstand a simultaneous axial load of 25,000 lb. 12 in. off centre on the leeward side and a wind moment of 35,000 ft.-lb.

Assume that the mass and form of the concrete pedestal is sufficient to accommodate the vertical loading and the overturning moment applied to it by the column base.

Rivets, $\frac{3}{4}$ in. Anchor bolt holes $\frac{1}{4}$ in. larger than the anchor bolts. p_f and p_t = 18,000 lb. per sq. in., p_s = 12,000 lb. per sq. in., p_b = 24,000 lb. per sq. in. and p_b on concrete pedestal = 600 lb. per sq. in.

Base Plate.—The area of the base plate must be such as to keep the pressure on the concrete at the leeward edge of the base plate within 600 lb. per sq. in. The plate must transmit a centric compression of 25,000 lb. and a moment of $(35{,}000 \times 12) + (25{,}000 \times 12) = 720{,}000$ in.-lb.

It is convenient to replace this system of forces by a single 25,000-lb. downward vertical force, R, acting on a line $720{,}000/25{,}000 = 28.8$ in. off centre on the leeward side, as shown in Fig. 35(b).

Since no tension can exist between the base plate and the top of the pedestal, this effect being entirely resisted by the windward anchor bolt, it may

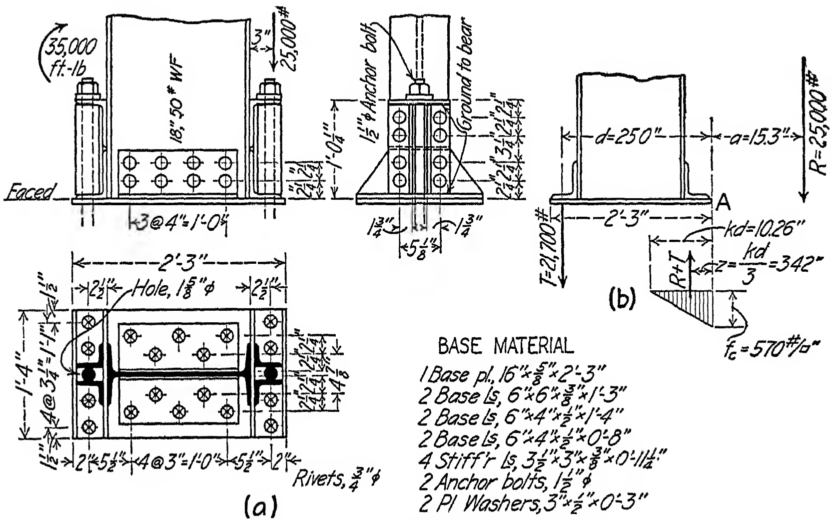

FIG. 35.—Base Subjected to Overturning.

be shown that, with reference to Fig. 35(b) the following equations apply approximately (see Appendix II):

$$z^3 \pm az^2 + 3\,Cz = Cd \tag{1}$$

$$C = \frac{10\,A_g(d \pm a)}{3\,b} \tag{2}$$

$$T = R \cdot \frac{z \pm a}{d - z} \tag{3}$$

$$f_c = \frac{T + R}{1.5\,bz} \tag{4}$$

The symbols involved have the following significance:

A_g = gross area of anchor bolt (or bolts) on windward side of column;
a = distance of resultant R from the toe A of the base plate;
b = breadth of base plate;
d = distance from toe A to windward anchor bolt or bolts;

kd = distance from toe A to inner limit of compressive area of pedestal;
f_c = maximum existing stress in compression per unit of area;
R = substitute vertical force producing same direct and turning effect as the actual forces;
T = total tension in windward anchor bolt (or bolts);
z = distance from toe A to centre of gravity of pressure triangle.

In order to apply Eqs. (1) to (4) to the design of the base, A_g, a, b and d must be assumed.

Approximation of Data.—To furnish data required in the exact design of the base, the simple theory of combined compression and flexure may be applied. Assume tentatively a 14 × 27-in. base plate. From Eq. (4) of Art. 36, the area required to satisfy a centric compression of 25,000 lb. and a bending moment of 720,000 in.-lb. is

$$A_c + A_f = \frac{25{,}000}{600} + \frac{720{,}000 \times 13.5}{\frac{1}{12} \times (27)^2 \times 600} = 41.7 + 266.7 = 308.4 \text{ sq. in.}$$

As this approximate method is in error considerably on the side of weakness a 16 × 27-in. base plate will be assumed in the more exact analysis.

Considering the arrangement of Fig. 35(a), in conjunction with Fig. 35(b) a = 15.3 in., b = 16 in. and d = 25 in.

To approximate the gross area of the windward anchor bolt (or bolts) compute the moment from Fig. 35(b) about the toe A. This is 25,000 × 15.3 = 382,500 in.-lb. Considering the turning as taking place about A, and neglecting the small amount of pull in the leeward bolt (or bolts) the tension in the windward bolt is 382,500/25 = 15,300 lb. Area required = 15,300/18,000 = 0.85 sq. in. One $1\frac{1}{4}$-in. bolt with an area at the root of thread of 0.89 sq. in. would suffice. However, since this method is in error on the unsafe side a $1\frac{1}{2}$-in. bolt will be assumed in the more exact analysis. Hence, A_g = 1.767 sq. in.

Revised Design of Base Plate.—Introducing in Eq. (2) the data established above, with the plus sign for a, since R falls outside the base,

$$C = \frac{10 \times 1.767(25 + 15.3)}{3 \times 16} = 14.85$$

Equation (1) then becomes, the plus sign applying for the second term,

$$z^3 + 15.3\, z^2 + 44.6\, z = 371.3$$

Solving for z by the method of approximation, z = 3.42, and the length of the compression area, kd, = 3 × 3.42 = 10.26 in.

From Eq. (3), again using the plus sign,

$$T = 25{,}000 \times \frac{3.42 + 15.3}{25.0 - 3.42} = 21{,}700 \text{ lb.}$$

From Eq. (4)

$$f_c = \frac{21{,}700 + 25{,}000}{1.5 \times 16 \times 3.42} = 570 \text{ lb. per sq. in.}$$

As the latter is within the safe compressive stress on concrete, the base plate is sufficiently large.

The thickness will be determined by the flexural strength at the leeward edge of the plate of the combined thickness of the base plate and the base angle on the flange. The most trying condition will be near a corner of the plate, where the influence of the stiffener angles on the flange is small. Consider a strip 1 in. wide at right angles to the 16-in. edge. Let the average upward pressure on this be 500 lb. per sq. in. The cantilever span, from the edge of the fillet of the base angle, assuming the latter to be $\frac{1}{2}$ in. thick, is 3.5 in. Hence, the moment on the strip $= \frac{1}{2} \times 500 \times (3.5)^2 = 3060$ in.-lb.

Section modulus required is

$$S = 3060/18{,}000 = 0.170 \text{ in.}^3$$

If the base plate be assumed as $\frac{5}{8}$ in. thick, and it and the $\frac{1}{2}$-in. base angle be assumed to be riveted together sufficiently well to act as one slab $1\frac{1}{8}$ in. thick, then the section modulus provided is

$$S = \tfrac{1}{6} \times 1(1.125)^2 = 0.211 \text{ in.}^3$$

which is sufficient. The ratio of span to thickness, which is $3.5/1.125 = 3.11$, is such as to satisfy the deflection requirements discussed in Arts. 56 and 57.

Bearing of Column Shaft.—Any column shaft which at a section immediately above the base will safely resist the forces specified will not be overstressed in end bearing on the base plate, since bearing compression may always be specified higher than the compressive stress in the shaft.

Base Angles.—The base angles connecting the flanges of the column to the base plate have already been fixed as $\frac{1}{2}$ in. thick. Angles 6×4 in. with the 4-in. legs horizontal will accommodate the necessary rivets and anchor bolt holes. The upper horizontal angles on the flange to provide seats for the anchor bolts will be made of the same size and will have the 4-in. legs horizontal.

Two vertical stiffener angles $3\frac{1}{2} \times 3 \times \frac{3}{8}$ in. will be provided on each flange to distribute the pull of the anchor bolts and to transfer the pressure on the ends of the base plate to the column flanges. Four $\frac{3}{4}$-in. rivets will be put in each stiffener angle. The sufficiency of these rivets may be checked easily and with safety by assuming that the whole of the moment on the column is to be absorbed by the attachments on the column flanges. Each group of 8 rivets has a single shearing value of $8 \times 5300 = 42{,}400$ lb., and as the two planes of rivet shear are 18 in. apart, the resisting moment of the riveting is $42{,}400 \times 18 = 763{,}000$ in.-lb. The total moment at the column base is only 720,000 in.-lb.

The $6 \times 6 \times \frac{3}{8}$-in. angles on the web of the column are provided to attach the web securely to the base plate and to aid in distributing the load in the web (which under the assumed loading is small) to the base plate.

Anchor Bolts.—The value of the tension in the windward anchor bolt has been found to be $T = 21{,}700$ lb.

Area required = 21,700/18,000 = 1.21 sq. in.

Area of $1\frac{1}{2}$-in. bolt at root of thread = 1.29 sq. in., which is adequate.

Washers about 6 × 6 in., or other suitable anchors, should be used at the bottom of the bolt. The latter should be of such length as to engage a mass of concrete with a bounding area on its sides sufficient to give a safe diagonal tensile resistance equal to the pull in the anchor bolt.

59. Column Bracket without Stiffeners.—Design a single angle column bracket without stiffeners to carry an I-beam reaction of 10,000 lb. The beam terminates $\frac{1}{4}$ in. from the flange of the column, which is $\frac{3}{8}$ in. thick.

Rivets, $\frac{3}{4}$ in. Permissible stresses on rivets: transverse shear, p_s = 12,000 lb. per sq. in.; bearing, p_b = 24,000 lb. per sq. in.; tension parallel to the rivet axis, p_t = 7000 lb. per sq. in. The latter is fixed at a very low value to allow for the combination of shearing and tensile stress which is considerably greater than the axial tensile stress, and also at the same time to allow for imperfections in the rivet heads. (See Art. 62 for example of combination of shearing and tensile stresses.) Permissible flexure on bracket angle, p_f = 16,000 lb. per sq. in.

Number of rivets necessary solely to transmit the load in single shear = 10,000/5300 = 1.9, say 2. By reason of the turning effect on the bracket, more will doubtless be necessary.

Adopt as bracket material a 6 × 4 × $\frac{1}{2}$-in. shelf angle with the 6-in. leg vertical. This will accommodate 4 rivets, as shown in Fig. 36.

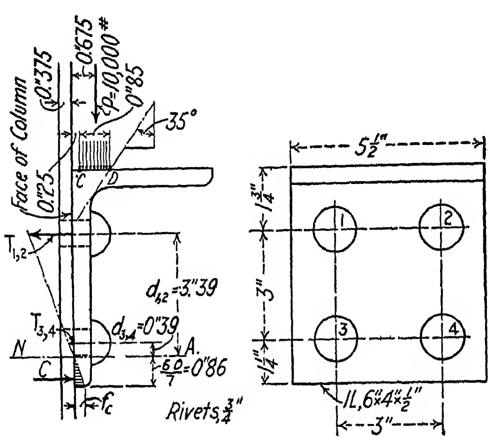

Fig. 36.—Bracket without Stiffeners.

Since the unstiffened outstanding leg tends to bend down easily under vertical loading, the beam reaction may be considered as applied very largely over that portion of the length of the leg marked CD. Let the distribution of load on it be taken as uniform. This length is determined by the position of the end of the beam and the intersection with the back of the 4-in. leg of a line tangent to the fillet curve and making an angle of 35 deg. with the vertical. As CD is about 0.85 in. long, the line of action of the force P is about 0.675 in. out from the face of the column.

It is assumed that for this bearing length the vertical compressive stress in the web of the supported beam and the bearing stress on the beam flange, or on the bracket, are within safe limits. (See Art. 60 for a consideration of these stresses.)

Turning will be assumed as tending to take place about a centre of compression $\frac{1}{21}$ of the depth of the bracket up from the bottom. (See *Engineering News*,

April 25, 1912, p. 786.) The neutral axis is thus $\frac{3}{21} = \frac{1}{7}$ of the bracket depth or $\frac{6}{7} = 0.86$ in. up from the bottom of the bracket.

The total turning moment on the bracket is $M = 10{,}000 \times 0.675 = 6750$ in.-lb.

Of this, the rivets in tension will, on the basis of the above assumption as to the position of the neutral axis, resist a moment of

$$M' = \frac{M}{1 + \dfrac{2\,h\Sigma d}{21\ \Sigma d^2}} \tag{1}$$

where h = depth of bracket, out to out, and d = vertical distance of each rivet from the neutral axis. See Appendix II for the derivation of Eqs. (1) and (2).

Since $h = 6$ in., $\Sigma d = 2(0.39 + 3.39) = 7.56$ in., and $\Sigma d^2 = 2(0.39^2 + 3.39^2) = 23.3$ in.^{2} Hence

$$M' = \frac{6750}{1 + \dfrac{2 \times 6 \times 7.56}{21 \times 23.3}} = 5690 \text{ in.-lb.}$$

Now, the tensile stress on any rivet, number "n," is, from analogy with Eq. (3) of Art. 21

$$T_n = \frac{M'd_n}{\Sigma d^2} \tag{2}$$

where d_n = vertical distance to any particular rivet, number "n." Since the most highly stressed rivets are 1 and 2, and since $d_{1,2} = 3.39$ in., and $\Sigma d^2 = 23.3$, the maximum tensions are

$$T_1 = T_2 = \frac{5690 \times 3.39}{23.3} = 830 \text{ lb.}$$

The safe tension on a rivet shaft is $0.442 \times 7000 = 3100$ lb., and consequently the bracket is amply strong according to the approximate method of design adopted. A bracket with two rivets in it would be inadequate.

Assuming that the vertical bearing pressure is uniformly distributed over the $\frac{1}{2}$-in. thickness of the vertical leg of the angle and on a length transverse to the beam axis of only 1 in.—an amount depending on the size of the beam—the bearing area would be 0.5 sq. in. This, at 24,000 lb. per sq. in. would develop a resistance of 12,000 lb., or more than the applied load.

Bending of the angle at the neutral axis NA due to the compression at the toe should be investigated.

Total compression = total tension in the rivets 1, 2, 3, 4. This is $2(830+95) = 1850$ lb. This acts at $0.86 \times \frac{2}{3} = 0.57$ in. from NA, and the moment there is consequently $1850 \times 0.57 = 1055$ in.-lb.

Required section modulus $= 1055/16{,}000 = 0.066$ in.^{3} Section modulus provided, if bracket is $5\frac{1}{2}$ in. long, is $\frac{1}{6} \times 5.5 \times (0.5)^2 = 0.229$ in.^{3}

Maximum bearing pressure at toe of angle.

$$f_c = \frac{1850}{\frac{1}{2} \times 0.86 \times 5.5} = 782 \text{ lb. per sq. in.}$$

60. Welded Column Bracket without Stiffeners.—A 12-in., 32-lb. W.F. section delivers a reaction of 12,000 lb. to a single shelf angle, without stiffeners, welded by two vertical $\frac{3}{8}$-in. fillets to the face of a column, as shown in Fig. 37. A clearance of $\frac{3}{8}$ in. exists between the end of the beam and the face of the column. Design the bracket.

Permissible stress on fillet welds not over 6 in. long subjected to parallel (longitudinal) shear per lineal inch is $p_s = 500\,b$, where b = short side of the fillet triangle expressed in sixteenths of an inch. To compensate for craters, consider the effective length of each fillet weld as $\frac{1}{2}$ in. less than the actual length. Permissible stress in bearing of beam on bracket, $p_b = 24{,}000$ lb. per sq. in.

Assume that bearing of shelf angle against the face of the column near the

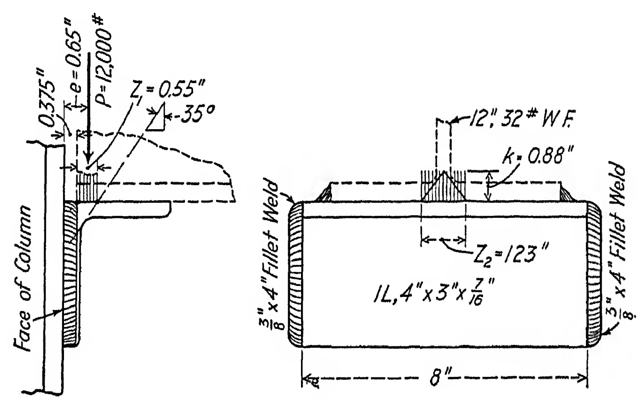

FIG. 37.—Welded Column Bracket without Stiffeners.

toe is imperfect, so that the welds must transfer all the moment as well as the shear.

If it be assumed that the vertical leg of the shelf angle is not over 6 in., the permissible shear on $\frac{3}{8}$-in. fillet welds will be $500 \times 6 = 3000$ lb. per lin. in.

Considering the effect of shear only, the required length of fillet on each end of the shelf $= 12{,}000/2 \times 3000 = 2.00$ in. Since the depth will need to be increased substantially to allow for the added effect of the moment, the angle will be assumed as $4 \times 3 \times \frac{7}{16}$ in., with the 4-in. leg vertical.

Bearing Stress.—The sufficiency of this angle for bearing of the beam upon it should first be investigated. While part of the reaction is transferred to the angle outside the fillet, it is probable that most of it, because of flexure in the angle, is applied on a very short zone. Let it be assumed, as has been done in Art. 59, that the load is delivered uniformly along a zone reaching from the end

of the beam to the point where a tangent to the angle fillet making an angle of 35 deg. with the vertical cuts the back of the outstanding leg.

Now, the distance of this intersection from the back of the angle is approximately equal to

$$c = 1.7\,t + 0.5\,r \tag{1}$$

in which t = thickness of angle and r = radius of the fillet. Since $t = \frac{7}{16}$ in. by assumption and $r = \frac{3}{8}$ in., $c = 0.93$ in.

As the end of the beam is $\frac{3}{8}$ in. from the face of the column, the length of the bearing zone, parallel to the length of the beam, is

$$z_1 = c - 0.38 = 0.93 - 0.38 = 0.55 \text{ in.}$$

At right angles to the length of an I-beam, the width of zone in which the bearing may be considered as approximately uniform may be found with sufficient exactness for a beam with circular or parabolic fillets from the expression

$$z_2 = 1.4\,k \tag{2}$$

in which k = vertical distance between the junction of the beam fillet with the beam web and the under side of the beam flange. For the 12-in., 32-lb. W.F., $k = 0.88$, and hence $z_2 = 1.23$ in.

The bearing stress under the beam flange is then

$$f_b = \frac{P}{z_1 z_2} = \frac{12{,}000}{0.55 \times 1.23} = 17{,}700 \text{ lb. per sq. in.}$$

This is much below the permissible bearing stress of 24,000 lb. per sq. in.

It has been assumed in the present problem that the vertical compressive stress in the beam web arising from the short bearing length z_1 is not excessive. Unless beams with thick webs, or webs reinforced for compression, are used, the safe reaction of beams resting on brackets without stiffness is comparatively small. Compressive stresses in beam webs are investigated in the chapter on beams.

Stress from Shear.—If the vertical fillet welds be each made the full depth of the 4-in. leg of the angle, the effective length d of each fillet will be $3\frac{1}{2}$ in., and the stress in shear is

$$f_s = \frac{P}{2\,d} = \frac{12{,}000}{2 \times 3.5} = 1715 \text{ lb. per lin. in.}$$

Stress from Moment.—In order to find the moment, the line of action of the 12,000-lb. load must be determined. As the length of the bearing zone is $z_1 = 0.55$ in., the eccentricity of the reaction with respect to the face of the column is

$$e = 0.375 + (\tfrac{1}{2} \times 0.55) = 0.65 \text{ in.}$$

The moment is then

$$M = 12{,}000 \times 0.65 = 7800 \text{ in.-lb.}$$

Now the stress *per square inch* on the welds due to moment is

$$f_m' = \frac{M}{S}$$

in which S = the section modulus of the two welds. If the width of fillet on a short side is a, expressed in inches, the stress *per lineal inch* due to moment is

$$f_m = \frac{Ma}{S} = \frac{Ma}{2 \times \frac{1}{6} ad^2} = \frac{3\,M}{d^2} \tag{3}$$

$$= \frac{3 \times 7800}{(3.5)^2} = 1910 \text{ lb.}$$

Combined Stress.—The combined, or resultant, stress due to shear and moment is

$$f_r = (f_s^2 + f_m^2)^{1/2}$$
$$= (1715^2 + 1910^2)^{1/2} = 2560 \text{ lb. per lin. in.}$$

This is within the safe limit. An angle with a $3\frac{1}{2}$-in. vertical leg would not give sufficient effective length of fillet.

Investigation of the strength of the 8-in. length of shelf angle acting as a beam shows that it is amply strong.

61. Simple Column Bracket with Stiffeners.—Design a simple column bracket with two stiffener angles to carry a load of 40,000 lb. applied at $2\frac{1}{4}$ in. from the face of the column, as shown in Fig. 38. Let the thickness of the column material be $\frac{1}{2}$ in.

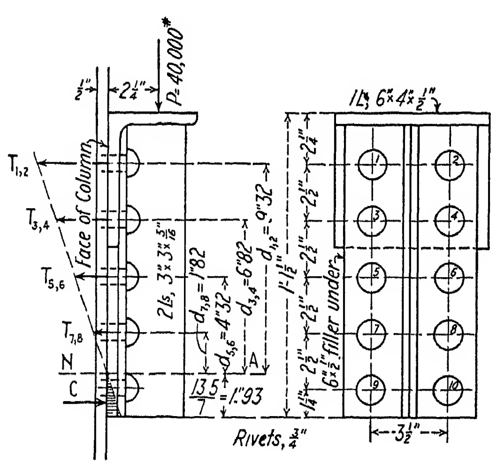

Fig. 38.—Bracket with Stiffeners.

Rivets, $\frac{3}{4}$ in. Permissible stresses on rivets: transverse shear, p_s = 10,000 lb. per sq. in.; bearing, p_b = 20,000 lb. per sq. in.; tension parallel to the rivet axis, p_t = 6000 lb. per sq. in. Neglect the combination of shearing and tensile stresses on the rivets. (See Art. 62 for example of combination of stresses.)

Adopt as bracket material a $6 \times 4 \times \frac{1}{2}$-in. shelf angle and two $3 \times 3 \times \frac{5}{16}$-in. stiffener angles with a $\frac{1}{2}$-in. filler underneath.

Number of rivets necessary solely to transmit the load in single shear = 40,000/4420 = 10, counting to the nearest even number. Assume a spacing of rivets as indicated, giving a total depth of bracket of 13.5 in.

As in the case of the bracket of Art. 59, assume the neutral axis as at $\frac{1}{7}$ of the total depth of the bracket up from the bottom, or $13.5/7 = 1.93$ in.

The total turning moment on the bracket is $M = 40{,}000 \times 2.25 = 90{,}000$ in.-lb.

The rivets in tension will, according to Eq. (1) of Art. 59, h being 13.5 in. Σd being $2\ (1.82 + 4.32 + 6.82 + 9.32) = 44.56$ in., and Σd^2 being $2\ (1.82^2 + 4.32^2 + 6.82^2 + 9.32^2) = 310.7$ in.^{2}, resist a moment of

$$M' = \frac{90{,}000}{1 + \dfrac{2 \times 13.5 \times 44.56}{21 \times 310.7}} = 75{,}900 \text{ in.-lb.}$$

According to Eq. (2) of Art. 59

$$T_1 = T_2 = \frac{75{,}900 \times 9.32}{310.7} = 2280 \text{ lb.}$$

Safe tension on a rivet is $0.442 \times 6000 = 2652$ lb., or more than the existing tension on the extreme rivets.

The low permissible tensile stress of 6000 lb. per sq. in. is chosen to allow for the combination of shearing and tensile stress, which is not made in this example. See Art. 62 for the effect of the combination. Conservative tensile stresses are also necessary because of the uncertainty concerning the value of the rivet heads.

62. Effect of Combined Shear and Tension on Bracket Rivets.—In the problem of Art. 61 let it be assumed that the maximum permissible tensile stress on a rivet shaft due to the combination of shearing and tensile stress is $p_t = 13{,}000$ lb. per sq. in. Find the safe capacity of the bracket.

For the 40,000-lb. load the existing transverse shearing stress on the rivets is $f_s = 40{,}000/10 \times 0.442 = 9040$ lb. per sq. in. The existing axial tensile force on rivets 1 and 2 has been found to be 2280 lb., and the tensile stress is $f_t = 2280/0.442 = 5150$ lb. per sq. in.

Now the maximum tensile stress due to the combination of shearing and tensile stresses is, from Eq. (24) of Art. 72,

$$\begin{aligned} f_{t\,\text{max.}} &= \tfrac{1}{2} f_t + (f_s^2 + \tfrac{1}{4} f_t^2)^{1/2} \\ &= \tfrac{1}{2} \times 5150 + (9040^2 + \tfrac{1}{4} \times 5150^2)^{1/2} = 11{,}960 \text{ lb. per sq. in.} \end{aligned}$$

As the limiting safe stress is 13,000 lb. per sq. in., the bracket is underloaded. Its safe capacity would be $40{,}000 \times 13{,}000/11{,}960 = 43{,}500$ lb.

The increase of shearing stress due to the combination with it of tensile stress is relatively very much smaller than the increase of tensile stress due to the combination with it of shearing stress, and hence it has not been necessary to consider the former case.

63. Crane Runway Column Bracket.—Design a crane runway column bracket for a beam reaction of 24,000 lb. applied 9 in. from the face of the column, as shown in Fig. 39. Column material $\frac{3}{8}$ in. thick.

Rivets, $\frac{3}{4}$ in. diameter. Permissible stresses on rivets; $p_s = 10{,}000$, $p_b = 20{,}000$ and $p_t = 6000$ lb. per sq. in. Assume that the applied load does not bear directly on the upper edge of the web plate and that the vertical edge of the plate does not bear against the side of the column.

Bracket Material.—To ensure lateral stiffness and to give reasonable bearing values for rivets through it, assume the web plate as $\frac{3}{8}$ in. The size of this plate will be determined by the necessary length of the horizontal seat angles to accommodate the beam and the depth of the vertical angles required to give an adequate number of rivets for connection to the column. The seat angles will be $6 \times 3\frac{1}{2} \times \frac{3}{8}$ in., with 6-in. legs vertical, so as to provide space for the necessary number of rivets through the web plate. For the vertical connection angles, use two $4 \times 3\frac{1}{2} \times \frac{3}{8}$-in. angles, with 4-in. legs against the column to suit the material of the column flange. If the ratio of the length of the unsupported outer edge of the web plate to its thickness is over 35, for a bracket of this type, stiffener angles should be used.

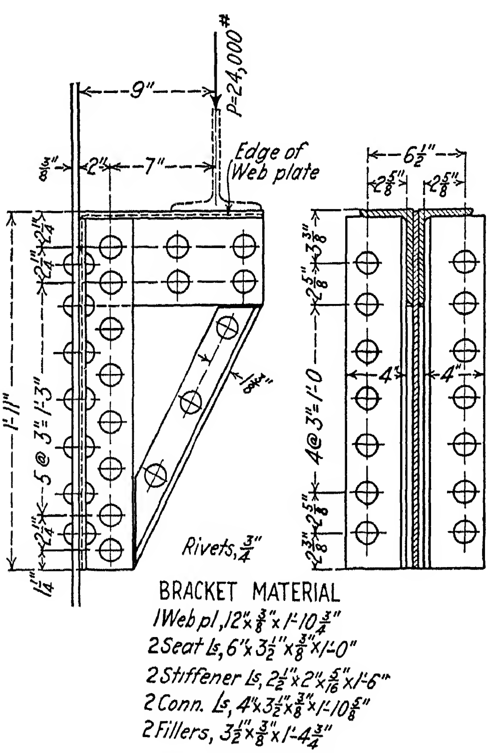

FIG. 39.—Crane Runway Column Bracket.

Riveting of Seat Angles.—Neglecting the bearing of the load on the horizontal upper edge of the web plate, enough rivets must be put through the seat angles to transfer 24,000 lb. to the $\frac{3}{8}$-in. web. Since the safe resistance of a rivet (bearing value) $= 0.75 \times 0.375 \times 20{,}000 = 5630$ lb., this necessitates $24{,}000/5630 = 4.3$ rivets. Put 6 rivets through the seat angles, 4 of them outside the vertical connection angles. Part of the load will be transferred by the seat angles to the vertical angles through bearing of the seat angles on them and by the two rivets directly connecting the seat angles to the vertical angles. The spacing of the rivets in the seat angles, and the length of the latter, are shown in Fig. 39.

Riveting of Vertical Angles to Web Plate.—Assume 8 rivets through the vertical angles and the web plate, as shown in Fig. 39. This line of rivets must resist 24,000 lb. applied parallel to it and at a distance of 7 in., as shown in

Fig. 40(*a*), that is a direct force of 24,000 lb. and a moment of 24,000 × 7 = 168,000 in.-lb.

Direct force D_n on any rivet, number "n," =24,000/8 = 3000 lb.

Turning force on extreme rivet 1 (or 8), distant d_1 from the centre of gravity G of the group, from Eq. (2) of Art. 59, is

$$T_1 = \frac{Md_1}{\Sigma d^2} = \frac{168{,}000 \times 9.75}{2(1.5^2 + 4.5^2 + 7.5^2 + 9.75^2)} = 4720 \text{ lb.}$$

Combining the vertical force of 3000 lb. with the horizontal force of 4720 lb., the resultant force of rivet 1 (or on 8) is found to be $\{(3000)^2 + (4720)^2\}^{1/2}$ =5600 lb. As the safe stress in a rivet in this situation is 5630 lb., the extreme rivets are safe and eight rivets are sufficient.

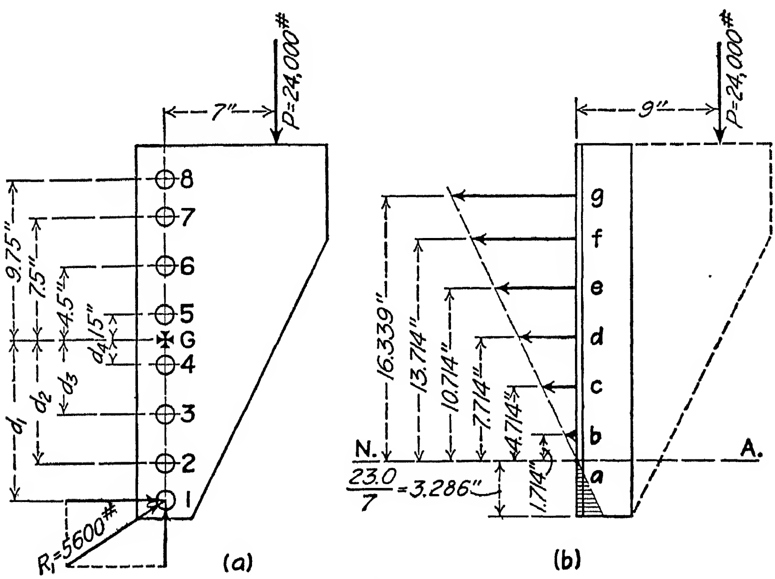

FIG. 40.—Turning Forces on Rivets.

Riveting of Vertical Angles to Column.—The rivets connecting the bracket to the column must, first, resist a vertical downward shear of 24,000 lb. If seven pairs of rivets are assumed, staggering with the rivets through the web plate, the direct shearing stress on each rivet is 24,000/14 = 1715 lb. Since the safe resistance of a rivet in this situation is its single shearing value, or 0.442 × 10,000 = 4420 lb., the 14 rivets are adequate for the shearing effect.

The total turning moment on the bracket is 24,000 × 9 = 216,000 in.-lb.

As in the case of the bracket of Art. 59, assume the neutral axis as at $\frac{1}{7}$ of the total depth of the bracket up from the bottom, that is, $\frac{23}{7}$ = 3.29 in.

The rivets in tension will, applying Eq. (1) of Art. 59, h being 23 in., Σd being 2(1.714 + 4.714 + 7.714 + 10.714 + 13.714 + 16.339) = 109.818, and

Σd^2 being $2(1.714^2 + 4.714^2 + 7.714^2 + 10.714^2 + 13.714^2 + 16.339^2) = 1310.3$, resist a moment of

$$M' = \frac{216{,}000}{1 + \dfrac{2 \times 23 \times 109.818}{21 \times 1310.3}} = 182{,}500 \text{ in.-lb.}$$

Applying Eq. (2) of Art. 59, the tension on each of the upper pair of rivets marked "g" will be

$$T_g = \frac{182{,}500 \times 16.339}{1310.3} = 2270 \text{ lb.}$$

As the safe tensile force on one rivet is $0.442 \times 6000 = 2652$ lb., the 14 rivets are adequate.

Stiffener Angles.—Length of unsupported outer edge of web plate = 19 in., approximately. Ratio of unsupported length to thickness = 19/0.375 = 51. Hence, stiffeners are necessary as this exceeds 35. Use two angles, $2\frac{1}{2} \times 2 \times \frac{5}{16}$-in., with 3 rivets, as shown in Fig. 39.

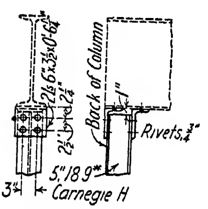

Fig. 41.—Column Cap with Flush Back.

64. Design of Column Cap with Flush Back.—A 5-in. 18.9-lb. Carnegie H-column used in a wall is to support the end of an 18-in., 54.7-lb. I delivering a load of 35,000 lb. to the top of the column with the web of the beam directly over and parallel to the web of the column, as shown in Fig. 41. The end of the beam is not to project more than $\frac{1}{2}$ in. past the outer flange of the column. Design a column cap.

Rivets, $\frac{3}{4}$ in. Permissible shear on shop rivets, $p_s = 12{,}000$ lb. per sq. in. Permissible bearing, $p_b = 24{,}000$ lb. per sq. in.

A very simple and economical cap, based on the standards of the Dominion Bridge Co., Ltd., consists of two angles arranged as shown in Fig. 41. For a gauge of 3 in. in the flanges of the column, the grip is $\frac{3}{8}$ in. If the thickness of the angles be assumed as $\frac{3}{8}$ in., the rivets will be in single shear and in bearing on $\frac{3}{8}$-in. metal. The safe resistance is the single shearing value, or $0.442 \times 12{,}000 = 5300$ lb.

Number of rivets required to connect cap angles to column flanges $= 35{,}000/5300 = 7$.

The least practicable number is 8, and they will be arranged as shown in Fig. 41.

As in the case of the bracket of Art. 59, the tension on the shafts of any rivets in either one of the cap angles is much less than the safe tensile strength of a rivet shaft computed by a representative specification, as for example, $p_t = 8000$ lb. per sq. in.

Four field rivets or bolts will be used through the flange of the supported beam into the cap angles.

65. Design of Symmetrical Cap for Double Beam Girders.—An 8-in., 35-lb. W.F.-column carries the ends of two girders each consisting of two 18-in.,

54.7-lb. I-beams, the girder webs being parallel to the column web and 9 in. centre to centre, as shown in Fig. 42. The reaction from each girder is 60,000 lb. Design a cap for the column.

Rivets, $\frac{3}{4}$ in. p_s = 12,000 lb. per sq. in. p_b = 24,000 lb. per sq. in.

As the girder flanges bear on the column section only on the outer $2\frac{1}{2}$ in. of each column flange, and an unstiffened cap plate alone could not distribute the girder reactions effectively over the column area, a pair of side plates will be employed on the flanges to transfer most of the load thereto.

The required thickness of the side plates may be taken as such that the bearing value of each beam flange on the column flange and on the side plate equals the beam reaction. This is too severe by the amount that the vertical leg of the side angle involved can transfer load to the side plate.

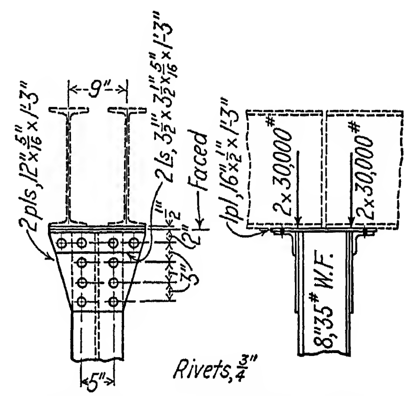

FIG. 42.—Cap for Double Beam Girders.

Required bearing area for one beam = 30,000/24,000 = 1.25 sq. in.

To increase the effective bearing width of each beam flange, a $\frac{1}{2}$-in. cap plate is used and this at the same time ties the side plates together from side to side of the column. If k = vertical distance between the junction of the beam fillet with the web and the underside of the cap plate, and t = web thickness, the effective bearing width of a standard beam flange is approximately

$$b' = 1.4\,k + 0.5\,t \tag{1}$$

For an 18-in., 54.7-lb. I bearing on a $\frac{1}{2}$-in. cap plate, b' under the cap plate is $1.4 \times 1.875 + 0.46/2 = 2.86$ in. This zone b' overlaps the flange by 0.93 in., and as the thickness of the latter near the end is about $\frac{1}{2}$ in., the flange affords $0.93 \times 0.5 = 0.47$ sq. in. of bearing area. The remainder, or $1.25 - 0.47 = 0.87$ sq. in., must be supplied by the side plate. As the bearing zone is 2.86 in. long, the plate thickness must be at least $0.78/2.86 = 0.27$ in. Since $\frac{5}{16}$ in. is usually the minimum thickness employed in such details, this will be used.

Side Plate Attachment.—The riveting of a side plate to the column flange will be made sufficient to transmit a girder reaction thereto less the amount taken in direct bearing on the column flange, that is for $60{,}000 - 2 \times 0.93 \times 0.5 \times 24{,}000 = 37{,}700$ lb. The safe resistance of a $\frac{3}{4}$-in. rivet in this situation is its single shearing value, or 5300 lb., and hence $37{,}700/5300 = 7.1$ rivets are needed. Eight is the nearest practicable number. The size of the plate will be 12×15 in.

Side angles, $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{5}{16}$ in., are employed to attach the cap plate and girders to the column and to assist to some extent in the transfer of the load to

the column shaft. Rivets in the outer ends of the angles are needed to secure them to the side plates, as shown in Fig. 42.

66. Pin Connection for Compression Member.—A compression member consists of two 12-in., 25-lb., channels 11 in. back to back, with flanges turned in, as shown in Fig. 43. It bears on a $4\frac{1}{2}$-in. pin through the webs. According to the governing specification it is to be connected for an axial stress equal to the gross sectional area in square inches multiplied by 12,500 lb. per sq. in. Design the end connection.

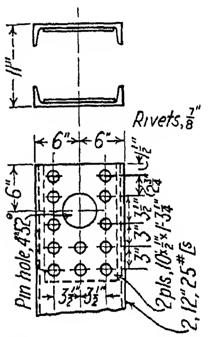

FIG. 43.—Pin Connection.

Rivets, $\frac{7}{8}$ in. Permissible shearing stress on pins and shop rivets, p_s = 12,000 lb. per sq. in. Permissible bearing stress on pins and shop rivets, p_b = 24,000 lb. per sq. in.

It will be assumed that the $4\frac{1}{2}$-in. pin is itself adequate.

As the gross area of the member is $2 \times 7.32 = 14.64$ sq. in., the connection must be designed for a load of $14.64 \times 12{,}500 = 183{,}000$ lb. Each channel must, therefore, transmit 91,500 lb.

The bearing thickness on the pin required for each channel $= 91{,}500/4.5 \times 24{,}000 = 0.85$ in. As the web is 0.39 in. thick, a pin plate $0.85 - 0.39 = 0.46$ in., or say $\frac{1}{2}$ in. thick, is required on the channel web. This plate will be made 10 in. wide, the maximum practicable width for a plate on the inside of the web, where it is best to place such reinforcement.

The stress taken from the pin by this plate is in proportion to its thickness, 0.5 in. As the web is 0.39 in. thick, the pin plate will take $91{,}500 \times 0.50/0.89 = 51{,}400$ lb.

Rivets connecting this plate to the web have as their least resistance their single shear value, or 7220 lb. Hence $51{,}400/7220 = 8$ rivets are needed for each channel.

As the force applied by the pin is resisted below it, these rivets are located on this side of the pin. Four rivets are used back of, or above, the pin merely to bind the plates together.

The member may be cut off at a distance back of the pin equal to one-half the maximum width of the member, or 6 in. The length of the pin plate depends on the pitch of the rivets.

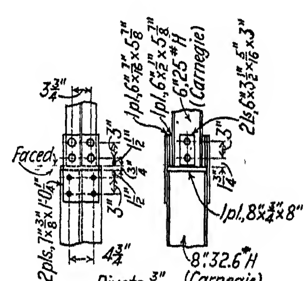

FIG. 44.—Bearing Splice for Dissimilar Columns.

67. Design of Bearing Splice for Dissimilar Columns.—An upper section of a column, consisting of a 6-in., 25.0-lb. Carnegie H rests centrically on a lower section consisting of an 8-in., 32.6-lb. Carnegie H, as in shown Fig. 44. The axial load in the upper section of the column is 60,000 lb., and there is no bending moment. Design a splice.

Rivets, $\frac{3}{4}$ in. For shop rivets, p_s = 12,000 and p_b = 24,000 lb. per sq. in. For field rivets, p_s = 10,000 and p_b = 20,000 lb. per sq. in.

As the flanges of the upper section do not coincide with the flanges of the lower section, a heavy cap plate, say $\frac{3}{4}$ in., is desirable to distribute the load as much as possible over the lower section.

Direct bearing of the upper section on the lower occurs over the full width of the web and for a length thereof of about $6 + 2 \times 0.7 \times t$, where t = thickness of the cap plate, that is 0.75 in.

This length is 7.05 in., and the bearing area on the web of the 8-in. H is $7.05 \times 0.313 = 2.21$ sq. in. At $p_b = 24{,}000$ lb. per sq. in. this would transmit safely a pressure of $2.21 \times 24{,}000 = 53{,}000$ lb., leaving only $60{,}000 - 53{,}000 = 7000$ lb. to be transmitted otherwise to the lower section.

It is not desirable to allow such a large proportion of the total column load to be transmitted to the lower section through the web. Two fillers (one $\frac{1}{2}$ in. and the other $\frac{7}{16}$ in., to allow for build-up) will be placed on each flange, with their lower ends bearing on the cap plate. If 4 shop rivets be placed through them into each flange, the fillers will transmit bearing pressure to the limit of strength of the rivets, or $8 \times 5300 = 42{,}400$ lb. The web would thus need to transmit in direct bearing only $60{,}000 - 42{,}400 = 17{,}600$ lb. The strength is, therefore, ample, not counting the uncertain element of beam action in the cap plate for distributing load from the flanges of the upper section to those of the lower one.

Splice plates on the flanges will be provided, $\frac{3}{8}$ in. thick, to keep the columns rigidly in line. Four field rivets through each flange of the lower section are shown in Fig. 44.

Two angles, $6 \times 3\frac{1}{2} \times \frac{5}{16}$ in. are used to shop rivet the cap plate to the upper section of the column.

68. Design of Shear and Moment Splice for Similar Columns.—Above a column splice the section consists of a 12-in., 72-lb. W.F. and below the splice of a 12-in., 92-lb. W.F., their inside profiles coinciding. At the splice there exists a transverse shear of 10,000 lb. and a bending moment of 65,000 ft.-lb. in the plane of the column web. Design a splice for shear and moment, assuming that the centric axial load is transferred completely by end bearing of the upper section on the lower.

Rivets, $\frac{3}{4}$ in. For shop rivets, $p_s = 12{,}000$ and $p_b = 24{,}000$ lb. per sq. in. For field rivets, $p_s = 10{,}000$ and $p_b = 20{,}000$ lb. per sq. in. Permissible shear and tension on plates, $p_s = 10{,}000$ lb. per sq. in. of gross area, and $p_t = 16{,}000$ lb. per sq. in. of net area, respectively.

If the shear is resisted wholly by the two splice plates on the web, Fig. 45, their required gross area is $10{,}000/10{,}000 = 1.0$ sq. in. Two $\frac{5}{16}$-in. plates, $7\frac{1}{2}$ in. long in a direction transverse to the column are indicated in Fig. 45, the area being $2 \times 7.5 \times 0.3125 = 4.69$ sq. in.

Two $\frac{3}{4}$-in. field rivets on each side of the splice have a bearing value on the 0.43-in. web of the upper section of $2 \times 0.75 \times 0.43 \times 20{,}000 = 12{,}900$ lb., or more than the total transverse shear.

Assume that the bending is resisted entirely by the flange splice plates. If the centre to centre distance apart of the plates on the opposite flanges is ten-

tatively assumed as 12.5 in., the stress in each plate is $M/12.5 = 65{,}000 \times 12/12.5 = 62{,}400$ lb.

Net area required in one plate $= 62{,}400/16{,}000 = 3.90$ sq. in.

A $12 \times \frac{7}{16}$-in. plate will give, allowing for 2 holes out (see Fig. 1(b)) a net area of $12 \times 0.4375 - 2 \times 0.875 \times 0.4375 = 4.48$ sq. in.

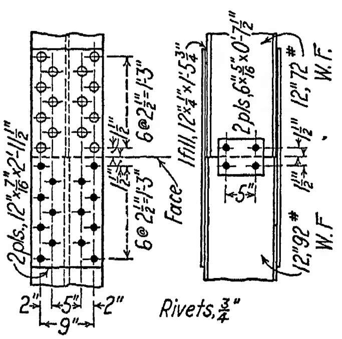

FIG. 45.—Shear and Moment Splice for Similar Columns.

As symmetrical riveting of the splice plates above and below the splice is desirable, the strength of the riveting will be based on the field rivets below the splice. In each plate $62{,}400/4420 = 14$ field rivets are required. Sixteen are shown, and the same number of shop rivets is shown above the splice.

The difference in actual depths of the column sections is $12\frac{5}{8} - 12\frac{1}{4} = \frac{3}{8}$ in. Hence for proper clearance in erection, each filler should be $\frac{1}{2} \times \frac{3}{8} + \frac{1}{16} = \frac{1}{4}$ in.

69. Design of Latticing for Column with Centric Axial Load.—A latticed column composed of two 12-in., 35-lb. channels with flanges turned out, as shown in Fig. 46, carries a centric axial load of 265,000 lb. The channels are $8\frac{1}{2}$ in. apart back to back. Proportion the lattice bars, using (1) single latticing at an angle of 60 deg. with the axis of the member, and (2) double latticing at 45 deg. with axis. The latticing must resist a maximum transverse shear of 3% of the axial load in the member, considered as equally divided amongst the bars cut by any right sectional plane.

Rivets, $\frac{3}{4}$ in. Minimum width of lattice bars, $2\frac{1}{2}$ in. Minimum thickness of bars $\frac{1}{40}$ of distance between end rivets for single latticing, and $\frac{1}{60}$ of this distance for double latticing, but in no case less than $\frac{5}{16}$ in. $p_c = 15{,}000 - 50\, l/r$ lb. per sq. in. $p_t = 16{,}000$ lb. per sq. in.

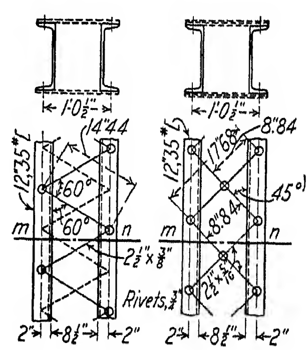

FIG. 46.—Design of Column Latticing.

Maximum transverse shear $= 0.03 \times 265{,}000 = 7950$ lb.

Single Latticing.—As two bars are cut by any right section plane $m - n$, Fig. 46(a), as the length of a bar between end connections is 14.44 in., and the distance between gauge lines in the channel flanges is 12.5 in., the stress in a bar is

$$\pm P = \frac{7950 \times 14.44}{2 \times 12.5} = \pm 4590 \text{ lb.}$$

Required area in tension

$$A = 4590/16{,}000 = 0.29 \text{ sq. in.}$$

The minimum width of bar is $2\frac{1}{2}$ in. and the minimum thickness allowable is 14.44/40 $\times$ 0.361, or say $\frac{3}{8}$ in. A $2\frac{1}{2} \times \frac{3}{8}$-in. bar gives a net area of (2.5 $-$0.875) $\times$ 0.375 $\times$ 0.61 sq. in., or over twice that required.

The compression bar will be assumed, by reason of a certain unknown fixity at its ends, as having an effective length of $\frac{3}{4}$ the length centre to centre of rivets, or 0.75 $\times$ 14.44 = 10.83 in. Assuming a bar $\frac{3}{8}$ in. thick, and considering buckling in the direction of the bar thickness

$$r = 0.288\,t = 0.288 \times 0.375 = 0.108 \text{ in.}$$

Hence

$$p_c = 15{,}000 - 50 \times 1083/0.108 = 9980 \text{ lb. per sq. in.}$$

Required compression area

$$A = 4590/9980 = 0.46 \text{ sq. in.}$$

For compressive stress the area provided is 2.5 $\times$ 0.375 = 0.94 sq. in., or much more than required.

Double Latticing.—Four bars are cut by any section plane $m - n$, Fig. 46(b). The distance between gauge lines is 12.5 in. and the distance between end rivets in a bar is 17.68 in. Hence stress in a bar is

$$\pm P = \frac{7950 \times 17.68}{4 \times 12.5} = \pm\, 2810 \text{ lb.}$$

Required tension area

$$A = 2810/16{,}000 = 0.18 \text{ sq. in.}$$

The minimum thickness of bar allowed is 17.68/60 = 0.29 in., or $\frac{5}{16}$ in. A $2\frac{1}{2} \times \frac{5}{16}$-in. bar gives a net area of (2.5 $-$ 0.875) $\times$ 0.3125 = 0.51 sq. in.

The effective length of a compression bar will, by reason of the partial fixity at the ends and a certain measure of support at the centre from the intersecting tension bar, be assumed as being $\frac{1}{2}$ the length centre to centre of end rivets, or 0.5 $\times$ 17.68 = 8.84 in. Assuming a $\frac{5}{16}$-in. bar, r = 0.288 $\times$ 0.3125 =0.09 in. Hence,

$$p_c = 15{,}000 - 50 \times 8.84/0.09 = 10{,}100 \text{ lb. per sq. in.}$$

Required compressive area

$$A = 2810/10{,}100 = 0.28 \text{ sq. in.}$$

The area provided by a $2\frac{1}{2} \times \frac{5}{16}$-in. bar is 2.5 $\times$ 0.3125 = 0.78 sq. in.

Computation of the weight of the latticing per foot of column will show that double latticing, Fig. 46(b) is considerably heavier than single latticing for the same duty. Double latticing is generally used only where the distance between gauge lines in opposite flanges is at least 15 in.

70. Single Latticing for Eccentrically Loaded Column.—Recommend a system of 60-deg. single latticing for the 4-angle column designed in Art. 48 and

loaded as shown in Fig. 27. The latticing is to resist both a transverse end shear of 3% of the total axial load and the shear incident to the moment of eccentricity.

Rivets, $\frac{3}{4}$ in. Minimum width of lattice bars, $2\frac{1}{2}$ in. Minimum thickness of bars $\frac{1}{40}$ of distance between end rivets, but in no case less than $\frac{5}{16}$ in. $p_c = 15{,}000 - 50\ l/r$, and $p_t = 16{,}000$ lb. per sq. in.

End shear for which latticing would be designed if there were no eccentricity $= 0.03 \times 72{,}000 = 2160$ lb.

Moment of eccentricity due to the resultant being 1.19 in. off centre (see Art. 48) is $M = 72{,}000 \times 1.19 = 85{,}700$ in.-lb. End shear due to a moment of eccentricity of an axial loading $= M/0.5\,l$, where $l =$ length of column. It is, therefore, $85{,}700/75 = 1145$ lb.

Total end shear $= 2160 + 1145 = 3305$ lb.

Distance between gauge lines in flange angles $= 7$ in. and length of bar between end connections $= 8.08$ in.

Stress in a lattice bar, there being only one plane of latticing, is

$$\pm P = 3305 \times 8.09/7 = 3820 \text{ lb.}$$

Required area in tension $= 3820/16{,}000 = 0.24$ sq. in.

As the minimum thickness of bar, according to the rule respecting the ratio of thickness to length, is $8.09/60 = 0.135$ in., the limit of $\frac{5}{16}$ in. will apply. For a $2\frac{1}{2} \times \frac{5}{16}$ in. bar the net area is $(2.5 - 0.875) \times 0.3125 = 0.51$ sq. in., or more than required.

For compression, the effective length will be assumed as $0.75 \times 8.09 = 6.07$ in. Assuming a thickness of $\frac{5}{16}$ in., $r = 0.288 \times 0.3125 = 0.09$ in. Hence $p_c = 15{,}000 - 50 \times 6.07/0.09 = 11{,}630$ lb. per sq. in.

Required compression area, $A = 3820/11{,}630 = 0.33$ sq. in.

Provided gross area of bar $= 2.5 \times 0.3125 = 0.78$ sq. in., or much more than required.

71. Exercise Problems on Compression Member Details.—The following exercise problems are based on the principles employed in the solution of the problems in this chapter. See Appendix I for the answers.

(1) A compression member in a truss is 9 ft. 2 in. long centre to centre of end connections and is composed of two angles $3\frac{1}{2} \times 3 \times \frac{5}{16}$ in. with the two $3\frac{1}{2}$-in. legs adjacent and $\frac{3}{8}$ in. apart. Assuming the permissible stress in pounds per square inch to be $p_c = 16{,}000 - 70\ l/r$, find the number of $\frac{3}{4}$-in. rivets required for an end connection of sufficient strength to develop the full capacity of the member, the connection to be made to a $\frac{3}{8}$-in. gusset plate. $p_s = 11{,}000$ lb. per sq. in. $p_b = 22{,}000$ lb. per sq. in.

(2) Find the maximum tension in the rivets of a $6 \times 4 \times \frac{5}{8}$-in. bracket angle attached by the 6-in. leg to a column and containing four rivets on gauge lines $2\frac{1}{2}$ and $4\frac{3}{4}$ in. from the back of the outstanding leg. The load on the bracket is 13,600 lb. and it is applied $1\frac{1}{8}$ in. from the face of the column. Assume turning to be about a point $\frac{1}{21}$ of the depth of the bracket up from the toe.

(3) A column bracket is to consist of a $6 \times 4 \times \frac{1}{2}$-in. shelf angle connected to a $\frac{1}{2}$-in. column flange by four $\frac{3}{4}$-in. rivets in the 6-in. leg driven in two rows on gauges of 2 in. and $2\frac{1}{2}$ in. Find the maximum safe reaction applied at 1 in. from the face of the column, assuming the turning to be about the toe of the angle. Allowable shearing, bearing and tensile stresses on rivets = 10,000, 20,000 and 6000 lb. per sq. in., respectively.

(4) A column bracket consists of a single shelf angle, $6 \times 4 \times \frac{5}{8}$ in. with the 4-in. leg outstanding. The 6-in. leg is connected to the column by two horizontal lines of rivets $2\frac{1}{4}$ and $4\frac{1}{2}$ in., respectively, from the back of the outstanding leg of the shelf angle, each line containing two $\frac{3}{4}$-in. rivets. A load of 18,000 lb. is applied at 1 in. from the back of the shelf angle. Find the greatest existing tension on a rivet, and state whether this tension is safe. Assume turning to be about the toe of the angle. $p_t = 6000$ lb. per sq. in.

(5) In the case of the bracket of Exercise Problem 2, compute the maximum tensile stress on one of the upper rivets due to the combination of axial tension and shear. Rivets, $\frac{3}{4}$ in.

(6) In a 20-ft. panel of a bridge truss the top chord consists of two 10-in., 20-lb. channels, $6\frac{1}{2}$ in. back to back, with the flanges turned out and a $12 \times \frac{5}{16}$-in. cover plate. Single latticing at 60 deg. with the axis of the member is to be used on the bottom flange. Design the latticing. Rivets, $\frac{3}{4}$ in. p_c for chord = 11,000 lb. per sq. in. Lattice bars must not be smaller than $2\frac{1}{4} \times \frac{1}{4}$ in., and the thickness must not be less than $\frac{1}{40}$ of the distance centre to centre of end connections. p_c for lattice bars $= 16,000 - 70\ l/r$, l being $\frac{3}{4}$ the distance centre to centre of end connections. p_t for bars = 16,000 lb. per sq. in. Latticing is to be designed for one-half the transverse shear which is to be taken as $Pl/8000\ x_e$, where P = capacity of chord, l = length of chord segment in inches, and x_e = distance from vertical gravity axis to extreme fibre.

CHAPTER V

BEAMS

72. Fundamental Formulae for Beams.—The following formulae are applicable to the design of steel beams.

Bending moment for simply supported and restrained beams

$$M = \frac{Wl}{C} = \frac{wl^2}{C} \tag{1}$$

Equation of three moments for beam containing an infinite series of equal, uniformly loaded spans

$$M_1 + 4\,M_2 + M_3 = -\frac{wl^2}{2} \tag{2}$$

For strength in symmetrical bending

$$\frac{f}{y} = \frac{f_e}{y_e} = \frac{M}{I} \tag{3}$$

$$f_f = f_e = \frac{My_e}{I} = \frac{M}{S} \tag{3a}$$

$$M = M_r = \frac{Ip_f}{y_e} = Sp_f \tag{4}$$

$$W = C \cdot \frac{Sp_f}{l} \tag{5}$$

$$S = \frac{M}{p_f} \tag{6}$$

$$A = \frac{My_e}{r^2 p_f} \tag{7}$$

For strength in unsymmetrical bending, that is in flexure not operating in the plane of one of the principal axes

$$f_e = \frac{M}{S'} \tag{8}$$

$$S' = \frac{I_x I_y}{I_y y \sin\theta + I_x x \cos\theta} \tag{9}$$

For shearing strength

$$f_s = \frac{QV}{It} \tag{10}$$

$$f_{as} = \frac{V}{dt} \tag{11}$$

$$A_w = dt = \frac{V}{p_s} \tag{12}$$

For diagonal compressive strength of web

$$f_{dc} = f_{as} = \frac{V}{dt} \tag{13}$$

$$A_w = dt = \frac{V}{p_{dc}} \tag{14}$$

For vertical compressive strength of web

$$f_{vc} = \frac{P}{(a + jd)t} \tag{15}$$

$$t = \frac{P}{(a + jd)p_{vc}} \tag{16}$$

Deflection of beams of uniform section

$$\Delta = K\frac{Wl^3}{EI} = K\frac{wl^4}{EI} \tag{17}$$

$$\Delta = \frac{Nf_eL^2}{E}\cdot\frac{1}{2y_e} \tag{18}$$

Deflection of beams of variable section

$$\Delta = \frac{1}{E}\Sigma\frac{M}{I}m\cdot dx \tag{19}$$

Shift of neutral axis of a section resulting from perforation

$$s = \frac{Q_h}{A_n} \tag{20}$$

Net moment of inertia of perforated section

$$I_n = I_g - I_h - A_ns^2 \tag{21}$$

Approximate uncorrected net moment of inertia of perforated section

$$I_n' = I_g - I_h \tag{22}$$

Correction factor to be applied to I_n'

$$F = 1.0 - \frac{0.25}{d^{1/2}} \tag{23}$$

Maximum combined stresses at a selected point.

$$f_{n\ \max.} = \tfrac{1}{2}f_f + (f_s^2 + \tfrac{1}{4}f_f^2)^{1/2} \tag{24}$$

$$f_{s\ \max.} = (f_s^2 + \tfrac{1}{4}f_f^2)^{1/2} \tag{25}$$

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The significance of the symbols employed in the above formulae, and elsewhere in this chapter, is as follows:

A = effective area required for the moment M;
A_n = net area of a section;
A_w = dt = area of the web of a beam;
a = bearing length of concentrated load on beam; length of a support, parallel to beam axis;
b = breadth of beam flange;
C = numerical coefficient in the general formula for bending moment in a beam. For simply supported beams its values are: With a single concentrated load at the centre, 4; with equal concentrated loads at the third-points, 6; with uniformly distributed load, 8. For beams restrained at one end and simply supported at the other, carrying a uniformly distributed load: At the restrained end, 8; at the point of maximum positive moment 128/9. For beams restrained at both ends uniformly loaded: At the restrained ends, 12; at the centre, 24.
c = clear distance between opposite fillets of a beam or channel;
Δ = maximum deflection of a beam;
d = out to out depth of a beam;
dx = length of any short segment of a beam;
E = modulus of elasticity of the material;
F = correction factor to be applied to the approximate net moment of inertia I_n' to compensate for the error in neglecting the shift of the neutral axis;
f = fibre stress, due to flexure, at a point distant y from the neutral axis of a beam;
f_e = fibre stress, due to flexure, at an extreme fibre, distant y_e from the neutral axis of a beam;
f_{as} = existing average shearing stress;
f_{dc} = existing diagonal compressive stress in a beam web;
f_f = in general, existing stress in flexure as distinguished from stress in compression or tension;
$f_{n\,\max.}$ = maximum normal stress existing on any plane at a selected point;

f_s = existing shearing stress;

$f_{s\ max.}$ = maximum shearing stress existing on any plane at a selected point;

f_{vc} = existing vertical compressive stress in a beam web;

I = moment of inertia of the cross section about its gravity axis normal to the plane of bending;

I_g = moment of inertia of a gross section about its own gravity axis;

I_h = moment of inertia of the holes in a section about the gravity axis of the gross section;

I_n = moment of inertia of a net section about the gravity axis of the net section;

I_n' = approximate uncorrected net moment of inertia of a perforated section, assuming that the neutral axis is at the centre of gravity of the gross section;

I_x = moment of inertia of the cross section about the x-axis;

I_y = moment of inertia of the cross section about the y-axis;

j = fraction of beam depth to be added to length of concentrated load in computing f_{vc};

K = numerical coefficient in the general formula for deflection of a beam. For simply supported beams its values are: With single concentrated load at the centre, $\frac{1}{48}$; with equal concentrated loads at the third points, 23/1296; with uniformly distributed load 5/384; with load increasing uniformly to centre, $\frac{1}{60}$. For beams restrained at one end and simply supported at the other, carrying a uniformly distributed load, 1/185. For beams wholly restrained at both ends and uniformly loaded, 1/384. For beams with uniform loading, with end moment 1/40 wl^2 and centre moment 1/10 wl^2, 3.8/384. For other than uniformly distributed loading, use the formula $\Delta = K \dfrac{Wl^3}{EI}$.

L = effective span of a beam in feet;

l = effective span of a beam in the same unit of length as is involved in w;

l' = spacing of lateral supports to a beam.

M = bending moment;

M_1, M_2, M_3 = bending moment at supports, 1, 2 and 3 of a continuous beam;

M_r = moment of resistance of a cross section;

m = bending moment at the centre of any segment of a beam due to a load of 1 lb. acting at the point where the deflection is required;

N = numerical coefficient in the formula for maximum deflection of a beam based on extreme fibre stress. For simply supported beams its values are: With single concentrated load at the centre, 24; with equal concentrated loads at the third points, 30.67; with uniformly distributed load, 30; with load increasing uniformly to centre, 28.8.

P = any concentrated load acting on a beam; the reaction at a support;

p_b = permissible stress in bearing;

p_{dc} = permissible stress in diagonal compression in a beam web;

p_f = permissible flexural stress;

p_{vc} = permissible stress in vertical compression in a beam web;
p_s = permissible shearing stress;
Q = statical moment of the area above or below a given point on a beam cross section, taken about the neutral axis;
Q_h = statical moment of holes in a section taken about neutral axis of gross section;
r = radius of gyration of a cross section in the direction of the plane of bending;
S = effective section modulus of a beam subjected to symmetrical bending, required or provided, as the expression may indicate;
S' = effective section modulus of a beam subjected to unsymmetrical bending, required or provided, as the expression may indicate;
s = shift of the neutral axis of a section brought about by perforation;
t = thickness of the web of a beam;
θ = angle which the plane of the moment makes with the x-axis of a cross section subjected to unsymmetrical bending;
V = total vertical shear at a cross section;
W = total load on a beam;
w = load per unit of length on a beam;
x = a distance in the x-direction, generally horizontal;
y = a distance in the y-direction, generally vertical; the distance of any fibre of a beam from the neutral axis;
y_e = the distance of an extreme fibre of a beam from the neutral axis.

73. Design for Moment of a Laterally Supported, Uniformly Loaded Beam. —An I-beam framing into girders 17 ft. 8 in. apart, centre to centre, is to carry a uniformly distributed load of 1250 lb. per lin. ft. in addition to its own weight. It is continuously supported along the top flange by a floor slab. Recommend a size to satisfy moment requirements. p_f = 18,000 lb. per sq. in. Neglect the restraining moment at the supports.

Assuming a standard 12-in., 31.8-lb. I-beam, the total load per lineal foot is 1281.8 lb.

Maximum moment, M, from Eq. (1) of Art. 72 $= \frac{1}{8} \times 1281.8 \times (17.67)^2 \times 12 = 600{,}000$ in.-lb.

$$S = M/p_f = 600{,}000/18{,}000 = 33.4 \text{ in.}^3$$

This may be met with a standard 12-in., 31.8-lb. I having $S = 36.0$, or a 12 in., 28-lb. W.F. with $S = 35.6$. Relative pound prices, availability and requirements respecting web thickness will determine the choice of section.

As a precautionary measure, the shearing and web buckling stresses should be investigated as in Art 79.

Tables of safe capacity of beams may be conveniently used to determine the necessary size.

The total superimposed load is $1250 \times 17.67 = 22{,}100$ lb. Adding 560 lb. to allow for the weight of the beam itself, the total load to be accommodated will be about 22,660 lb. The tables show that, for p_f = 18,000 lb. per sq. in.,

a 12-in., 31.8-lb. I has a total capacity of 24,500 lb. and a 12-in., 28-lb. W.F. one of 24,200 lb.

74. Design for Moment of Laterally Supported Beam with Single Central Load.—If the load applied to the beam of Art. 73 is a single load of 23,000 lb. applied at the centre and the other data are as given in Art. 73, recommend a size.

The uniform load giving the same moment as a 23,000-lb. central load is 46,000 lb. From the capacity tables it is apparent that a 16-in., 45-lb. W.F. with a capacity (by adjustment) of 49,200 lb. would be adequate. There is ample margin in the capacity to allow for the deduction of the weight of the beam, which would be 800 lb.

75. Effect of Restraining Moment of End Connections.—Redesign the beam of Art. 73, assuming that by reason of the restraining moment of the end connections, the maximum positive moment in the beam is reduced by 10%.

Maximum moment M, assuming the weight of the beam as 25 lb. per lin. ft., is $\frac{9}{10} \times \frac{1}{8} \times 1275 \times (17.67)^2 \times 12 = 537{,}000$ in.-lb.

$$S = 537{,}000/18{,}000 = 29.9 \text{ in.}^3$$

The lightest beam meeting this requirement is a 12-in., 25-lb. W.F. with a section modulus of 30.9 in.^{3}

Shearing and web buckling stresses should be investigated, as in Art. 79.

76. Flexural Capacity of Beam Laterally Supported and Loaded at the Third Points.—An 18-in., 47-lb. W.F., of 21-ft. span, supported continuously along the top flange by a slab, is loaded at the third-points by two equal concentrated loads. What may these safely be, so far as flexure is concerned? $p_f =$ 18,000 lb. per sq. in.

From the tables of safe capacity under uniform loading, which are based on $p_f = 18{,}000$ lb. per sq. in., the total safe uniformly distributed load is 47,000 lb. The net uniform loading would then be this less the weight of the beam, or $47{,}000 - 47 \times 21 = 46{,}010$ lb.

Now, since the moment in a beam due to a load W applied in two equal concentrated loads at the third-points is $Wl/6$, while for the same load applied uniformly it is $Wl/8$, the net loading applied as two equal third-point loads would be $46{,}010 \times \frac{6}{8} = 34{,}500$ lb., or 17,250 lb. for each load, approximately.

This loading has no relation to the shearing or web buckling capacity. See Art. 79.

77. Capacity of an I-Beam without Intermediate Lateral Support.—A 10-in., 25.4-lb. I-beam has a span of 10 ft. centre to centre of bearings and is without lateral support at any intermediate point. Find the safe superimposed uniformly distributed load per lineal foot of beam, so far as bending is concerned.

$p_f = 16{,}000 - 150\, l'/b$, where l' = maximum spacing of points of lateral support in the region of the point of maximum moment, and b = breadth of beam flange.

From the tables of safe capacity, the total gross safe load, based on $p_f =$ 18,000 lb. per sq. in. is 29,300 lb.

But $p_f = 16{,}000 - 150 \times 120/4.66 = 12{,}140$ lb. per sq. in.

Hence the total gross safe capacity, taking account of the tendency of the top flange to buckle is $29{,}300 \times 12{,}140/18{,}000 = 19{,}740$ lb.

Deducting the weight of the beam or $10 \times 25.4 = 254$ lb., the total net safe capacity is $19{,}740 - 254 = 19{,}486$ lb., or $19{,}486/10 = 1949$ lb. per lineal ft.

78. Design of I-Beam with Imperfect Lateral Support.—A steel beam of 16 ft. 6 in. span supported laterally at the ends and at the third-points is to carry a concentrated load of 14,000 lb. at each of the third-points, in addition to its own weight. Recommend a size to satisfy flexural requirements only.

$p_f = 19{,}000 - 300\, l'/b$, where l' = maximum spacing of points of lateral support in the region of the point of maximum moment, b = breadth of beam flange, but p_f is not to exceed 16,000 lb. per sq. in.

Since from Eq. (1), Art. 72, the moment in a beam due to a load W applied as two equal concentrated loads at the third-points is $Wl/6$, the tabular safe loads for beams uniformly loaded may be used for beams symmetrically loaded at the third-points by multiplying the tabular loads by $\frac{3}{4}$.

In selecting a beam from the capacity tables, therefore, one with a total net capacity for uniform load of $2 \times 14{,}000 \times \frac{4}{3} = 37{,}300$ lb. ought to be chosen.

Examination of the tables shows that for $p_f = 18{,}000$ lb. per sq. in. a 15-in., 42.9-lb. I has a gross capacity for uniform load of 42,900 lb. But $p_f = 19{,}000 - 300 \times 66/5.5 = 15{,}400$ lb. per sq. in. Hence the gross capacity would be reduced to $42{,}900 \times 15{,}400/18{,}000 = 36{,}700$ lb. and the net capacity would be $36{,}700 - 16.5 \times 42.9 = 35{,}990$ lb. This beam is insufficient.

Assuming a 16-in., 36-lb. W.F., with a flange width of 7.0 in., $l'/b = 66/7.0 = 9.6$ and hence $p_f = 16{,}000$ lb. per sq. in. The gross capacity for uniform load is 40,900 lb. and the net capacity is $40900 - 16.5 \times 36 = 40{,}300$ lb., which is sufficient.

79. Design of Steel Beam for Bending, Shear and Web Buckling.—A steel beam with top flange continuously supported laterally rests on two columns 18 ft. apart centre to centre and carries a uniformly distributed load of 2000 lb. per lin. ft., including the weight of the beam, in addition to a column load of 85,000 lb. applied 1 ft. 8 in. from one support, as shown in Fig. 47. Proportion the beam.

$p_f = 16{,}000$ lb. per sq. in. $p_s = 10{,}000$ lb. per sq. in., considered as uniformly distributed over the gross area dt of the web. Permissible diagonal compressive stress in web, $p_{dc} = 15{,}000 - 150\, c/t$ (c = clear distance between fillets). Permissible vertical compressive stress in web, $p_{vc} = 15{,}000 - 120\, c/t$. Assume j in Eqs. (15) and (16), Art. 72, as 0.125 for end supports, and 0.25 for the interior concentrated load.

Moment—The maximum moment occurs at a concentrated load.

Reaction due to concentrated load $= 85{,}000 \times 16.33/18 = 77{,}100$ lb. Moment $= 77{,}100 \times 20 = 1{,}542{,}000$ in.-lb.

Moment at the column load due to uniform load $= (2000 \times 9) \times 20 - \frac{1}{2} \times 2000 \times (1.67)^2 \times 12 = 326{,}700$ in.-lb.

Total moment = 1,542,000 + 326,700 = 1,868,700 in.-lb.

Required section modulus for beam having adequate lateral support = 1,868,700/16,000 = 116.8 in.[3] One 20-in., 65.4-lb. I with S = 116.95 would be adequate for bending.

Shear.—The total end reaction = 77,100 + (9 × 2000) = 95,100 lb.

Average shearing stress, Eq. (11), Art. 72, = 95,100/(20 × 0.5) = 9510 lb. per sq. in., which is within the specification.

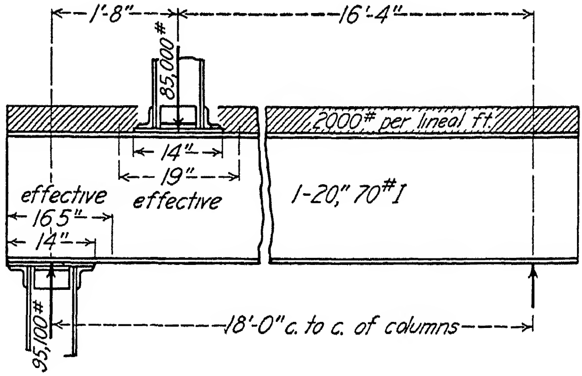


FIG. 47.—Beam Subjected to Heavy Shear and Web Buckling Stresses.

Web Compression.—Since the existing diagonal compressive stress at the support is approximately equal to the average shearing stress, Eq. (13), Art. 72, it is 9510 lb. per sq. in. (For an investigation of combined shearing and flexural stress, see Art. 103.)

Permissible diagonal compressive stress = 15,000 − 150 × 17/0.5 = 9900 lb. per sq. in., the clear distance between fillets being approximately 17 in. and the web thickness 0.5 in.

Existing vertical compressive stress over the assumed 14-in. bearing Eq. (15), Art. 72, = 95,100/(14 + 20/8) × 0.5 = 11,530 lb. per sq. in.

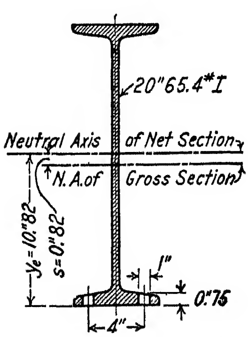


FIG. 48.—Net Section Modulus of Punched Beam.

Permissible vertical compressive stress on web = 15,000 − 120 × 17/0.5 = 10,920 lb. per sq. in. The section is not adequate. End web stiffeners might be used, but it is generally more economical to use a beam with thicker web.

Assuming a 20-in., 70-lb. I, the existing vertical compression stress is 95,100/(14 + 20/8) × 0.567 = 10,150 lb. per sq. in. Permissible stress = 15,000 − 120 × 17/0.567 = 11,400 lb. per sq. in. The revised section is adequate.

Since the concentrated load is less than the reaction, and the length of web effective for the resistance of vertical compression is 14 + 20/4 = 19 in., as compared with 16.5 in. at the support, the web of the adopted beam is sufficient for the web compression under the column load.

80. Net Section Modulus of Punched Beam.—Calculate the net or effective section modulus of a 20-in., 65.4-lb. I with holes for two $\frac{7}{8}$-in. rivets through the tension flange, as shown in Fig. 48. Assume the neutral axis to pass

through the centre of gravity of the net section. What is the percentage reduction of flexural strength?

It is necessary, first, to find the position of the neutral axis of the net section. The distance of the neutral axis of the net section from the neutral axis of the gross section or the "shift," is, from Eq. (20) of Art. 72

$$s = \frac{Q_h}{A_n}$$

Statical moment, Q_h, of two 1-in. holes through $\frac{3}{4}$-in. metal of flange taken about neutral axis of gross section which is 9.625-in. distance from the centre of gravity of the holes is $2 \times 1 \times 0.75 \times 9.625 = 14.44$ in.^{3}

Net area of section, A_n, $= 19.08 - 2 \times 1 \times 0.75 = 17.58$ sq. in.

Hence, $s = 14.44/17.58 = 0.82$ in.

Moment of inertia of net section taken about the centre of gravity of the net section is, from Eq. (21) of Art. 72

$$I_n = I_g - I_h - A_n s^2$$

Inserting numerical values

$$I_n = 1169.5 - 2(1 \times 0.75 \times 9.625^2) - 17.58 \times 0.82^2 = 1018.7 \text{ in.}^4$$

Section modulus of reduced section

$$S_n = \frac{I_n}{d/2 + s} = \frac{1018.7}{10.82} = 94.1 \text{ in.}^3$$

Section modulus of gross sections $S_g = 116.9$ in.^{3}

Ratio of section modulus of gross section to that of net section

$$S_n/S_g = 94.1/116.9 = 0.805.$$

That is, the holes reduce the section modulus by 19.5%, if the neutral axis is at the centre of gravity of the net section.

A rough approximation for the net section modulus, and one sufficiently accurate for some purposes, may be made by considering the neutral axis as at the centre of gravity of the gross section.

The approximate net moment of inertia is then, dropping the last term of Eq. (21) of Art. 72,

$$I_n' = I_g - I_h$$

which becomes

$$I_n' = 1169.5 - 2(1 \times 0.75 \times 9.625^2) = 1030.5 \text{ in.}^4$$

Approximate net section modulus is

$$S_n' = \frac{I_n'}{d/2} = \frac{1030.5}{10} = 103.05 \text{ in.}^3$$

Ratio of section modulus as computed on assumption that neutral axis is at centre of gravity of net section to that found on the assumption that the neutral axis is at the centre of gravity of the gross section

$$S_n/S_n' = 94.1/103.05 = 0.913.$$

81. Correction of Approximate Section Modulus.—For the problem of Art. 80 find the correction which should be applied to the net section modulus as found on the assumption that the neutral axis is at the centre of gravity of the gross area in order to give the most probable value of the true net section modulus.

There is reason to believe that the neutral axis of an unsymmetrically punched beam does not shift from the centre of gravity of the gross section to the centre of gravity of the net section, but that it lies at some intermediate point. As close a determination of the true section modulus of beams with flange holes as it is possible to make may be made by finding the section modulus on the assumption that the neutral axis is at the centre of gravity of the gross section and then multiplying it by the efficiency factor of Eq. (23), Art. 72,

$$F = 1.0 - \frac{0.25}{d^{1/2}}$$

where d = overall depth of the beam or girder in inches.

For the case of Art. 80

$$F = 1.0 - 0.25/(20)^{1/2} = 0.944$$

The most probable value of the true net section modulus is therefore $0.944 \times 103.05 = 97.4$ in.^{3}

82. Reinforcement of Beam for Flexure.—A 15-in., 42.9-lb. I of 20-ft. span centre to centre of bearings is to be reinforced by flange plates so as to carry safely in flexure a total uniformly distributed load of 50,000 lb., including the weight of the beam. Recommend a section for the flange plates. Rivets, $\frac{7}{8}$ in. $p_f = 18{,}000$ lb. per sq. in.

$$\text{Maximum moment in beam} = Wl/8 = 50{,}000 \times 240/8 = 1{,}500{,}000 \text{ in.-lb.}$$
$$S = 1{,}500{,}000/18{,}000 = 83.33 \text{ in.}^3$$

A rough estimate of the amount of section modulus contributed by a $6 \times \frac{1}{2}$-in. plate on each flange, as shown in Fig. 49, indicates that these will probably be sufficient. Assume that they are used and that there are two lines of staggered rivets in each plate.

Moment of inertia of gross section of 15-in., 42.9-lb. I = 441.8 in.^{4}

Moment of inertia of two flange plates $= 2\{1/12 \times 6 \times (0.5)^3 + 6 \times 0.5 \times (7.75)^2\} = 360.5$ in.^{4}

Total gross moment of inertia of reinforced section $= 441.8 + 360.5 = 802.3$ in.^{4}

Moment of inertia of *one* hole through the $\frac{5}{8}$-in. metal of the beam flange and $\frac{1}{2}$-in. flange plate, taken about the neutral axis of the gross section of the reinforced beam $= 1 \times 1.125 \times (7.44)^2 = 62.3$ in.^{4}

Net moment of inertia of section $= 802.3 - 62.3 = 740.0$ in.^{4} and net section modulus $= 740/8 = 92.5$ in.^{3}

As pointed out in Art. 81, this should be corrected by multiplying it by $F = 1.0 - 0.25/(16)^{1/2} = 0.9375$. The corrected or adjusted section modulus, therefore, $= 0.9375 \times 92.5 = 86.7$ in.^{3}

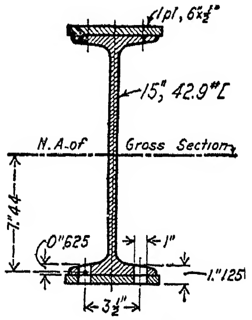


FIG. 49.—Reinforcement for Flexure.

This is somewhat in excess of the requirement, or 83.33 in.^{3}, but $6 \times \frac{7}{16}$-in. plates are insufficient.

83. Length of Reinforcing Plates.—Find the theoretical length required, and recommend a practical length, for the flange plates employed in the reinforced beam of Art. 82.

The required theoretical length of each flange plate (see Appendix II) is

$$x_1 = l\left(\frac{s_1'}{S}\right)^{1/2} \tag{1}$$

in which l = span of beam, centre to centre of bearings; s_1' = required section modulus of the two plates in order to make up the difference between the section modulus of the punched beam at the plate ends and S; and S = total corrected net section modulus required at the centre of the reinforced beam.

Now the corrected net section modulus required at the point of maximum moment is, from Art. 82, $S = 83.3$ in.^{3}

Corrected net section modulus of the beam with two opposite 1-in. holes deducted from tension flange (see Fig. 50), taken with respect to the beam section alone, that is, not as part of a reinforced beam, using the data of Art. 82, is

$$= \left\{1.0 - \frac{0.25}{(15)^{1/2}}\right\}\left\{\frac{441.8 - 2 \times 1 \times 0.625 \times (7.19)^2}{7.5}\right\} = 47.1 \text{ in.}^3$$

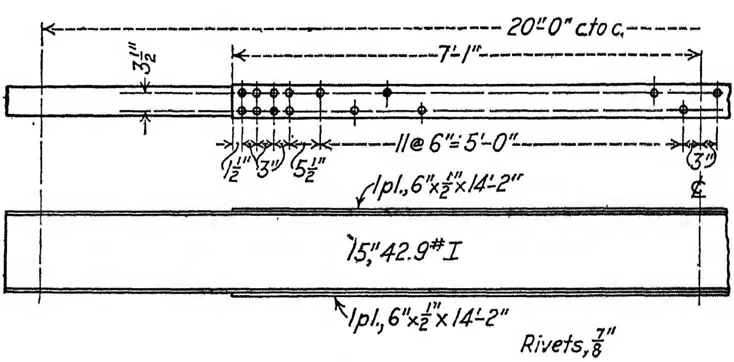


FIG. 50.—Riveting of Reinforcing Plates.

Required corrected net section modulus s_1' which must be contributed by the two plates in order to make up the difference between the section modulus of the punched beam alone and the total required section modulus = 83.3 − 47.1 = 36.2 in.³

Hence theoretical length of plates is

$$x_1 = 20\left(\frac{36.2}{83.3}\right)^{1/2} = 13.15 \text{ ft.}$$

By making the plates 6 in. longer at each end, or say 14 ft. 2 in. long, two rows of rivets, 3 in. apart, may be driven outside the theoretical point of cut-off, as shown in Fig. 50.

84. Riveting of Reinforcing Plates.—Determine the theoretical and practical rivet spacing in the reinforcing plates of the beam of Art. 82. Rivets, $\frac{7}{8}$ in. p_s = 12,000 lb. per sq. in. p_b = 24,000 lb. per sq. in. Maximum spacing in the line of stress must not exceed 16 times the thickness of the plate, nor 6 in.

The correct theoretical pitch of the flange rivets at any point is

$$p = \frac{nvI}{QV} \tag{1}$$

where n = number of rivets in a transverse row; v = least value of one rivet; I = moment of inertia of the compound section; Q = statical moment of plate or plates *on one flange* taken about the neutral axis of the compound section; and V = total vertical shear at the section. (Reference: *Engineering News-Record*, March 10, 1927, p. 401.)

In this case rivets will be placed opposite each other near the ends of the plates, giving a value of $n = 2$, while over the greater part of the length of the plates the rivets will be staggered, making $n = 1$.

The least value, v, of a rivet is its single shearing value = 7220 lb.

I and Q may without material error be taken for the gross areas involved, whether it be the tension or the compression flange that is being considered. From Art. 82, I = 802.3 in.⁴, and $Q = 6 \times 0.5 \times 7.75$ = 23.25 in.³

Let the pitch be investigated at a point near the end of the reinforcing plates, say 3 ft. from a support. Since the total loading per lineal foot = 50,000/20 = 2500 lb., the shear V at this point = 2500(10 − 3) = 17,500 lb.

Hence in this region where the rivets are in pairs

$$p = \frac{2 \times 7220 \times 802.3}{23.25 \times 17{,}500} = 28.4 \text{ in.}$$

If the rivets were staggered, the pitch p would be 14.2 in.

In any case, it is evident that the theoretical pitch is far greater than would be permitted under the practical rule respecting maximum spacing. Consequently, as shown in Fig. 50, the rivets will be arranged with a 6-in. staggered pitch for the greater part of the length of the plates, with a few closer spaces

near each end to provide for the larger increments of stress in a given length as the supports are neared.

85. Design of Beam Overhanging Both Supports.—An I-beam is to rest on two supports 12 ft. apart and is to overhang each support far enough to give equal positive and negative moments. The total applied uniform load is to be 120,000 lb. The beam is continuously supported laterally. Recommend a section.

$p_f = 18{,}000$ lb. per sq. in. $p_s = 10{,}000$ lb. per sq. in. $p_{dc} = 15{,}000 - 150\,c/t$. $p_{vc} = 19{,}000 - 173\,d/t$. The supports are 12 in. long, and the value of j in Eq. (15) of Art. 72 is 0.25. See Art. 79, respecting design for diagonal and vertical compression in the web. Neglect deflection.

If the total uniformly distributed load be W, the distance between supports will be l and the overchange be l_1, then equating the maximum positive and negative moments, we have

$$W\left(\frac{l}{8} - \frac{l_1}{4}\right) = W\left(\frac{l_1^2}{2\,l + 4\,l_1}\right)$$

Solving, $l_1 = 0.354\,l = 0.354 \times 12 = 4.25$ ft.

Assuming a 12-in., 31.8-lb. I, the total load per lineal foot is $120{,}000/20.5 + 31.8 = 5882$ lb. per lin. ft.

The maximum negative (equal to the maximum positive) moment $= \frac{1}{2} \times 5882 \times (4.25)^2 \times 12 = 638{,}000$ in.-lb.

$$S = 638{,}000/18{,}000 = 35.5 \text{ in.}^3$$

A 12-in., 31.8-lb. I with $S = 36.0$ will do, as will a 12-in., 28-lb. W.F., provided the web is sufficiently thick for shear and web buckling.

The maximum shear occurs at the inner edge of the 12-in. support, and is $5882(10.25 - 4.75) = 32{,}400$ lb.

Area required $= 32{,}400/10{,}000 = 3.24$ sq. in.

For the W.F. section, the area provided is $12 \times 0.240 = 2.88$ sq. in., which is not sufficient. For the standard 12-in., 31.8 lb. I, area of web $= 12 \times 0.35 = 4.20$ sq. in., which is sufficient.

The existing maximum diagonal buckling stress, f_{dc}, for a cantilever beam only roughly equals the existing average shearing stress on the web. From Eq. (13), Art. 72 it $= 32{,}400/4.20 = 7720$ lb. per sq. in. for the 31.8-lb. I. (An illustration of the method of determining this stress more exactly is given in Art. 103.)

The permissible diagonal compressive stress is

$$p_{dc} = 15{,}000 - 150 \times 9.75/0.35 = 10{,}820 \text{ lb. per sq. in.}$$

Hence the section is satisfactory for diagonal buckling.

Total reaction $= 5882 \times 10.25 = 60{,}300$ lb. Maximum existing vertical compressive stress, Eq. (15), Art. 72 $= f_{vc} = 60{,}300/(12 + 12/4)0.35 = 11{,}480$ lb. per sq. in.

$p_{vc} = 19{,}000 - 173 \times 12/0.35 = 13{,}070$ lb. per sq. in. This exceeds the existing stress, and hence the assumed section is in all respects adequate.

86. Design of Cantilever Beam Unsupported Laterally.—A cantilever beam of 6-ft. span built securely into a masonry wall carries in addition to its own weight a concentrated load of 1500 lb. applied at the free end. The beam is unsupported laterally. Recommend a size.

$p_f = 16{,}000 - 120\, l'/b$. $p_s = 10{,}000$. $p_{dc} = 20{,}000 - 200\, c/t$, with a maximum of 12,000 lb. per sq. in. Neglect deflection.

Moment due to concentrated loading $= 1500 \times 72 = 108{,}000$ in.-lb. Moment due to weight of beam itself, assuming a 6-in., 12.5-lb. I, is $\frac{1}{2} \times 12.5 \times (6)^2 \times 12 = 2700$ in.-lb. Total moment $= 110{,}700$ in.-lb.

For a 6-in., 12.5-lb. I, the flange width is 3.33 in., and hence $p_f = 16{,}000 - 120 \times 72/3.33 = 13{,}400$ lb. per sq. in.

$$S = 110{,}700/13{,}400 = 8.3 \text{ in.}^3$$

The assumed size, with $S = 7.3$, is not sufficient, but a 7-in., 15.3-lb. I with $S = 10.3$ will do.

Maximum shear $= 1500 + 6 \times 15.3 = 1592$ lb. Web area required $= 1592/10{,}000 = 0.16$ sq. in. Area provided $= 7 \times 0.25 = 1.75$ sq. in., which is far more than is needed.

Existing stress in diagonal compression, according to Eq. (13), Art. 72, is roughly $f_{dc} = 1592/1.75 = 910$ lb. per sq. in., which obviously is very much less than the permissible stress p_{dc}. A more accurate determination of this stress, as in Art. 103, is not necessary here.

The 7-in., 15.3-lb. I is, therefore, adequate in all respects.

87. Design of Laterally Supported Beam Restrained at One End and Simply Supported at the Other.—A beam of 15-ft. span continuously supported laterally and restrained at one end and freely supported at the other is to carry a uniformly distributed load of 1600 lb. per lin. ft. in addition to its own weight. Recommend a size.

$p_f = 18{,}000$ lb. per sq. in. $p_s = 12{,}000$ lb. per sq. in. $p_{dc} = 12{,}000$ lb. per sq. in., where the clear depth of web between the fillets does not exceed 60 times the web thickness.

Assume a weight of beam of 31.8 lb. per lin. ft. The total loading is then 1631.8 lb. per lin. ft.

Maximum moment, which is at the restrained end, Eq. (1), Art. 72, is $\frac{1}{8} \times 1631.8 \times (15)^2 \times 12 = 552{,}000$ in.-lb.

$$S = 552{,}000/18{,}000 = 30.6 \text{ in.}^3$$

A 12-in., 25-lb. W.F. with $S = 30.9$, is the most economical section.

Maximum end shear is at the restrained end and is $\frac{5}{8}\, wl = \frac{5}{8} \times 1625 \times 15 = 15{,}200$ lb. Web area required $= 15{,}200/12{,}000 = 1.27$ sq. in. For the W.F. section, $A_w = 12 \times 0.24 = 2.88$ sq. in., which is more than adequate.

Maximum diagonal buckling stress, Eq. (13), Art. 72, is $f_{dc} = 15{,}200/2.88 = 5280$ lb. per sq. in., as compared with 12,000 lb. per sq. in. allowed, the ratio of clear depth to web thickness being for the W.F. section $10.375/0.24 = 43$.

88. Design of Beam Restrained at Both Ends and without Intermediate Lateral Support.—A beam of 10-ft. span perfectly restrained at both ends, but unsupported laterally between end supports, is to carry a total uniformly distributed load of 3000 lb. per lin. ft. including the weight of the beam. Recommend a size.

$p_f = 20{,}000/(1 + l'^2/2000\,b^2)$ lb. per sq. in. $p_s = 12{,}000$ lb. per sq. in. $p_{dc} = 12{,}000$ lb. per sq. in., where the clear depth of web between the fillets does not exceed 60 times the web thickness. Neglect deflection.

Maximum moment which is at the restrained ends, Eq. (1), Art. 72, is $1/12 \times 3000 \times (10)^2 \times 12 = 300{,}000$ in.-lb.

Assuming a 10-in., 21-lb. W.F., the flange width $b = 5.75$ in. and $p_f = 20{,}000/(1 + 120^2/2000 \times 5.75^2) = 16{,}410$ lb. per sq. in.

Section modulus required $= 300{,}000/16{,}410 = 18.3$ in.^{3}

Section modulus provided $= 21.5$ in.^{3}, which is sufficient.

End shear $= \frac{1}{2} \times 3000 \times 10 = 15{,}000$ lb.

Area required for shear $= 15{,}000/12{,}000 = 1.25$ sq. in. Area provided $= 10 \times 0.24 = 2.40$ sq. in.

The section is secure against diagonal web buckling, as f_{dc}, Eq. (13), Art. 72, is only $15{,}000/2.40 = 6250$ lb. per sq. in.

89. Design of Three-Span Continuous Beam for Moving Load.—A steel beam is continuous over three equal spans of 15 ft. each. It carries a uniformly distributed dead load of 200 lb. per lin. ft., including the weight of the beam, and a uniformly distributed moving load of 1000 lb. per lin. ft. Determine the necessary section of the beam. The beam rests freely on all four supports and is continuously supported laterally.

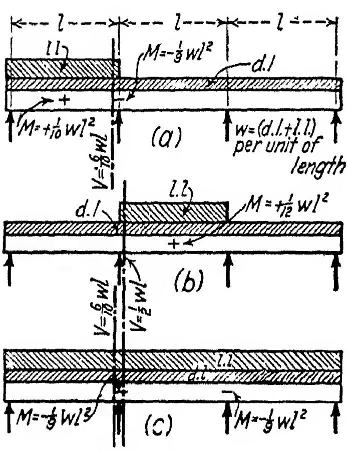


Fig. 51.—Maximum Moments and Shears in Three-Span Continuous Beam.

$p_f = 16{,}000$ lb. per sq. in. $p_s = 10{,}000$ lb. per sq. in. $p_{dc} = 15{,}000 - 150\,c/t$ lb. per sq. in. Neglect the deflection.

From works on Mechanics (see also Hool and Johnson's "Concrete Engineers' Handbook") it is found that when the ratio of live load to dead load is as in this case, that is $1000/200 = 5$, the maximum moments at span centres and at supports, and the shears at supports for beams of three spans are approximately as follows:

End Spans

Moment near centre	$+\frac{1}{10}\,wl^2$
Moment at interior support	$-\frac{1}{8}\,wl^2$
Shear adjacent to interior support	$\frac{6}{10}\,wl$

Interior Span

Moment at centre.......................... $+\frac{1}{12}wl^2$
Moment at supports........................ $-\frac{1}{9}wl^2$
Shear adjacent to supports.................. $\frac{1}{2}wl$

In these formulae, w = total uniformly distributed load, that is, dead load + live load, and l = length of one span. For maximum positive moment in an end span, the live load is placed as in Fig. 51(a), while for maximum positive moment in the interior span, it is placed as in Fig. 51(b). For maximum negative moment over the supports, and for maximum shears at supports the loading is as shown in Fig. 51(c).

From the above it is seen that the greatest moment occurs over a support and is

$$M = \frac{1}{9} \times 1200 \times (15)^2 \times 12 = 360{,}000 \text{ in.-lb.}$$

Required section modulus

$$S = 360{,}000/16{,}000 = 22.5 \text{ in.}^3$$

For this a 10-in., 23-lb. W.F. with $S = 24.1$ in.^{3} will do, if the shearing and web compressive stresses are not excessive.

Maximum shear, which occurs on the outside of an inner support is

$$V = 0.6 \times 1200 \times 15 = 10{,}800 \text{ lb.}$$

Required web area for shear is 10,800/10,000 = 1.08 sq. in.

As the beam has a web area of $A_w = dt = 10 \times 0.240 = 2.40$ sq. in., it is amply strong.

Since the permissible stress in diagonal compression for a 10-in., 23-lb. W.F., is $p_{dc} = 15{,}000 - 150 \times 8.50/0.24 = 9690$ lb. sq. in., the right sectional area required to meet the diagonal compression is 10,800/9690 = 1.08 sq. in.

The assumed beam is entirely adequate.

90. Deflection of Uniformly Loaded Beam Based on Loading.—A 10-in., 25.4-lb. I-beam of 15-ft. effective span carries a total uniformly distributed superimposed load of 20,000 lb. Calculate the maximum deflection. E = 29,000,000 lb. per sq. in.

Maximum deflection is from Eq. (17) of Art. 72.

$$\Delta = \frac{5}{384} \cdot \frac{Wl^3}{EI}$$

The quantities l and I must be in the same unit of length as is involved in W and E, that is the inch. Therefore, l = 180 in. and I = 122.1 in.^{4} As the weight of the beam is $15 \times 25.4 = 381$ lb., W = 20,381 lb. Hence,

$$\Delta = \frac{5}{384} \cdot \frac{20{,}381 \times (180)^3}{29{,}000{,}000 \times 122.1} = 0.437 \text{ in.}$$

91. Deflection of Uniformly Loaded Beam Based on Fibre Stresses.—Two $5 \times 4 \times \frac{3}{8}$-in. angles with the 5-in. legs vertical and stitch riveted together back to back carry a uniformly distributed load over an effective span of 8 ft. at an extreme fibre stress of 16,000 lb. per sq. in. What is the maximum deflection? $E = 29{,}000{,}000$ lb. per sq. in.

The deflection of a uniformly loaded beam may, according to Eq. (18) of Art. 72, be expressed in terms of the stress f_e at the extreme fibre as follows:

$$\Delta = \frac{30 f_e L^2}{E} \cdot \frac{1}{2 y_e}$$

L being the span in feet and y_e beng the distance from the neutral axis to the extreme fibre. Values of the coefficient $30 f_e L^2/E$ for various values of f_e and L and for $E = 29{,}000{,}000$ lb. per sq. in. are listed in the structural handbooks as "coefficients of deflection." In this case the coefficient is 1.059. Hence

$$\Delta = 1.059 \times \frac{1}{2 \times 3.47} = 0.153 \text{ in.}$$

92. Deflection Based on Fibre Stress of Beam with Triangular Loading.—A lintel carrying a mass of brickwork of the form of an isosceles triangle consists of two $4 \times 3 \times \frac{5}{16}$-in. angles with the 4-in. legs vertical and stitch riveted together back to back. The effective span is 8 ft. and the extreme fibre stress is 16,000 lb. per sq. in. Find the maximum deflection. $E = 29{,}000{,}000$ lb. per sq. in.

According to Eq. (18) of Art. 72, the deflection of a beam carrying a triangular loading increasing uniformly from the supports to the centre is, in terms of the extreme fibre stress f_e

$$\Delta = \frac{28.8 f_e L^2}{E} \cdot \frac{1}{2 y_e}$$

where the symbols have the meaning assigned in Art. 72.

Values of this coefficient of $1/2\, y_e$ are not listed in the handbooks. Inserting numerical values, therefore

$$\Delta = \frac{28.8 \times 16{,}000 \times (8)^2}{29{,}000{,}000} \cdot \frac{1}{2 \times 2.74} = \frac{1.017}{2 \times 2.74} = 0.186 \text{ in.}$$

93. Limiting Span for Given Deflection.—Find the greatest ratio of span to depth that may be employed for an I-beam carrying equal loads at the third-points in order that the deflection may not exceed 1/360 of the span.

$f_e = 18{,}000$ lb. per sq. in. $E = 29{,}000{,}000$ lb. per sq. in. Assume the weight of the beam to be applied in two equal loads at the third-points.

For third-point loading

$$\Delta = \frac{23}{216} \cdot \frac{f_e l^2}{E y_e}$$

Equating this to $l/360$, replacing f_e and E by their values, and solving for l

$$l = 42.03\, y_e = 21.01\, d.$$

For spans greater than 21.01 times the depth the deflection will be excessive.

94. Design of King-Post Trussed Beam.—A king-post trussed beam of 15-ft. span centre to centre of bearings and continuously supported laterally is to carry a superimposed load of 1300 lb. per lin. ft. There is no restriction as to head room. Design the beam.

Permissible stress in combined flexure and compression, p_f = 16,000 lb. per sq. in. Permissible compressive stress on strut, $p_c = 15{,}000 - 50\, l/r$ lb. per sq. in. p_t = 16,000 lb. per sq. in. p_s = 10,000 lb. per sq. in. p_b = 20,000 lb. per sq. in. Neglect deflection. Rivets, $\frac{3}{4}$ in.

Assume that the tie rod is prestressed sufficiently to hold the centre of the top chord at the same elevation as the end supports under full load. The continuous-beam theory will then apply.

Let the depth of the trussing from the centre of the top chord to the centre line of the truss rod at the lowest point be 2 ft. 6 in., as shown in Fig. 52. This gives a slope of the tie rod of 1 vertical to 3 horizontal.

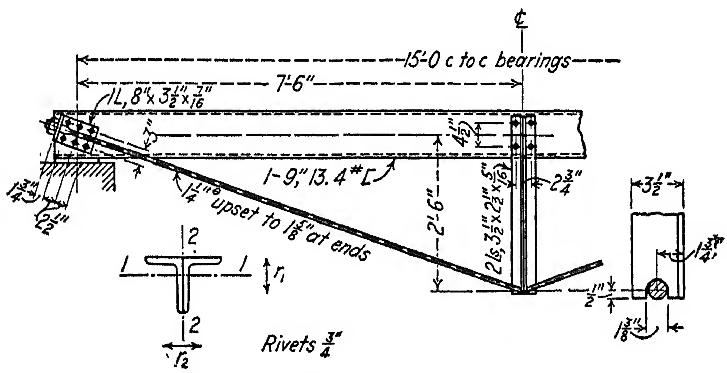


FIG. 52.—King-Post Trussed Beam.

Top Chord Flexure and Compression.—For a continuous beam of two spans, assuming no restraint at the end supports, the maximum moment occurs at the intermediate support, and from Eq. (2) of Art. 72, it is

$$M = \frac{1}{8} wp^2$$

where w is the load per unit of length and p is the panel length. In this case, assuming the weight of the beam to be 20 lb. per lin. ft., $M = \frac{1}{8} \times 1320 \times (7.5)^2 \times 12$ = 111,500 in.-lb.

Extreme fibre stress, assuming the top chord to consist of one 9-in., 13.4-lb. channel with a section modulus of 10.5 in.^{3}, is

$$f_f = \frac{M}{S} = \frac{111{,}500}{10.5} = 10{,}600 \text{ lb. per sq. in.}$$

Horizontal, or axial compression in the top chord is

$$H = \tfrac{5}{8}\, wp \cot \alpha$$

where α = angle of slope with the horizontal of the inclined portions of the tie rod. For the assumed data

$$H = \tfrac{5}{8} \times 1320 \times 7.5 \times 3.0 = 18{,}550 \text{ lb.}$$

Maximum compressive stress in top chord

$$f_c = \frac{H}{A} = \frac{18{,}550}{3.89} = 4770 \text{ lb. per sq. in.}$$

Total maximum compressive stress due to combined flexure and compression is

$$f_f + f_c = 10{,}600 + 4770 = 15{,}370 \text{ lb. per sq. in.}$$

This is well within the limit of 16,000 permitted.

Top Chord Shear.—From the theory of continuous beams, the maximum shear in the top chord occurs immediately adjacent to the strut and is

$$V = \tfrac{5}{8}\, wp = \tfrac{5}{8} \times 1320 \times 7.5 = 6190 \text{ lb.}$$

The average shearing stress on the web is, from Eq. (11) of Art. 72

$$f_{as} = \frac{V}{A} = \frac{6190}{9 \times 0.23} = 2990 \text{ lb. per sq. in.}$$

which is very much below the permissible stress of 10,000 lb. per sq. in.

Tie Rod.—Initial tension not having to be considered as an extra stress, the tension in the tie rod is

$$T = \tfrac{5}{8}\, wp \operatorname{cosec} \alpha = \tfrac{5}{8} \times 1320 \times 7.5 \times 3.16 = 19{,}550 \text{ lb.}$$

Required net area A = 19,550/16,000 = 1.22 sq. in.

Use one $1\frac{1}{4}$-in. rod, upset, having an area of 1.27 sq. in. in the body and of 1.515 sq. in. at the root of the thread of the $1\frac{5}{8}$-in. upset. Adjustment is provided by means of the nuts at the end connections of the rods.

Struts.—Compression in strut is

$$P = \tfrac{5}{4}\, wp = \tfrac{5}{4} \times 1320 \times 7.5 = 12{,}380 \text{ lb.}$$

Assume two $3\frac{1}{2} \times 2\frac{1}{2} \times \frac{5}{16}$-in. angles with the $3\frac{1}{2}$-in. legs tight together and outstanding, as shown in Fig. 52. Area = 3.56 sq. in. Moment of inertia about axis $1 - 1 = 4.4$ in.^{4} Radius of gyration about axis $1 - 1 = 1.11$ in. Radius of gyration about axis $2 - 2 = 0.96$ in.

The 12,380-lb. load is applied at the free bottom end of the strut with an eccentricity of $1.75 - 1.14 = 0.61$ in., and hence the moment at the point of

maximum buckling is $M = 12{,}380 \times 0.61 = 7550$ in.-lb. At this point the maximum combined stress is, Eq. (3) of Art. 36,

$$f_c + f_f = \frac{12{,}380}{3.56} + \frac{7550 \times 2.36}{4.4}$$
$$= 3480 + 4050 = 7530 \text{ lb. per sq. in.}$$

Permissible compressive stress in strut is $p_c = 15{,}000 - 50 \times 30/0.96 = 13{,}440$ lb. per sq. in.

The tilting effect on the strut is resisted by a bending moment in the angles, which reaches its maximum at the lower pair of rivets. The arm of the force is the distance of the centre of the tie rod from the plane of the back of the angles, say 1.75 in. $M = 12{,}380 \times 1.75 = 21{,}650$ in.-lb. Neglecting the effect of the rivet holes, the maximum combined compressive stress, which is at the outer edge of the $3\frac{1}{2}$-in. legs, is

$$f_c + f_f = \frac{12{,}380}{3.56} + \frac{21{,}650 \times 2.36}{4.4}$$
$$= 3480 + 11{,}650 = 15{,}130 \text{ lb. per sq. in.}$$

As the angles are held against buckling at the connection to the top chord, $p_f = 16{,}000$ lb. per sq. in., and the section assumed is adequate.

The outstanding legs of the angles will be notched to semicircular form at the bottom so as to receive the rod.

Bearing area required for the rod is

$$A = \frac{P}{p_b} = \frac{12{,}380}{20{,}000} = 0.62 \text{ sq. in.}$$

Area provided is $1.25 \times 0.625 = 0.78$ sq. in.

Details.—The suggested details are shown in Fig. 52.

For the end connections of the tie rod, use an $8 \times 4 \times \frac{7}{16}$-in. angle cut to $8 \times 3\frac{1}{2} \times \frac{7}{16}$ in. with the $1\frac{1}{4}$-in. rod passing through the $3\frac{1}{2}$-in. leg.

Value of one rivet in bearing on 0.23-in. web of the channel $= 0.75 \times 0.23 \times 20{,}000 = 3450$ lb.

Number of rivets required in angle cleat $= 19{,}550/3450 = 6$.

Number of rivets required in top of strut $= 12{,}380/3450 = 4$.

95. Maximum Stress in I-Beam Subjected to Unsymmetrical Bending.—A 6-in., 12.5-lb. purlin with a span of 16 ft. is, before the roofing or sag rods are put in place, subjected to a vertical central load of 600 lb. The slope of the roof is 30 deg. What is the maximum flexural stress in the purlin? Assume the plane of loading to pass through the centre of gravity of the beam cross section.

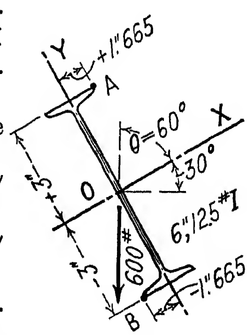


Fig. 53.—I-beam Subjected to Unsymmetrical Bending.

Moment due to concentrated load $= Wl/4 = 600 \times 192/4 = 28{,}800$ in.-lb.
Moment due to weight of purlin $= Wl/8 = (16 \times 12.5) \times 192/8 = 4800$ in.-lb.
Total moment $= 33{,}600$ in.-lb.

Flexural Modulus Method.—From Eq. (9) of Art. 72, the flexural modulus is

$$S' = \frac{I_x I_y}{I_y y \sin\theta + I_x x \cos\theta}$$

Now I_x and I_y are from tables 21.8 in.^{4} and 1.8 in.^{4}, respectively. The coordinates of the fibre at A, which is the one under greatest compressive stress, are $x = +1.665$ in. and $y = +3.0$ in. For the point B, where the greatest tensile stress occurs, $x = -1.665$ in. and $y = -3.0$ in. The angle $\theta = 60$ deg. Hence, with respect to the point A

$$S' = \frac{21.8 \times 1.8}{1.8 \times 3.0 \times 0.866 + 21.8 \times 1.665 \times 0.5} = 1.72 \text{ in.}^3$$

With respect to the point B, $S' = -1.72$ in.^{3}

Hence, maximum fibre stress at A and B is, from Eq. (8) of Art. 72, so far as magnitude is concerned

$$f_e = \frac{M}{S'} = \frac{33{,}600}{1.72} = 19{,}550 \text{ lb. per sq. in.}$$

Moment Resolution Method.—Component of moment in plane of purlin web or about the axis of X, is

$$M_x = M \sin\theta = 33{,}600 \times 0.866 = 29{,}100 \text{ in.-lb.}$$

Component of moment at right angles to plane of web, or about the axis of Y, is

$$M_y = M \cos\theta = 33{,}600 \times 0.5 = 16{,}800 \text{ in.-lb.}$$

The fibre stresses, f_x resulting from M_x, and f_y resulting from M_y, give for either the point A or the point B, the sum

$$f_x + f_y = \frac{M_x}{S_x} + \frac{M_y}{S_y} = \frac{29{,}100}{7.3} + \frac{16{,}800}{1.1} = 19{,}300 \text{ lb. per sq. in.}$$

This result, allowing for slide-rule error, is identical with that obtained by employing the flexural modulus, as it should be.

96. Design of Purlin Subjected to Unsymmetrical Bending.—A channel purlin of 14-ft. span on a roof of $\frac{1}{4}$ pitch carries a vertical uniformly distributed load of 70 lb. per lin. ft. and a uniformly distributed load of 80 lb. per lin. ft. normal to the roof. Sag or tie rods connect to the web of the purlin at mid span. Find the required size of the purlin.

Permissible flexural stress, $p_f = 14{,}000$ lb. per sq. in., being fixed low to cover the indeterminate flange buckling effect.

Assume the applied forces to act through the centre of gravity of the channel. Assume that the supporting value of the sag rod is such as to give a coeffi-

cient for lateral moment at the sag rod connection $\frac{8}{10}$ of that arising in a continuous beam of two spans. Assume that the component of moment normal to the roof is taken by the purlin as a simple beam of 14-ft. span; that the component parallel to the roof is taken by the 7-ft. spans between the trusses and the centre tie rod, acting with the above indicated partial restraint at the tie rod, as indicated in Fig. 54(a). The tendency to buckle laterally on 7-ft. lengths is to be allowed for partly in the staying value of the roof covering and partly in low permissible flexural stress. The deflection is assumed not to be a determining factor in this case.

Component of vertical loading normal to roof = 70 cos α, where the α = angle of roof slope, or 26°-34′, and component parallel to roof = 70 sin α. These are respectively $70 \times 0.8944 = 62.6$ lb., and $70 \times 0.4472 = 31.3$ lb. per lin. ft.

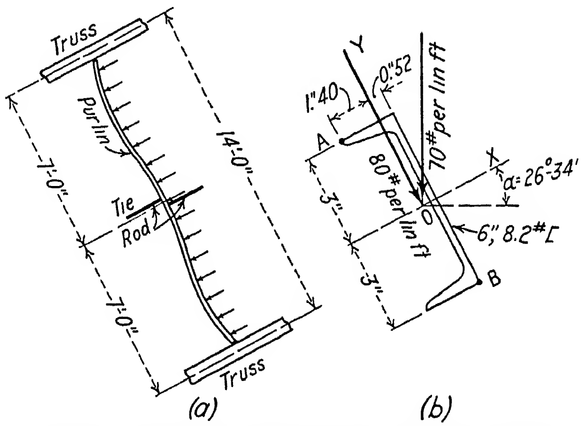


FIG. 54.—Purlin Subjected to Unsymmetrical Bending.

Moment in plane of purlin web is due to combined loading of $62.6 + 80 = 142.6$ lb. per lin. ft. This is $M_x = \frac{1}{8} \times 142.6 \times (14)^2 \times 12 = 41{,}900$ in.-lb.

Maximum moment normal to plane of web, for a span of 7 ft. (see assumptions), is $M_y = \frac{1}{10} \times 31.3 \times (7)^2 \times 12 = 1840$ in.-lb. This occurs either as a positive moment about 2.7 ft. from a truss, or a negative moment at the tie-rod connection. The most serious combination of moments is at the tie-rod connection.

Maximum compressive stress occurs at the upper left-hand corner A, Fig. 54(b), and maximum tensile stress at lower right-hand corner B.

Assume a 6-in., 8.2-lb. channel, with moments of inertia I_x in plane of web = 1.30 in.^{4} and I_y normal to plane of web = 0.70 in.^{4} Lateral distance from neutral axis to point A = 1.40 in. and to point B = 0.52 in.

Compressive stress at point A due to moment $M_x = 41{,}900 \times 3/13.0 = 9700$ lb. per sq. in. Due to M_y it $= 1840 \times 1.40/0.70 = 3680$. Total compressive stress $= 9670 + 3680 = 13{,}350$ lb. per sq. in., which is within the allowable stress.

Tensile stress at point B due to moment $M_x = 9670$ lb. per sq. in. Due to M_y it is $1840 \times 0.52/0.70 = 1370$ lb. per sq. in. Total = 11,040 lb. per sq. in., or well under the prescribed 14,000 lb. sq. per in.

The assumed section is, therefore, adequate. It could not, however, be reduced.

97. Design of Floor Beam Subjected to Both Vertical and Lateral Loading.—A floor beam of 15-ft. span is subjected to a vertical uniformly distributed load of 700 lb. per lin. ft. including its own weight and a resultant horizontal arch thrust of 800 lb. per lin. ft., assumed as applied at mid-height of the beam. The beam is divided into three 5-ft. segments by tie rods as shown in Fig. 55. Find the required section. p_f = 17,000 lb. per sq. in.

Assume a 10-in., 25.4-lb. I for which the section modulus S_x about the horizontal axis OX is 24.4 in.³ and the section modulus S_y about the vertical axis OY is 3.0 in.³

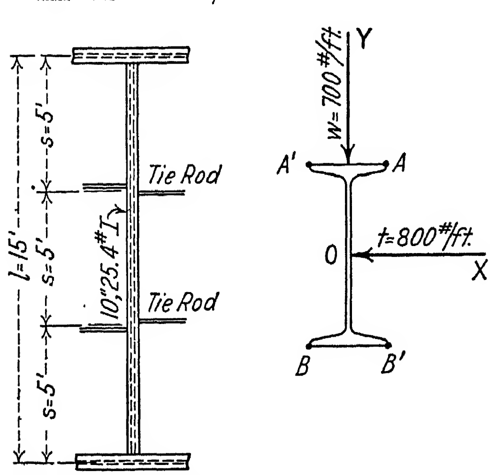


Fig. 55.—Floorbeam Subjected to Both Vertical and Lateral Loading.

Centre of Span.—Vertical moment at centre of span

$$M_x = \frac{wl^2}{8}$$

$$= \frac{700 \times (15)^2 \times 12}{8}$$

$$= 236{,}000 \text{ in.-lb.}$$

Horizontal moment at centre of span, assuming the tie rods to afford unyielding lateral supports to the beam, if t= lateral thrust per lineal foot and s = spacing of tie rods in feet

$$M_y = \frac{ts^2}{40} = \frac{800 \times (5)^2 \times 12}{40} = 6000 \text{ in.-lb.}$$

Maximum fibre stress in compression on fibre at A and in tension on fibre at B, at mid-span

$$f_x + f_y = \frac{M_x}{S_x} + \frac{M_y}{S_y} = \frac{236{,}000}{24.4} + \frac{6000}{3.0} = 9680 + 2000 = 11{,}680 \text{ lb. per sq. in.}$$

Tie Rod Connection.—Vertical moment at a tie-rod connection

$$M_x' = \tfrac{1}{2}\,wls - \tfrac{1}{2}\,ws^2 = \tfrac{1}{2} \times 700 \times 15 \times 5 - \tfrac{1}{2} \times 700 \times (5)^2$$

$$= 17{,}500 \text{ ft.-lb.} = 210{,}000 \text{ in.-lb.}$$

Horizontal moment at the tie-rod connection, from theory of continuous beams

$$M_y' = \frac{ts^2}{10} = \frac{800 \times (5)^2 \times 12}{10} = 24{,}000 \text{ in.-lb.}$$

Maximum fibre stress in compression on fibre at A' and in tension on fibre at B', at tie-rod connection

$$f_x' + f_y' = \frac{M_x'}{S_x} + \frac{M_y'}{S_y} = \frac{210,000}{24.4} + \frac{24,000}{3.0}$$
$$= 8610 + 8000 = 16,610 \text{ lb. per sq. in.}$$

The beam is, therefore, more seriously stressed at the tie-rod connection than at the centre. The assumed section is sufficient.

98. Reinforcement of Beam for Web Crippling.—A girder composed of two 15-in., 42.9-lb. I-beams, having a total reaction of 32,000 lb. is supported on the top of a 6-in., 27.5-lb. H-column, as shown in Fig. 56. The height of the building is increased and a 6-in., 22.5-lb. H-column with a load of 35,000 lb. is seated on the top flanges of the beams directly above the lower column section. Investigate the vertical web buckling stresses in the beams between the upper and lower column sections and recommend such reinforcement as may be necessary.

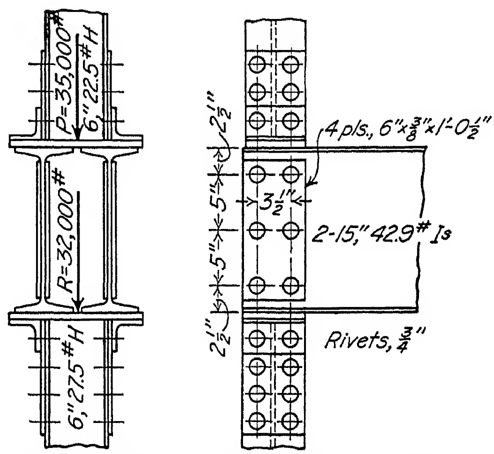


FIG. 56.—Reinforcement of Beam for Web Crippling.

Permissible vertical compressive stress in the web $p_{vc} = 19,000 - 173\ d/t$ lb. per sq. in., where d = depth of beam and t = web thickness in inches.

Assuming that the total vertical compression in the beam webs over the bearing is resisted by a length of 6 in., that is, the depth of the column section, j in Eq. (15) of Art. 72 thus being zero, a hypothesis on the side of severity, the compressive stress on the webs is

$$f_{vc} = \frac{67,000}{2 \times 6 \times 0.41} = 13,620 \text{ lb. per sq. in.}$$

The maximum permissible compressive stress is

$$p_{vc} = 19,000 - 173 \times 15/0.41 = 12,670 \text{ lb. per sq. in.}$$

Hence the web is overstressed.

As reinforcement, rivet two $6 \times \frac{3}{8}$-in. plates $12\frac{1}{2}$ in. long on the web of each of the beams, with 6 rivets in each pair of plates as shown in Fig. 56. These do not need to be ground to fit the fillet, as they effectively increase the web thickness for buckling from fillet to fillet to 1.16 in. and in the very short unsup-

ported vertical zones at the edges of the fillets, the web can safely accommodate a pressure (bearing) of, say, 20,000 lb. per sq. in.

With 5-in. vertical rivet pitch the $\frac{3}{8}$-in. plates are in no danger of lateral buckling.

99. Reinforcement of Beam Web for Shear.—Near its left-hand end a beam is subjected to the vertical forces shown in Fig. 57. Investigate the shearing stresses in the web, and recommend any necessary reinforcement, it being assumed that a beam with a thicker web (which would afford a more economical solution of the problem) is not available.

Permissible average shearing stress, $p_s = 12{,}000$ lb. per sq. in., where the clear distance c between its fillets is not over 50 times the web thickness t. Permissible stress on fillet welds not over 6 in. long, subjected to parallel (longitudinal) shear per lineal inch $= 500\,b$, where b = short side of the fillet triangle expressed in sixteenths of an inch. Permissible stress is flexure, $p_f = 18{,}000$ lb. per sq. in.

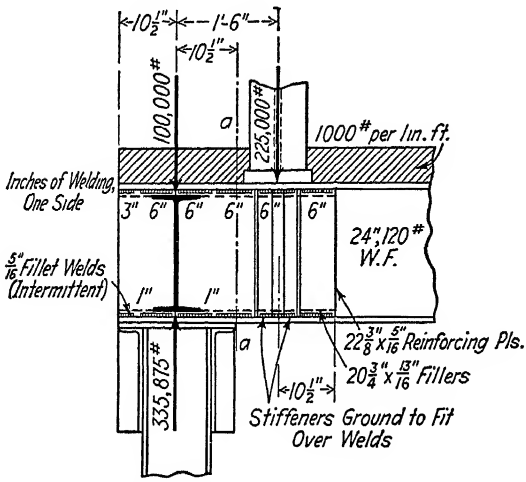


FIG. 57.—Reinforcement of Beam Web for Shear.

Vertical shear at a–a, the inner edge of the column cap, $10\frac{1}{2}$ in. from the centre line of the lower column, is

$$V = 335{,}900 - 100{,}000 - 1.75 \times 1000 = 234{,}150 \text{ lb.}$$

As the ratio c/t for the beam indicated is $20.875/0.556 = 37.5$, the permissible shearing stress is $p_s = 12{,}000$ lb. per sq. in.

Area of web required $= V/p_s = 234{,}150/12{,}000 = 19.52$ sq. in.

Area of web of 24×12-in., 120-lb. W.F. $= 24.31 \times 0.556 = 13.53$ sq. in. This is insufficient.

Area of reinforcing plates required $= 19.52 - 13.53 = 5.99$ sq. in.

If a $20 \times 5/16$-in. plate were riveted to the web on each side, a gross area of 12.5 sq. in. would be added, and if adequately riveted would satisfy the requirements for both vertical and horizontal shear in the depth between the upper and lower horizontal lines of connecting rivets. Outside these rivet lines, that is in those portions of the web adjacent to the fillets, the shearing stress, both horizontal and vertical, in pounds per square inch may be found by applying Eq. (10) of Art. 72,

$$f_s = \frac{QV}{It}$$

The statical moment of the area of the beam on one side of the neutral axis and lying outside the beginning of the fillet, and neglecting the fillets, is

$$Q = (12.088 \times 0.930 \times 11.690) + (0.556 \times 0.785 \times 10.832) = 136.0 \text{ in.}^3$$

For the beam, $I = 3635.3$; and for two $20 \times \frac{5}{16}$-in. plates it is $2 \times \frac{1}{12} \times \frac{5}{16} \times (20)^3 = 416.7$. Hence total $I = 4052.0$, and the actual stress is

$$f_s = 136.0 \times 234{,}150/(4052.0 \times 0.556) = 14{,}110 \text{ lb. per sq. in.}$$

or more than the permissible stress.

To meet this high stress, angles might be riveted to the inner face of the flanges and to the web, but this would introduce several objectionable features. Instead, the two $\frac{5}{16}$-in. plates will be made $22\frac{3}{8}$ in. wide, so as to fit, with a small clearance, between the flanges, to which they will be welded by horizontal fillet welds. Fillers, $20\frac{3}{4} \times \frac{13}{16}$ in., will be used under the reinforcing plates.

The average shearing stress, vertical and horizontal, throughout the clear depth between the flanges is then, adding the section of the reinforcing plates to the whole web section,

$$f_{sa} = \frac{234{,}150}{(13.53 + 13.98)} = 8520 \text{ lb. per sq. in.}$$

As the vertical distance between fillet welds will be about 22 in., the horizontal shear per lineal inch to be transferred to top and bottom flanges by the two reinforcing plates will be $234{,}150/22 = 10{,}660$ lb. diminished by whatever shear the web can carry. Neglecting fillets, this is $0.556 \times 12{,}000 = 6680$ lb., and hence the welds at the top and bottom must resist 3980 lb. per lin. in.

If $\frac{5}{16}$-in. single fillet welds 6 in. long and separated by gaps of about 1 in. be used, the shear borne by each 6-in. length of fillet would be $7 \times 3980/2 = 13{,}930$ lb. The shear per lineal inch of fillet would then be $13{,}930/6 = 2320$ lb. As the safe stress is $500\,b = 500 \times 5 = 2500$ lb., the proposed welding is adequate.

It will be ample, and much in excess of requirements, if, beyond the centre of the concentrated load, the reinforcing plates are extended a sufficient distance to develop the flexural strength of the plates working at a stress proportionate to the distance of their edges from the neutral axis of the beam. Their moment of resistance, Eq. (4), Art. 72, is

$$M_r = Sp'_f$$

The section modulus of the two plates being $S = \frac{1}{6} \times 0.625 \times (22.375)^2 = 52.2 \text{ in.}^3$, $M_r = 52.2 \times 18{,}000 \times 11.19/12.16 = 863{,}000$ in.-lb.

Aggregate length of weld for the top and for the bottom, assuming a safe value of 2500 lb. per lin. in., would be

$$l = \frac{863{,}000}{22 \times 2500} = 15.7 \text{ in.}$$

The plates will be carried $10\frac{1}{2}$ in. past the centre of the column, and 18 in.

of intermittent welding top and bottom will be employed, as compared with 15.7 in. required. The excess will more than allow for crater effect.

The stiffeners under the supported column will be ground to fit over the welds.

In certain cases, as in grillage beams, the maximum stresses due to the combination of shear and flexural stresses should be determined by the use of Eqs. (24) and (25) of Art. 72. In the present instance this is scarcely necessary, as the margin between existing and permissible shearing stress is wide.

100. Design of a Multiple-Section Lintel Carrying Brick Wall.—A solid, well-seasoned $13\frac{1}{2}$-in. brick wall (Toronto brick standards) weighing 125 lb. per cu. ft. is to be carried over a clear opening of 16 ft. There is ample mass of brickwork at either end of the span to withstand any probable thrust. The lintel must be covered by brickwork on its outer face, but may be exposed in the inner face. Design a section.

p_f = 16,000 lb. per sq. in. Shearing and web buckling stresses may be neglected (they are generally negligible for lintels). Minimum depth to obviate cracking of brickwork = $l/25$.

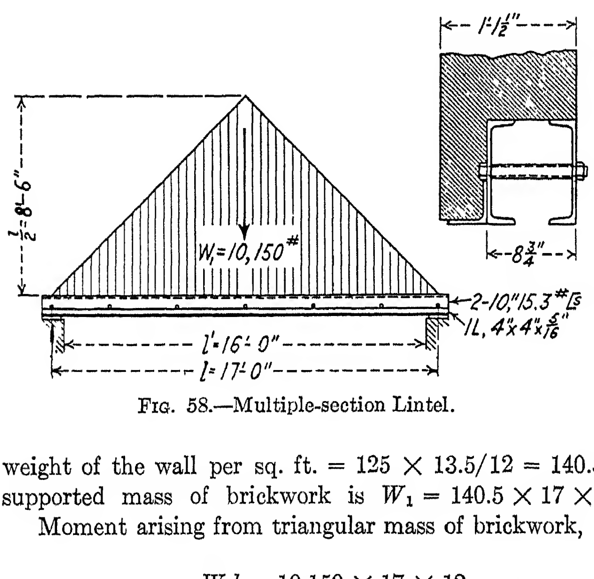


FIG. 58.—Multiple-section Lintel.

The brickwork supported will be a triangular mass having a maximum height of, say, one-half the centre-to-centre span = 8.5 ft., if the centre-to-centre span be taken as 17 ft. The weight of the wall per sq. ft. = 125 × 13.5/12 = 140.5 lb. Weight of the supported mass of brickwork is W_1 = 140.5 × 17 × 8.5/2 = 10,150 lb.

Moment arising from triangular mass of brickwork,

$$M_1 = \frac{W_1 l}{6} = \frac{10{,}150 \times 17 \times 12}{6} = 345{,}100 \text{ in.-lb.}$$

If the weight of the lintel, including the shelf angle (Fig. 58) on the outer face, separators, bolts, etc., be assumed as 40 lb. per lin. ft., the moment due to this is

$$M_2 = \frac{40 \times (17)^2 \times 12}{8} = 17{,}340 \text{ in.-lb.}$$

Total moment

$$M = M_1 + M_2 = 345{,}100 + 17{,}340 = 362{,}440 \text{ in.-lb.}$$

Section modulus required is

$$S = 362{,}440/16{,}000 = 22.7 \text{ in.}^3$$

In selecting a section the depth must not be less than $17 \times 12/25 = 8.2$ in. The $4 \times 4 \times \frac{5}{16}$-in. shelf angle riveted along the bottom of the outer face is not counted upon as contributing to the flexural value of the lintel.

Two 10-in., 15.3-lb. channels with section modulus provided amounting to $S = 2 \times 13.4 = 26.8$ in.³ will suffice. A lighter section would not do.

The channels will be placed with their flanges inwards and spaced $8\frac{3}{4}$ in. back to back, as shown in Fig. 58. The shelf angle will be riveted to the outer channel by rivets about 18 in. apart. Gas pipe separators in one tier spaced approximately 3 ft. apart will be used, as shown in Fig. 58.

101. Sufficiency of I-Beam Subjected to Flexure and Torsion.—A 10-in., 25.4-lb. I-beam of 15-ft. span, web connected at both ends, carries a 4500-lb. load at each third-point, applied in each case by a hanger bolt through the flanges, located $1\frac{3}{8}$ in. from the central plane of the web, as shown in Fig. 59. The two loads are on the same side of the web. Report on the shearing and flexural sufficiency of the beam neglecting the effect of the bolt holes.

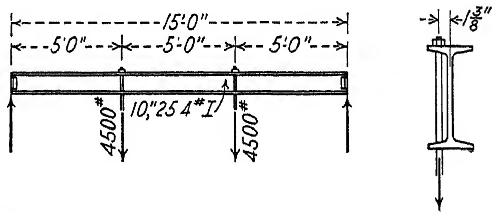


Fig. 59.—I-Beam Subjected to Flexure and Torsion.

Permissible stresses in shear and flexure, $p_s = 12{,}000$ lb. per sq. in., and $p_f = 18{,}000$ lb. per sq. in., respectively.

Shear.—Shearing stresses are produced on the beam section by both the vertical sharing force and the torsional moment.

The maximum (end) vertical shear is $4500 + 25.4 \times 7.5 = 4690$ lb. This produces an average shearing stress in the web, according to Eq. (11) of Art. 72, of $4690/10 \times 0.31 = 1510$ lb. per sq. in. Under the common assumption the vertical shearing stress in the flange is negligible.

The torsional moment, constant in the end thirds of the span, is $T = 4500 \times 1.375 = 6190$ in.-lb., but it is zero in the middle third.

On the basis of experimental investigations (Young and Hughes—Torsional Strength of Steel I-Sections, Bulletin 4, Section 3, School of Engineering Research, University of Toronto), the maximum torsional shearing stress in the web of a standard I-beam may be found from the formula

$$q_{mw} = \frac{2\,Tt}{J'}\left(\frac{d}{t}\right)^{0.45} \tag{1}$$

and the maximum torsional shearing stress in the flange from the formula

$$q_{mf} = \frac{2\,Tt'}{J'}\left(\frac{d}{t}\right)^{0.45} \tag{2}$$

where the effective polar moment of inertia is

$$J' = \left(0.40 + 0.017\frac{d}{t}\right)A_w^{1.25} \tag{3}$$

and t' is the average thickness of the flange.

In the present case, $t = 0.31$ in., $d = 10$ in., and $A_w = 3.1$ sq. in. Consequently

$$J' = \left(0.40 + 0.017 \times \frac{10}{0.31}\right)3.1^{1.25} = 3.7 \text{ in.}^4$$

Therefore, for $T = 6190$ in.-lb.

$$q_{mw} = \frac{2 \times 6190 \times 0.31}{3.7}\left(\frac{10}{0.31}\right)^{0.45} = 4930 \text{ lb. per sq. in.}$$

Similarly, from Eq. (2) of this article the maximum torsional shearing stress in the flange, the average thickness of the latter being 0.50 in., is

$$q_{mf} = \frac{2 \times 6190 \times 0.50}{3.7}\left(\frac{10}{0.31}\right)^{0.45} = 7950 \text{ lb. per sq. in.}$$

The maximum combined shearing stress on a vertical section of the web is, therefore, $1510 + 4930 = 6440$ lb. per sq. in., or little more than one-half the permissible stress.

The maximum shearing stress in the flange, 7950 lb. per sq. in., is also well within the prescribed limit.

Flexure.—At the centre of the span the vertical moment due to the weight of the beam is $25.4 \times (15)^2 \times 12/8 = 8570$ in.-lb. Due to the concentrated loads the moment is $4500 \times 5 \times 12 = 270{,}000$ in.-lb. Total vertical moment $= 278{,}570$ in.-lb.

Extreme fibre stress due to this vertical moment is, from Eq. (3) of Art. 72,

$$f_v = \frac{M}{S} = \frac{278{,}570}{24.4} = 11{,}420 \text{ lb. per sq. in.}$$

An effect of the torsion is to create a transverse moment in each flange having the same amount as would arise from the application of two lateral forces to the flange at the loading points, each amounting to

$$Q = \frac{T}{d'}$$

where d' = distance between centres of flanges. As d' = approximately 9.5 in., $Q = 6190/9.5 = 652$ lb.

The lateral moment in each flange is $Q \times 5 \times 12 = 39{,}120$ in.-lb., it being in opposite directions in the two flanges. If it be resisted wholly by the flange, which has an approximate lateral section modulus of $\frac{1}{6} \times 0.5 \times (4.66)^2 = 1.81$ in.^{3}, the lateral flexural stress is $f_l = 39{,}120/1.81 = 21{,}600$ lb. per sq. in,

The maximum compressive fibre stress and the maximum tensile fibre stress will each be

$$f_v + f_l = 11{,}420 + 21{,}600 = 33{,}020 \text{ lb. per sq. in.}$$

The section is, therefore, overstressed in flexure. One with a broader flange and a thinner web would be more efficient.

102. Design of Wall Girder Subjected to Eccentric Loading.—A wall girder of 18-ft. span supports a 12-in. brick wall, eccentrically placed with respect to the girder, and also a 10-in. strip of a ribbed concrete floor, as shown in Fig. 60. The weight of brick and concrete within the limits of the 12-in. wall thickness is 800 lb. per lin. ft. and the weight of the 2-in. floor slab plus the $1\frac{1}{2}$-in. floor finish plus the live load on the slab is 120 lb. per sq. ft. Design a girder to consist of a wide-flange beam with a plate riveted eccentrically to the bottom flange.

Permissible stresses in combined vertical and torsional shear, and in combined vertical and torsional flexure, where the construction prevents free torsional deformation, will be taken as $p_s' = 18{,}000$ and $p_f' = 27{,}000$ lb. per sq. in., respectively. The section shall, however, be sufficient to resist the vertical shear and the vertical moment only at $p_s = 12{,}000$ and $p_f = 18{,}000$ lb. per sq. in., respectively.

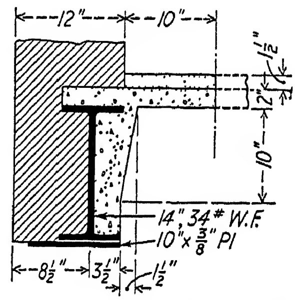


FIG. 60.—Wall Girder with Eccentric Loading.

Vertical Load.—An estimate of the vertical load, in pounds per lineal foot of girder, assuming the girder to be of the section shown, would be as follows:

Weight of brick and concrete within the	
12-in. thickness of wall	800
10-in. strip of floor slab, finish and floor live load, $\frac{10}{12} \times 120$	100
Concrete bracket, at 145 lb. per cu. ft.	8
Assumed weight of girder	50
Total	958

Total load on girder $= 18 \times 958 = 17{,}240$ lb.

The eccentricity of loading may be found by taking moments as follows about, say, the outer face of the wall, assuming the floor-slab reaction to be applied at the middle of the bracket projection:

Wall, 800×6	$= 4800$
Floor and live load, 100×12.75	$= 1275$
Bracket, 8×12.5	$= 100$
14-in., 34-lb. W.F., 34×8.5	$= 289$
$10 \times \frac{3}{8}$-in. plate, 12.8×6.9	$= 88$
	6552

Distance of centre of gravity from outer face of wall = 6552/958 = 6.84 in.
Eccentricity of load with respect to centre of I-beam = 8.50 − 6.84 = 1.66 in.
Moments.—The maximum vertical beam moment is

$$M = \tfrac{1}{8} \times 958 \times (18)^2 \times 12 = 466{,}000 \text{ in.-lb.}$$

The torsional moment, or torque, is a maximum at the supports. Assuming for the time being that free torsion is possible, it is equal to half the load multiplied by the eccentricity, or

$$T = \frac{17{,}240}{2} \times 1.66 = 14{,}300 \text{ in.-lb.}$$

Shearing Stresses.—Average stress in the web, having thickness t, due to the end vertical shear is, from Eq. (11), Art. 72,

$$f_{sa} = \frac{V}{dt} = \frac{8620}{14 \times 0.287} = 2141 \text{ lb. per sq. in.}$$

The torsional shearing stress in both web and flanges will be calculated by a method alternative to that of Art. 101, based on Föppl, and suggested by L. B. Tuckerman (*Engineering News-Record,* Nov. 27, 1924, p. 882). In accordance with this, the maximum shearing stress in the web is approximately

$$q_{mw} = \frac{T}{S_w} \tag{1}$$

where S_w is the " web stress factor " and is given by the relation

$$S_w = \frac{J}{t} \tag{2}$$

J being the " torsional stiffness factor."

A comparatively simple expression for J for I-beams devised by W. B. Campbell (*Engineering News-Record,* Oct. 11, 1928, p. 541) from tests on 16 beams, which also applies fairly well to wide-flange beams, is

$$J = 0.4\, dt^3 + 0.1(m + n)^3(b - t) \tag{3}$$

In this, m = thickness of flange at the point where the inner plane of the flange would, if produced, cut the face of the web; n = thickness of flange at its edge, and b = flange breadth.

Neglecting the effect of the 10 × $\frac{3}{8}$-in. plate in stress calculations, we find

$$J = 0.4 \times 14 \times (0.287)^3 + 0.1(0.906)^3(6.46) = 0.614$$

Hence, from Eq. (2) above, $S_w = 0.614/0.287 = 2.14$

The maximum torsional shearing stress in the web is then

$$q_{mw} = 14{,}300/2.14 = 6680 \text{ lb. per sq. in.}$$

The total combined shearing stress in the web is, therefore, 2141 + 6680

$=8821$ lb. per sq. in., which is much below the permissible limit, for either free or partially restrained torsion.

Although there is no appreciable stress in the flanges due to vertical shear, the maximum torsional shearing stress may be large. It may be expressed approximately as

$$q_{mf} = \frac{T}{S_f} \tag{4}$$

where S_f = the " flange stress factor " and is

$$S_f = \frac{J}{t'} \tag{5}$$

t' being the average thickness of the flange.

In the present case, $S_f = 0.614/0.453 = 1.355$, and $q_{mf} = 14{,}300/1.355 =$ 10,540 lb. per sq. in., which is within the safe limit, even for free torsion.

Flexural Stress.—Owing to the vertical moment the flexural stress at mid-span is

$$f_v = \frac{M}{S} = \frac{466{,}000}{48.5} = 9600 \text{ lb. per sq. in.}$$

The torque at each support may be considered as resisted by a couple Qd', as in the problem of Art. 101. Since d' = distance between centres of flanges $= 14.00 - 0.45 = 13.55$ in., $Q = 14{,}300/13.55 = 1055$ lb.

To produce $Q = 1055$ lb. at each end of a flange, there must be a virtual horizontal load of $1055/9 = 117$ lb. per lin. ft. The lateral moment from this is, for each flange,

$$M_l = \tfrac{1}{8} \times 117 \times (18)^2 \times 12 = 56{,}900 \text{ in.-lb.}$$

If the lateral moment is resisted by half the section, then

$$f_l = \frac{56{,}900}{3.15} = 18{,}050 \text{ lb. per sq. in.}$$

The total combined flexural stress is then $f_v + f_l = 9600 + 18{,}050 =$ 27,650 lb. per sq. in., compared with a permissible stress of 27,000 lb. per sq. in. for combined vertical and torsional flexure. This is sufficiently near the requirement. The stress due to vertical moment only is well within the prescribed limit of 18,000 lb. per sq. in.

It is of interest to note that the section is determined not by combined shearing stress by by combined flexural stress.

103. Design of Steel Grillage Footing.—A steel column delivers to the 2 ft. square steel plate base on which it rests, as indicated in Fig. 61, a maximum load of 250 tons. The footing under the base is to be of the steel grillage type, with the limitation that it cannot be over 6 ft. 8 in. wide in one direction. Design a 2-tier rectangular grillage footing to meet the situation.

Safe soil pressure, 5 tons per sq. ft. Permissible flexural stress on encased

beams, $p_f = 20{,}000$ lb. per sq. in. Permissible shearing stress on beam webs, $p_s = 13{,}000$ lb. per sq. in. Permissible stress in diagonal web compression, $p_{dc} = 19{,}000 - 155\,d/t$ and permissible stress in vertical web compression, $p_{vc} = 23{,}000 - 220\,d/t$ lb. per sq. in., d being depth of beam, and t the web thickness. Length of web resisting vertical compression will be taken as the length of applied load, a; that is j in Eqs. (15) and (16), Art. 72, $=0$.

Let the weight of the footing, including the steel plate column base, be 8 tons. Total load on the soil $= 250 + 8 = 258$ tons.

Required area of footing in contact with soil $= 258/5 = 51.6$ sq. ft. Use a footing 6 ft. 8 in. $\times$ 7 ft. 9 in., having an area of 51.7 sq. ft.

Upper Tier.—The beams of the upper tier being fewer in number than those of the lower tier, and consequently deeper and stiffer, should run in the

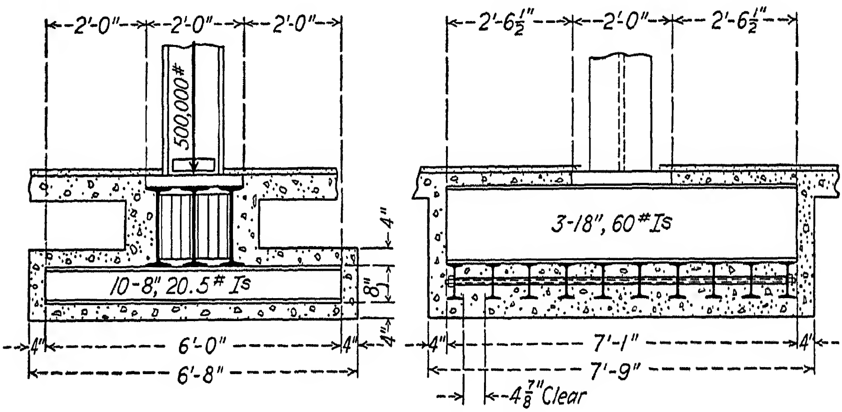


FIG. 61.—Steel Grillage Footing.

long direction of the footing. Allowing for 4-in. concrete encasement of all metal, as shown in Fig. 61, these beams will be 7 ft. 1 in. long.

Consider the beams of the tier as one girder. Disregard the weight of the footing and steel base in calculating stresses and consider the cantilever span of the beams on either side of the steel base as $(85 - 24)/2 = 30.5$ in., thus neglecting the effect of the encasement, except in the prescription of allowable working stresses.

Upward reaction on upper tier per lineal inch of tier $= 250 \times 2000/85 = 5880$ lb.

It being possible for curvature of the central beam of the upper tier to develop throughout its length, the maximum moment will be assumed as occurring at the center of the column. Neglecting the effect of the beam weight, it will be

$$M = \tfrac{1}{2}\,wl^2 = \tfrac{1}{2} \times 5880 \times (42.5)^2 = 5{,}315{,}000 \text{ in.-lb.}$$

Section modulus required

$$S = \frac{M}{p_f} = \frac{5{,}315{,}000}{20{,}000} = 265.7 \text{ in.}^3$$

Three 18-in., 54.7-lb. I's with a combined S of 265.2 would satisfy the requirement of flexure. Three beams are preferable to more by reason of the greater rigidity, while two would require a deeper footing and would necessitate very heavy webs to resist web compression. Standard beams are, in general, more satisfactory for grillages than wide-flange beams because of their relatively thicker webs. The narrower flanges also facilitate encasement.

The beams of the tier should be tested for shear and web compression.

Maximum vertical shear through the upper tier is at the edge of the plate base and is $5880 \times 30.5 = 179{,}400$ lb. The average intensity of shearing stress on the beam webs, from Eq. (11) of Art. 72, is

$$f_{as} = \frac{V}{dt} = \frac{179{,}400}{3 \times 18 \times 0.46} = 7220 \text{ lb. per sq. in.}$$

As $p_s = 13{,}000$ lb. per sq. in., the webs are safe in vertical or horizontal shear.

The maximum existing diagonal shearing stress arises at the beginning of the fillet, and will be computed from Eq. (25) of Art. 72, that is

$$f_{s\,\text{max.}} = (f_s^2 + \tfrac{1}{4} f_f^2)^{1/2}$$

Without appreciable error f_s may be taken as the average shearing stress, that is $f_{as} = 7220$ lb. per sq. in.

At the edge of the base plate the moment on the upper tier is

$$M_e = \tfrac{1}{2} \times 5880 \times (30.5)^2 = 2{,}740{,}000 \text{ in.-lb.}$$

and the flexural stress at the extreme fibre is $2{,}740{,}000/265.2 = 10{,}310$ lb. per sq. in. At the beginning of the fillet, which is 7.63 in. from the neutral axis, it is $f_f = 10{,}310 \times 7.63/9.0 = 8750$ lb. per sq. in.

The maximum diagonal shearing stress is, therefore,

$$f_{s\,\text{max.}} = \{(7220)^2 + \tfrac{1}{4}(8750)^2\}^{1/2} = 8440 \text{ lb. per sq. in.}$$

which is also within the allowable limit.

The maxium existing diagonal compressive stress, which also arises at the beginning of the fillet, is, according to Eq. (24) of Art. 72,

$$\begin{aligned} f_{n\,\text{max.}} &= \tfrac{1}{2} f_f + (f_s^2 + \tfrac{1}{4} f_f^2)^{1/2} \\ &= \tfrac{1}{2} \times 8750 + \{(7220)^2 + \tfrac{1}{4}(8750)^2\}^{1/2} = 12{,}815 \text{ lb. per sq. in.} \end{aligned}$$

The permissible diagonal compressive stress is

$$p_{dc} = 19{,}000 - 155 \times 18/0.46 = 12{,}940 \text{ lb. per sq. in.}$$

and hence the webs are adequate for this stress.

The maximum existing vertical compressive stress in the webs under the plate base, according to the specification, is, from Eq. (15) of Art. 72,

$$f_{vc} = \frac{500{,}000}{3 \times 24 \times 0.46} = 15{,}100 \text{ lb. per sq. in.}$$

As the permissible stress in vertical compression is

$$p_{vc} = 23{,}000 - 220 \times \frac{18}{0.46} = 14{,}400 \text{ lb. per sq. in.}$$

the webs are, therefore, inadequate in vertical compression.

Assuming three 18-in., 60-lb. I's for the upper tier

$$f_{vc} = \frac{500{,}000}{3 \times 24 \times 0.55} = 12{,}620 \text{ lb. per sq. in.}$$

and

$$p_{vc} = 23{,}000 - 220 \times 18/0.55 = 15{,}800 \text{ lb. per sq. in.}$$

These beams are then adequate for moment, shear and web compression.

Bottom Tier.—Whereas those beams of the lower tier that lie in the belt directly under the column shaft are necessarily kept from curving upward in about the central 14.7 in. of their length, those beams of the tier that lie outside the column shaft can curve upward throughout their length. For these the maximum moment will be at the centre. For uniformity the whole tier will be assumed to have its maximum moment at the centre.

To the left of the centre of the footing, the upward load on the lower tier, uniformly distributed, amounts to a total of 250,000 lb., neglecting the effect of the weight of the beams themselves. The downward weight from that portion of the upper tier to the left of the centre is also 250,000 lb. Hence, the moment at the tier centre is

$$M = 250{,}000(18 - 6) = 3{,}000{,}000 \text{ in.-lb.}$$

Section modulus required,

$$S = 3{,}000{,}000/20{,}000 = 150.0 \text{ in.}^3$$

Try 10 lines of 8-in., 20.5-lb. I's, with an aggregate section modulus of $10 \times 15.1 = 151$ in.^{3}

Maximum shear through tier $= 500{,}000 \times 24/72 = 166{,}700$ lb. Average vertical or horizontal shearing stress on beam webs, Eq. (11), Art. 72, is

$$f_{as} = \frac{166{,}700}{10 \times 8 \times 0.35} = 5950 \text{ lb. per sq. in.}$$

which is within the limit of 13,000 lb. per sq. in.

At the edge of the upper tier the moment in the lower tier is

$$M_e = \tfrac{1}{2} \times \frac{500{,}000}{72} \times (24)^2 = 2{,}000{,}000 \text{ in.-lb.}$$

and the flexural stress at the beginning of the fillet of one of the 8-in. beams is

$$f_f = \frac{3.125}{4} \times \frac{2{,}000{,}000}{151} = 10{,}340 \text{ lb. per sq. in.}$$

Combining with this the average vertical shearing stress in the web, the maximum diagonal shearing stress is

$$f_{s\ \mathrm{max.}} = \{(5950)^2 + \tfrac{1}{4}(10{,}340)^2\}^{1/2} = 7885 \text{ lb. per sq. in.}$$

which is safe.

The maximum existing diagonal compressive stress is

$$\begin{aligned} f_{n\ \mathrm{max.}} &= \tfrac{1}{2} \times 10{,}340 + \{(5950)^2 + \tfrac{1}{4}(10{,}340)^2\}^{1/2} \\ &= 13{,}055 \text{ lb. per sq. in.} \end{aligned}$$

as compared with a permissible stress of

$$p_{dc} = 19{,}000 - 155 \times 8/0.35 = 15{,}460 \text{ lb. per sq. in.}$$

The webs of the beams of the lower tier receive compression from the flanges of the beams of the upper tier. Each beam web resists the compression from the upper tier in three zones, each of a length that will be assumed as equal to the flange width of an 18-in., 60-lb. I, or 6.09 in. If the upper tier is assumed to distribute its load equally to all beams of the lower tier, then the maximum vertical compression in the webs of the beams of the lower tier is

$$f_{vc} = \frac{500{,}000}{3 \times 10 \times 6.09 \times 0.35} = 7820 \text{ lb. per sq. in.}$$

The maximum permissible vertical compressive stress is

$$p_{vc} = 23{,}000 - 220 \times 8/0.35 = 17{,}970 \text{ lb. per sq. in.}$$

The clear spacing between flanges of the beams of the lower tier is $(85 - 10 \times 4.08)/9 = 4.91$ in., which gives sufficient clearance for effective concreting.

Ten lines of 8-in., 21-lb. W.F.'s would also be satisfactory for the lower tier.

Channel or built-up separators are employed to hold the beams of the upper tier to their proper spacing and gas-pipe separators are employed for a similar purpose for the lower tier.

104. Design of Interior Floor Panel with Concrete Joists and Steel Girders.—Design a 17 × 19 ft. 5¾-in. interior floor panel of a steel building with concrete joists and steel girders for a live load of 75 lb. per sq. ft. The girders run in the long direction, as shown in Fig. 62.

A 2-in. concrete floor slab, constituting flange material for the joists, will be used. Upon it will be placed a 3½-in. thickness of light-weight-aggregate concrete weighing 90 lb. per cu. ft. in which conduits will be embedded. Over this will be a 1½-in. granolithic surfacing. Suspended from the joists will be a ceiling weighing 8 lb. per sq. ft. Removable metal forms with a top width, for the straight units, of 19 in. and a bottom width of 20.5 in. will be employed.

Joists are to be calculated for a maximum positive central moment of $\frac{1}{12}\,wl^2$ and a maximum negative moment at supports of $\frac{1}{12}\,wl^2$, where w = uniformly

distributed load per unit of span length and l = the effective span. Girders and tie beams are to be considered as simply supported.

Permissible stresses on concrete with a crushing strength of 2000 lb. per sq. in. at an age of 28 days: flexural compression, $p_c = 800$; flexural compression at face of supports, $p_c' = 800 + 15\% = 920$; shear, where tension steel is adequately anchored, $p_s = 60$ lb. per sq. in. Ratio of moduli of elasticity of steel and concrete, $n = 15$.

Permissible tension on reinforcing steel (hard or intermediate grade), $p_t = 20{,}000$ lb. per sq. in.

Permissible stresses on structural steel: flexure, $p_f = 18{,}000$; shear (where ratio of depth of web between fillets to web thickness is not over 50), $p_s = 12{,}000$.

Floor Slab.—Under uniformly distributed load, the slab must carry, including its own weight, the following loading:

Live load........................	75 lb. per sq. ft.
$1\frac{1}{2}$ in. granolithic finish............	18 lb. per sq. ft.
$3\frac{1}{2}$ in. light concrete................	26 lb. per sq. ft.
2 in. slab........................	25 lb. per sq. ft.
Total........................	144 lb. per sq. ft.

The effective span, assuming simple support, will be the top width of the form plus, say, 2 in., or 21 in., approximately.

In a 1-ft. strip of slab, the moment is then

$$M = \tfrac{1}{8} \times 144 \times (1.75)^2 \times 12 = 662 \text{ in.-lb.}$$

The required depth from the top of the slab to the centre of the reinforcing is, from the theory of reinforced concrete,

$$d = \left(\frac{M}{12\,R}\right)^{1/2} \tag{1}$$

For the materials and stresses prescribed, the value of R for balanced reinforcement is 131, so that

$$d = \left(\frac{662}{12 \times 131}\right)^{1/2} = 0.65 \text{ in.}$$

Adding 0.75 in. as the distance from the centre of steel to the bottom of the slab, a total thickness of only 1.4 in. would be required. However, since the actual live load will generally be applied in the form of concentrations, and since great precision in concrete construction is not possible, the 2-in. slab assumed will be adopted.

The area of reinforcement per foot width of slab will be

$$A_t = \frac{M}{p_t j d} \tag{2}$$

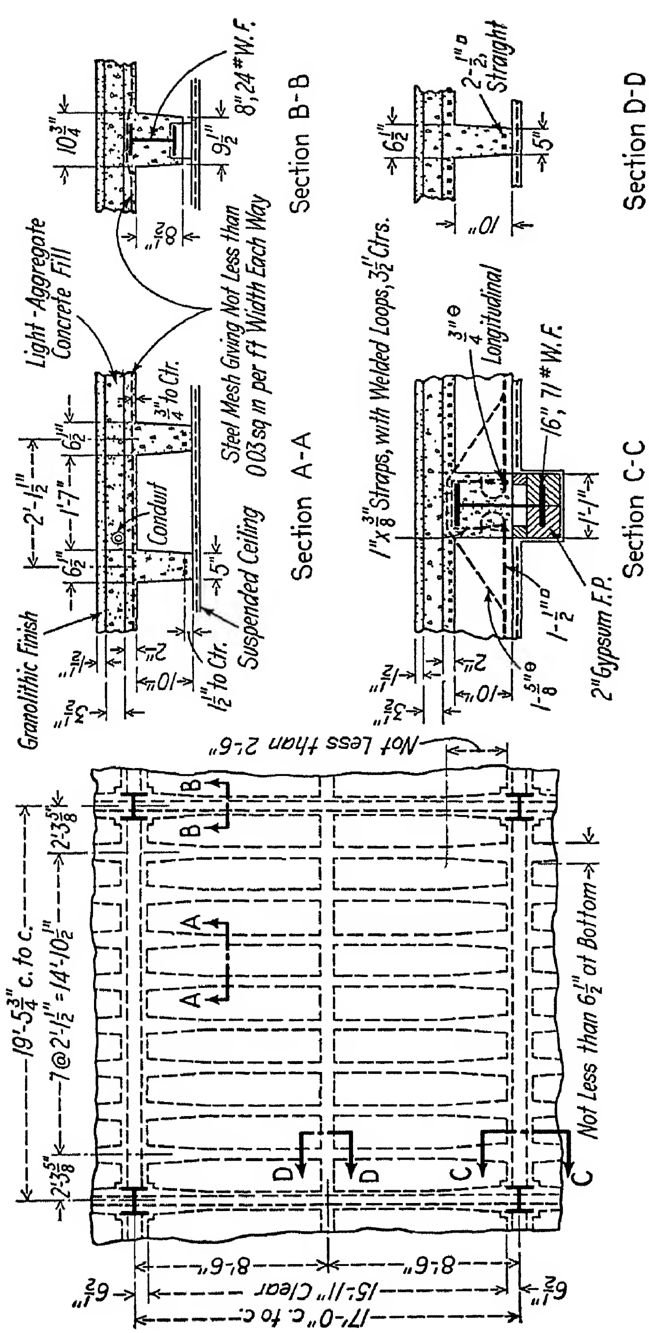


Fig. 62.—Interior Floor Panel with Concrete Joints and Steel Girders.

in which the approximate value of 0.875 may be assigned to j, and d should be given the actual constructed value, or $2 - 0.75 = 1.25$ in. Hence,

$$A_t = \frac{662}{20{,}000 \times 0.875 \times 1.25} = 0.03 \text{ sq. in.}$$

A mesh material, yielding at least this area on a section at right angles to the direction of the moment, and preferably somewhat more, to look after concentrations of loading, may be used. An equal area on a section transverse to the joists should be provided in order to provide against shrinkage stresses.

Concrete Joists.—Each concrete joist carries a uniformly distributed load due to dead load and live load, plus a small central concentrated load arising from the weight of a portion of the central transverse bridging rib, shown in Fig. 62.

Assuming joists 5 in. wide at the bottom, 10 in. deep below the slab, $6\frac{1}{2}$ in. wide at the junction with the slab and $25\frac{1}{2}$ in. centre to centre, the uniformly distributed load, including 8 lb. per sq. ft. for a suspended ceiling, but exclusive of that due to the bridging joist, is $2.13(144 + 8) + \dfrac{\frac{1}{2}(5 + 6.5) \times 10}{144} \times 150$ $= 384$ lb. per lin. ft.

Assuming the section of the bridging joist as the same as that of the main joists, there will be an additional central load on each main joist due to a net length of 19.75 in. of a reinforced concrete stem 10 in. deep and averaging 5.75 in. in width. This amounts to

$$P = \frac{10 \times 5.75}{144} \times \frac{19.75}{12} \times 150 = 100 \text{ lb.}$$

As this is a small quantity, its moment effect may be looked after by increasing it 50% and adding the load so augmented to the total uniformly distributed load when calculating both the maximum positive and maximum negative moments.

The effective span of the joists, they being restrained at the ends, may be taken as the clear distance between the concrete encasements of the top flange of the girders, or 17 ft. 0 in. less 1 ft. 1 in., that is 15 ft. 11 in., or 15.94 ft.

The total equivalent uniformly distributed load borne by a joist is, then,

$$W = (384 \times 15.94) + 1.5 \times 100 = 6270 \text{ lb.}$$

The maximum moment is

$$M = \pm \tfrac{1}{12} \times 6270 \times 15.94 \times 12 = 100{,}000 \text{ in.-lb.}$$

The maximum end shear for a joist is

$$V = \tfrac{1}{2} \times 15.94 \times 384 + \tfrac{1}{2} \times 100 = 3110 \text{ lb.}$$

Flexure in Concrete Joists, Approximate Method.—Anticipating the fact that the neutral axis of the joist lies below the slab, and that, therefore, a true T-beam exists, an approximate design may be conveniently made by assuming the centre

of compression as at the mid-depth of the slab. The moment arm is consequently

$$jd = d - \tfrac{1}{2}\, t \tag{3}$$

and if the distance from the centre of steel to the bottom surface of the concrete is 1.5 in., $jd = (2 + 10 - 1.5) - 1 = 9.5$ in.

Area of steel required is, from Eq. (2) above, $100{,}000/20{,}000 \times 9.5 = 0.53$ sq. in. This might be met by assigning one $\frac{5}{8}$-in. round rod and one $\frac{1}{2}$-in. square rod, giving a provided area of 0.55 sq. in. The $\frac{5}{8}$-in. round would be bent up and carried over the support at the top to a point about 4.5 ft. past the centre of the girder into the adjacent spans, lapping similar rods coming from these spans. The reinforcement for the end negative moment is thus $2 \times 0.30 = 0.60$ sq. in.

The sufficiency of the concrete in compression at mid-span may be tested also by the approximate theory. Considering the compression as taken wholly by the flange, with a breadth b, the compressed area is bt and the *average* stress in the flange at mid-span is

$$f_{ca} = \frac{M}{bt(d - \frac{1}{2}\, t)} \tag{4}$$

$$= \frac{100{,}000}{25.5 \times 2 \times 9.5} = 206 \text{ lb. per sq. in.}$$

So long as the neutral axis lies in the stem, the maximum stress is less than twice this amount, or less than 412 lb. per sq. in., and hence far below the permissible limit.

Where the end moment is the same as the moment at mid-span, obviously the critical section for compression in the concrete is at the face of a support, since the flange is on the tension side and the joist becomes practically a rectangular beam with a breadth $b = 5$ in. at the most highly compressed surface. The extreme fibre stress is, then, from the theory of reinforced concrete,

$$f_c = \frac{2\,M}{jkbd^2} \tag{5}$$

Assuming that $j = \frac{7}{8}$, $k = \frac{3}{8}$ and $d = 10.5$ in. (as at mid-span), this gives

$$f_c = \frac{2 \times 100{,}000}{0.875 \times 0.375 \times 5 \times (10.5)^2} = 1110 \text{ lb. per sq. in.}$$

Since the permissible flexural compressive stress at the face of supports is only 920 lb. per sq. in., the joists will need to be widened at the supports, or compressive reinforcement inserted. The first method will be adopted as the more convenient in this case.

If the flexural stress is to be reduced to below 920 lb. per sq. in., the breadth of the joist should be increased at the bottom to $5 \times 1110/920 = 6$ in.

It appears that, on the basis of the approximate method, to achieve this any standard metal form may be used that gives a bottom width of joist of not

less than 6 in. at the support, tapering down to 5 in. at a point not closer than 2 ft. 6 in. to the end of the effective span.

Flexure in Concrete Joists, Exact Method.—Proceeding by the more exact theory of reinforced concrete, the value of the moment arm, jd, is found to be 9.75 in., as against 9.5 in. used above. The revised area of steel required is 0.51 sq. in., as against 0.53 sq. in., and the average compressive stress in the flange of the T-beam at mid-span is 353 lb. per sq. in., as compared with 206 lb. per sq. in. At the support the compressive stress at the extreme fibre of the under side of the beam for a bottom width of 6 in. is 938 lb. per sq. in., as against 920 permitted.

The only revision of design necessary would, therefore, be a slight widening of the beam at the support to, say, $6\frac{1}{2}$ in.

Shear in Concrete Joists.—As has been found above, the end shear in a joist, neglecting the taper at the ends, is 3110 lb. Adding the weight of the additional material required for the taper, $V = 3110 + 20 = 3130$ lb. The shearing stress on the composite section is

$$f_s = \frac{V}{bjd} \tag{5}$$

or $3130/6.5 \times 0.877 \times 10.5 = 52$ lb. per sq. in. While shear reinforcement is, consequently, not necessary, the $\frac{5}{8}$-in. round rod will be bent up at 30 deg. at each end and carried into the adjacent spans as shown in Fig. 62, thus giving additional security.

Tie Beams.—In order to stay the steel columns laterally in the direction of the joists, and also to serve as a joist, a steel tie beam encased in concrete will be used.

So long as the columns are straight and the eccentricity of loading in the direction of the stay or tie member is not great, the axial load in the tie beam will be very small (see Art. 46). Any steel section with a slenderness ratio l/r not over 130 will be ample. For example, an 8×6.5, 24-lb. W.F. with a slenderness ratio of $204/1.61 = 127$ and an area of 7.06 sq. in. will at a working compressive stress of, say, 9000 lb. per sq. in. (the A.I.S.C. column formula would permit 9490) resist safely over 60,000 lb., or far more than would be necessary to preserve column alignment.

Let the assumed steel section be encased as shown in Fig. 62, giving a width of encasement at the bottom and at the under side of the slab of 9.5 and 10.75 in., respectively. The standard metal forms used elsewhere may be used to form the space between this encased beam and the adjacent concrete joist.

The transverse load borne by the tie beam, exclusive of the steel section and its haunching, is as follows:

Live load	75 lb. per sq. ft.
Granolithic finish	18 lb. per sq. ft.
Light-weight concrete	26 lb. per sq. ft.
Ceiling	8 lb. per sq. ft.
2-in. slab	25 lb. per sq. ft.
Total	152 lb. per sq. ft.

The weight of the steel section and the haunching (without deducting the volume of concrete replaced by the steel section) is $24 + \frac{1}{2}(9.50 \times 10.75) \times 8.5 \times 150/144 = 114$ lb. per lin. ft.

Total uniformly distributed load on the beam, the width served being 2.30 ft., is

$$w = 152 \times 2.30 + 114 = 464 \text{ lb. per lin. ft.}$$

At the centre, a weight of $P = 100$ lb. is received from a 19.75-in. length of the transverse bridging joist. For a simply supported beam the equivalent uniform load so far as producing moment is concerned is 200 lb.

The steel beam being riveted to the column, the effective span is 17 ft., and hence the total equivalent uniformly distributed load is

$$W = (464 \times 17) + 200 = 8100 \text{ lb.}$$

The maximum moment is

$$M = \tfrac{1}{8} \times 8100 \times 17 \times 12 = 206{,}300 \text{ in.-lb.}$$

The maximum shear is

$$V = \tfrac{1}{2} \times 17 \times 464 + \tfrac{1}{2} \times 100 = 3995 \text{ lb.}$$

Considering the lever arm of the resultant compression and the resultant tension as the distance from the mid-thickness of the slab to the mid-thickness of the lower flange of the I-beam, $jd = 8.0 - 0.20 = 7.8$ in.

If only the bottom flange be regarded as tension reinforcement of the composite section, $A_t = 6.5 \times 0.398 = 2.59$ sq. in. The stress in it would then be

$$f_t = \frac{206{,}300}{2.59 \times 7.8} = 10{,}210 \text{ lb. per sq. in.}$$

which is within the prescribed limit.

Applying Eq. (4) of this article, the average stress in the concrete flange is

$$f_{ca} = \frac{206{,}300}{29.75 \times 2 \times 7.8} = 444 \text{ lb. per sq. in.}$$

By the simple test of taking about the under surface of the slab the relative statical moments of the 29.75×2-in. flange and the transformed area of the steel section, or 7.06×15 sq. in., it is seen that the neutral axis of the composite section lies well within the stem. Consequently, even neglecting the compressive value of the top flange of the steel section, the maximum stress in the concrete is substantially less than $2 \times 442 = 888$ lb. per sq. in., and within the prescribed limit.

The stress due to the small probable axial load on the section may be safely superimposed on those due to flexure.

A mental calculation indicates that the end shear is well within the capacity of the I-beam web.

Bridging Joist.—With a view to distributing possible concentrated loads

amongst the various joists, a transverse bridging joist, of the same dimensions as the main joists, will be run across the panel at the middle of the 17-ft. span. It will be reinforced with two $\frac{1}{2}$-in. square straight rods.

Girders.—The load borne by a girder may be considered as uniformly distributed. It is as follows:

End shear from joists, $2 \times 3130/2.13$	= 2937 lb. per lin. ft.
Live load, floor finish, filling and 2-in. slab for a 13-in. strip = 144×1.08	= 156 lb. per lin. ft.
Concrete encasement of top flange of girder $10 \times 13 \times 150/144$	= 136 lb. per lin. ft.
Gypsum fireproofing of web and bottom flange, including plaster	= 42 lb. per lin. ft.
Steel girder, assumed	64 lb. per lin. ft.
Total	3335 lb. per lin. ft.

The conventional span used in design being 19.48 ft., the distance centre to centre of columns, the moment is

$$M = \tfrac{1}{8} \times 3335 \times (19.48)^2 \times 12 = 1{,}895{,}000 \text{ in.-lb.}$$

The maximum end shear is

$$V = \tfrac{1}{2} \times 3335 \times 19.48 = 32{,}500 \text{ lb.}$$

Required section modulus

$$S = \frac{1{,}895{,}000}{18{,}000} = 105.3 \text{ in.}^3$$

Use a 16-in., 71-lb. W.F., with $S = 115.9$ in.^{3} The 64-lb. section is insufficient.

From inspection it is evident that the beam is ample for the shear.

Details.—Details are indicated in Fig. 62. To transfer the reactions of the concrete joists to the girders, $1 \times \frac{3}{8}$-in. soft steel straps or stirrups are carried over the top flange of the girders at about 3 ft. 6 in. centres and loop around a $\frac{3}{4}$-in. longitudinal round rod at each end running parallel to the girder in the concrete encasement of the top flange. These loops are welded shut.

105. Riveted Beam Connection Subjected to Reaction without Moment.—Investigate the sufficiency of the beam web rivets of the connection shown in Fig. 63 for a 20-in., 65.4-lb. I for a reaction of 50,000 lb., if the turning moment and the moment of restraint developed by the rivets in the outstanding legs of the connection angles be assumed to offset each other.

Rivets, $\frac{3}{4}$ in. $p_s = 12{,}000$ lb. per sq. in. $p_b = 30{,}000$ lb. per sq. in.

Double shearing value of one $\frac{3}{4}$-in. rivet $= 2 \times 0.442 \times 12{,}000 = 10{,}600$ lb.
Safe reaction, based on shear is $5 \times 10{,}600 = 53{,}000$ lb.

Bearing value of $\frac{3}{4}$-in. rivet on $\frac{1}{2}$-in. web of beam = 0.75 × 0.5 × 30,000 = 11,250 lb.

Safe reaction, based on bearing, is 5 × 11,250 = 56,250 lb.

As both these safe values for the reaction exceed 50,000 lb., the connection is safe.

106. Combined Stress in Girder Web Rivets.—Find the maximum resultant tensile stress in the rivets in the outstanding legs of the angles of the connection investigated in Art. 105, Fig. 63. Assume the neutral axis as at one-seventh of the depth of the connection up from the lower end of the angles (see Arts. 59, 61 and 63).

The moment on the connection, if the reaction be assumed as applied in the plane of the back of the outstanding legs of the angles, is $M = 50{,}000 \times 2.25 = 112{,}500$ in.-lb.

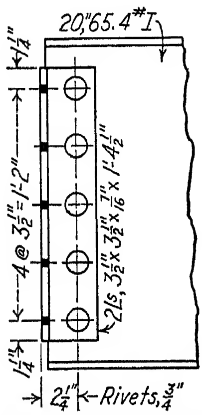


FIG. 63.—Beam connection Designed for Direct Force (No Moment).

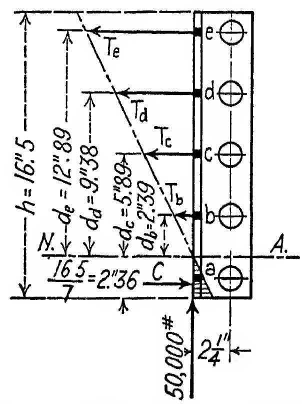


FIG. 64.—Combined Stress in Girder Web Rivets.

Of this, the girder web rivets passing through the outstanding legs of the angles will resist, as was pointed out in Art. 59, a moment of

$$M' = \frac{M}{1 + \dfrac{2\,h\Sigma d}{21\ \Sigma d^2}}$$

The depth of the connection, h, being 16.5 in., it follows that the neutral axis is 16.5/7 = 2.36 in. up from the bottom of the angles, as shown in Fig. 64, and the distances to the various rivets in tension are: $d_b = 2.39$ in., $d_c = 5.89$ in., $d_d = 9.39$ in., and $d_e = 12.89$ in. Hence, $\Sigma d = 2(2.39 + 5.89 + 9.39 + 12.89) = 61.12$, and $\Sigma d^2 = 2(2.39^2 + 5.89^2 + 9.39^2 + 12.89^2) = 589.28$.

Consequently

$$M' = \frac{112{,}500}{1 + \dfrac{2 \times 16.5 \times 61.12}{21 \times 589.28}} = 96{,}700 \text{ in.-lb.}$$

Tensile stress, due to turning, on one of the extreme web rivets is, according to Eq. (2) of Art. 59

$$T_e = \frac{M'd_e}{\Sigma d^2} = \frac{96{,}700 \times 12.89}{589.28} = 2115 \text{ lb.}$$

The tensile unit stress in one of these rivets is, therefore, $f_t = 2115/0.442 = 4780$ lb. per sq. in.

The shearing unit stress on any one of the girder web rivets is $f_s = 50{,}000/10 \times 0.442 = 11{,}320$ lb. per sq. in.

From Eq. (24) of Art. 72, the maximum resultant tensile stress on an extreme rivet is

$$\begin{aligned} f_{t\,\text{max.}} &= \tfrac{1}{2} f_t + (f_s^2 + \tfrac{1}{4} f_t^2)^{1/2} \\ &= \tfrac{1}{2} \times 4780 + (11{,}320^2 + \tfrac{1}{4} \times 4780^2)^{1/2} = 14{,}000 \text{ lb. per sq. in.} \end{aligned}$$

107. Beam Connection Subjected to Both Direct Shear and Moment.—Find the safe capacity of the beam rivets of a typical beam connection (Fig. 65) for a 12-in., 31.8 lb. I-beam. Neglect the restraining moment due to the tension in the shanks of the rivets through the outstanding legs of the connection angles. Rivets, $\frac{3}{4}$ in. diameter. $p_s = 10{,}000$ lb. per sq. in. $p_b = 20{,}000$ lb. per sq. in.

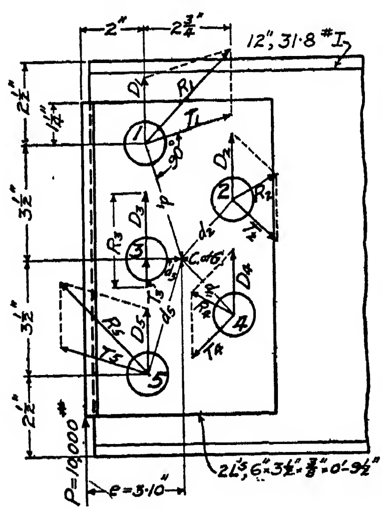


FIG. 65.—Analysis of Beam Connection.

Assume a reaction for the beam of $P = 10{,}000$ lb. The centre of rotation of the angles will be assumed as at the centre of gravity of the rivet group.

Direct force, D_n, on any rivet number "n" $= P/m$, where $m =$ number of rivets through the beam web. Hence $D_n = 10{,}000/5 = 2000$ lb.

Turning moment on connection is $M = Pe$, where $e =$ eccentricity of force P with respect to the centre of rotation. Hence, $M = 10{,}000 \times 3.10 = 31{,}000$ in.-lb.

According to Eq. (3) of Art. 21, the turning force on any rivet is

$$T_n = \frac{Mr_n}{\Sigma r^2}$$

T_n for each rivet is given in the accompanying table. Resultant force on each rivet is found by combining D_n and T_n vectorially as shown in Fig. 65. Resultants thus obtained are given in last column of Table 1.

TABLE 1

Analysis of Connection for P = 10,000 Lb.

Rivet	r, Inches	r^2, Inches^2	Direct Force D_n, Pounds	Turning Force T_n, Pounds	Resultant R_n, Pounds
1	3.67	13 45	2000	2870	3950
2	2.41	5 80	2000	1880	1520
3	1.10	1.21	2000	860	2860
4	2 41	5.80	2000	1880	1520
5	3.67	13.45	2000	2870	3950

$\Sigma r^2 = 39.71$

Safe capacity of connection = assumed reaction $P \times$ safe resistance of rivet $\div$ maximum resultant.

Safe resistance of rivet is bearing on 0.35-in. web of beam = $0.75 \times 0.35 \times 20{,}000 = 5250$ lb.

Maximum resultant (from table) = 3950 lb. Therefore, safe capacity of connection = $10{,}000 \times 5250/3950 = 13{,}300$ lb.

108. Double Duty of Girder Web Rivets.—Two 10-in., 25.4-lb. I's are connected to the web of a 15-in., 42.9-lb. I directly opposite each other, as shown in Fig. 66, thus utilizing the same field rivets through the girder web. If the reaction of each 10-in. I is 8500 lb., and the connection angles are $\frac{3}{8}$ in. thick, find the number of field rivets required, neglecting the effect of tension on the rivet shanks.

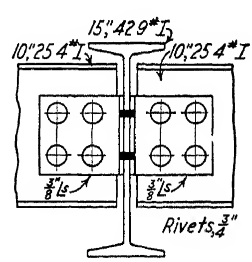


Fig. 66.—Double Duty of Girder Web Rivets.

Rivets, $\frac{3}{4}$ in. $p_s = 8000$ lb. per sq. in. $p_b = 16{,}000$ lb. per sq. in.

For the attachment of *one* 10-in. I to the girder, enough rivets in single shear (this value being less than the bearing value on the $\frac{3}{8}$-in. angles) to transmit 8500 lb. will be necessary.

Safe resistance of $\frac{3}{4}$-in. rivet in single shear = $0.442 \times 8000 = 3530$ lb.

Number of rivets required = $8500/3530 = 1.9$, or 2.

For the attachment of the *two* 10-in. I's enough rivets in bearing on the 0.41-in. web of the girder to transmit 17,000 lb. will be required.

This will be $17{,}000/4920 = 3.5$, or 4.

109. Design of Welded Beam Connection.—Design a welded beam connection for an 18-in., 47-lb. W.F. to transmit to a girder a reaction of 50,000 lb. Neglect any restraining moment developed by the attachment of the outstanding legs to the web of the girder.

Permissible stresses in fillet welds in pounds per lineal inch will be as follows:

Parallel (longitudinal) shear, $p_s = \frac{b}{6}(3300 - 50\,l)$, but not over $500\,b$; normal (transverse) shear, $p_s = 500\,b$; combined direct and turning stress at the part of the weld most remote from the centre of gravity, $p_s = 500\,b$. In these formulae, b = side of the largest isosceles triangle contained in the cross section of the fillet expressed in sixteenths of an inch and l = actual length in inches of the longitudinal fillet under consideration.

Provide 5% excess capacity in the connection to compensate for craters.

If too great a concentration of stress in the beam web is to be avoided, the sum of the throat distances of the two fillets at the vertical edges of the legs connecting to the web must not be greater than the web thickness. In other words:

$$a \leqq 0.707\,t \tag{1}$$

in which a = the short side of the fillet triangle in inches and t = web thickness in inches. As the web is 0.35 in., the fillets must not be over 0.25 in., if they

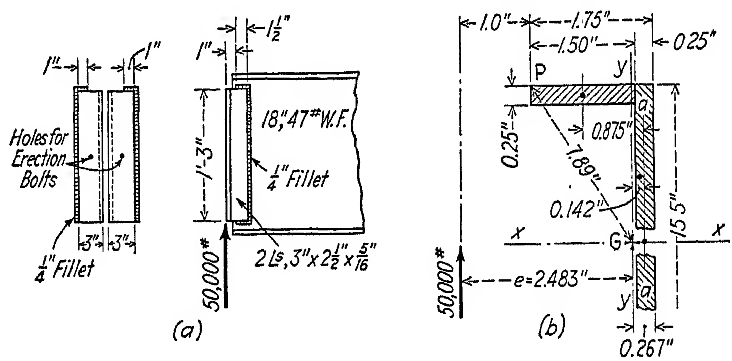


FIG. 67.—Design of Welded Beam Connection.

are to be worked up to the permissible limit. It is thus evident that for beams having very thin webs it is not practicable to use other than very narrow fillets.

To look after the effect of eccentric application of the load efficiently, the depth of the connection to the beam web should not be less than about two-thirds the beam depth. It will be assumed as 15.5 in., which is practically as much as can be accommodated between fillets.

Two $3 \times 2\frac{1}{2} \times \frac{5}{16}$-in. angles will be used for the connection, with $\frac{1}{4}$-in. fillet welds assumed as shown in Fig. 67.

Attachment to Girder Web.—Assuming fillet welds along the vertical edges of the legs of the angles connected to the girder web, a gross length of $2 \times 15 = 30$ in. of $\frac{1}{4}$-in. fillet is available. At the permitted stress of

$$p_s = \tfrac{4}{6}(3300 - 50 \times 15) = 1700 \text{ lb. per lin. in.}$$

the capacity is $30 \times 1700 = 51{,}000$ lb. Adding 1 in. along the top edge of each angle at the outer corner the necessary 5% excess strength is more than provided and additional security is afforded against tension at the top arising from the tilting action of the beam away from the girder.

Attachment to Beam Web.—For the attachment of each of the angles to the beam web 15.5 in. of fillet will be run up the vertical edge and 1.5-in. fillets will be run along each of the two horizontal edges.

The method of investigating the sufficiency of this assumed U-shaped weld will involve the simple combination of the stresses at the extreme corner due to the direct vertical shear and to the rotational effect about the centroid, proceeding in much the same manner as for the riveted connection, Art. 107. The direct shear will be assumed as uniformly distributed over the aggregate length of weld, while the stress due to rotation will be computed by the simple round shaft theory, using the polar moment of inertia.

The centre of gravity of the U-shaped weld, having an area of 4.625 sq. in. projected on the face of the web, is found by the usual methods to be 0.267 in. from the back of the U.

The polar moment of inertia is $I_x + I_y$, where I_x = ordinary moment of inertia of the hatched figure about the axis x–x and I_y = ordinary moment of inertia about the axis y–y.

I_x may conveniently be taken as the moment of inertia of a rectangle 1.75 × 15.5 in. less the moment of inertia of one 1.5 × 15 in. It is then

$$I_x = \tfrac{1}{12}(1.75 \times 15.5^3 - 1.5 \times 15^3) = 121.5 \text{ in.}^4$$

I_y will be calculated in the usual way, by first finding the I about the vertical axis a–a at the centre of the long fillet and then subtracting the product of the total area and the square of the distance between axes a–a and y–y.

$$I_y = \tfrac{1}{12} \times 15.5 \times (0.25)^3 + 2 \times \tfrac{1}{12} \times 0.25 \times (1.5)^3 + 2 \times 1.5 \times 0.25 \times (0.875)^2 = 4.625 \times (0.142)^2 = 0.6 \text{ in.}^4$$

The polar moment of inertia about the centroid G for the weld serving one angle is then 121.5 + 0.6 = 122.1 in.^{4}

For an applied force of 50,000 lb. applied at 2.483 in. from the centroid of the weld, giving a moment of 124,150 in.-lb., the direct and moment stresses at critical point P are, respectively,

$$f_d = \frac{50{,}000}{2 \times 4.625} = 5405 \text{ lb. per sq. in.}$$

$$f_m = \frac{124{,}150 \times 7.89}{2 \times 122.1} = 4055 \text{ lb. per sq. in.}$$

Combining these stresses graphically there results a maximum stress of 7700 lb. per sq. in., or 7700 × 0.25 = 1925 lb. per lin. in. of weld. This is sufficiently within the permissible stress of 2000 lb. per lin. in. to allow for craters. The assumed weld is consequently adequate for the reaction.

110. Reduced Depth End Bearing for Beam.—A 12-in., 31.8-lb. I-beam with a reaction of 20,000 lb. rests on the top flange of a 20-in., 75-lb. I with the bottom of the former 4½ in. below the top of the latter, as shown in Fig. 68. Design an end bearing for the 12-in. beam.

p_f = 16,000 lb. per sq. in. p_s = 10,000 lb. per sq. in. of gross area. Rivets, $\frac{3}{4}$ in. For rivets, p_s = 12,000 lb. per sq. in. p_b = 24,000 lb. per sq. in.

Assume that the cut in the lower side of the beam is 7 in. long and $4\frac{3}{4}$ in. deep. Two $6 \times 3\frac{1}{2} \times \frac{3}{8}$-in. angles will be used to receive all the reaction, as shown in Fig. 68, and they will be set with the back of their outstanding legs $7\frac{1}{2}$ in. down from the top of the beam, that is $\frac{1}{4}$ in. below the horizontal cut line. Shear and moment at the point of depth change will, therefore, be resisted by an I-shaped section.

Required area for shear is 20,000/10,000 = 2.00 sq. in. Assuming that all the shear is borne by the intact portion of the beam web without aid from the angles or beam flanges, the area provided = 7.25×0.35 = 2.54 sq. in., which is ample.

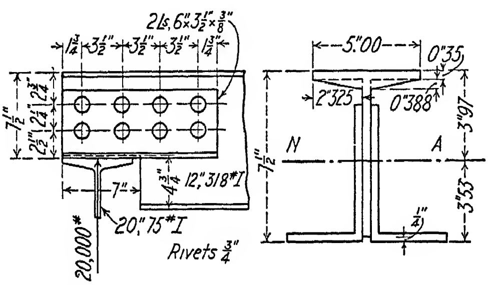


FIG. 68.—Reduced Depth End Bearing for Beam.

Moment at the point of depth change, assuming the reaction as applied at the centre of the flange of the supporting girder = $20{,}000 \times 3.8$ = 76,000 in.-lb.

Required section modulus = 76,000/16,000 = 4.75 in.^{3} A rough calculation shows that the intact part of the web alone would give a much smaller section modulus than this. Hence the seat angles will be required in some measure for the resistance of the moment as a tension flange of the $7\frac{1}{2}$-in. I-section. The section modulus of this section will have to be determined.

Taking the statical moment of the various parts of this section about, say, the top horizontal limit of the section, the centre of gravity, and hence the neutral axis, is found to be 3.97 in. down from the top. The moment of inertia about the neutral axis is found to be 86.5 in.^{4} Hence S = 86.5/3.97 = 21.8 in.^{3}, which is far more than is required.

Riveting.—As the horizontal cut surface of the beam web is not considered as bearing directly on the girder flange, there should be immediately above the girder flange enough rivets in the seat angles to transmit the full reaction to the beam web. The value of a rivet is its bearing value = $0.75 \times 0.35 \times 24{,}000$ = 6300 lb. Hence 20,000/6300 = 4 rivets are required. These are shown in Fig. 68.

To the right of the point of depth change enough rivets are required to transmit out of the angles any longitudinal force they may resist by reaction of their acting in part as a tension flange for the $7\frac{1}{2}$-in. I-section. A sufficiently accurate estimate of this force may be made by neglecting the compression in the angles above the neutral axis and assuming that the tension in the lower portions of the angles is in no degree offset by the compression in the upper portion of their vertical legs.

At the extreme lower fibre of the angles the stress is 3.53/3.97 of the maximum stress on the upper fibre of the beam flange, or, since $M = 76{,}000$ in.-lb.,

$$f_t = \frac{76{,}000}{21.8} \times \frac{3.53}{3.97} = 3100 \text{ lb. per sq. in.}$$

The average stress in the outstanding legs is, on the straight line basis, 2935 lb. per sq. in., and these two legs resist $2935 \times 2 \times 3.125 \times 0.375 = 6850$ lb. The portions of the vertical legs bearing tension resist $1550 \times 3.53 \times 2 \times 0.375 = 4120$ lb. The total tension, therefore, is $6850 + 4120 = 10{,}980$ lb. To resist this, $10{,}970/6300 = 2$ rivets are required. Two are shown in the lower gauge line to carry this tension and two in the upper gauge line to accommodate the compression in the upper part of the vertical legs of the angles.

111. Design of Simple Detached Bearing Plate.—A 15-in., 50-lb. I-beam with a reaction of 24,000 lb. is to deliver its load to a brick wall which can safely withstand a bearing pressure of 175 lb. per sq. in. Find the size and thickness of the detached bearing plate necessary. Thickness of wall, 18 in. (Toronto brickwork standards). $p_f = 16{,}000$ lb. per sq. in. Maximum overhang permitted to satisfy deflection requirements $= 4\,t$.

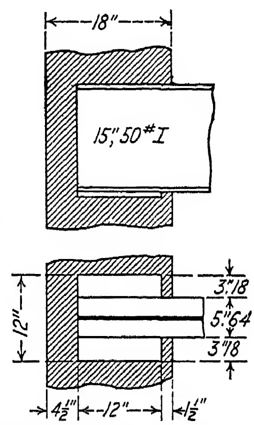


FIG. 69.—Bearing Plate.

Area of plate required $= 24{,}000/175 = 137$ sq. in. Adopt a plate 12×12 in. placed in the wall as shown in Fig. 69. Cantilever projection of plate past edge of beam flange $= (12 - 5.64)/2 = 3.18$ in.

Maximum moment on cantilever strip 1 in. wide due to actual maximum pressure

$$M = \frac{1}{2} \times \frac{24{,}000}{144} \times (3.18)^2 = 844 \text{ in.-lb.}$$

Section modulus required is $844/16{,}000 = 0.053$ in.^{3}

Assuming a $\frac{3}{4}$-in. plate which will satisfy the requirement as to deflection closely enough ($4 \times \frac{3}{4} = 3$ in. permissible overhang), the section modulus provided is $\frac{1}{6} \times 1 \times (0.75)^2 = 0.094$ in.^{3} This is more than adequate, but the permissible ratio of thickness to overhang will not admit of a lesser thickness.

Owing to the bending strength of the beam flange, the flexural stress at the centre of the plate is less than at the edge of the flange.

112. Exercise Problems on the Design of Beams.—The following exercise problems are based on the principles employed in the solution of the problems of this chapter. See Appendix I for the answers.

(1) What should be the section of an I-beam to carry a superimposed uniformly distributed load of 880 lb. per lin. ft. over an effective span of 11 ft.? $p_f = 16{,}000$ lb. per sq. in. $p_s = p_{dc} = 10{,}000$ lb. per sq. in.

(2) A pair of 6000-lb. loads are applied at the third-points of a beam of 12-ft.

span. What should be the section? $p_f = 18{,}000$ lb. per sq. in. $p_s = p_{dc} = 12{,}000$ lb. per sq. in.

(3) If, owing to the restraint of end connections, the maximum moment of the beam of Exercise Problem (2) above may be taken as 85% of that of a simply supported beam, what would then be the required section?

(4) A king-post trussed beam of 16-ft. span, centre to centre of end bearings, carries a uniformly distributed load, including its own weight, of 800 lb. per lin. ft. Find (a) the maximum moment and (b) the maximum shear.

(5) A trussed beam of 30-ft. span is divided into three equal panels by two struts spaced 10 ft. apart, centre to centre. If the total load carried by the beam be 1000 lb. per lin. ft., find the maximum shear in the beam.

(6) An I-beam is to carry a uniformly distributed load, including its own weight of 1500 lb. per lin. ft. over a span of 15 ft. centre to centre of bearings, with no intermediate lateral support. Recommend a size. $p_f = 16{,}000 - 150\,l'/b$. $p_s = p_{dc} = 10{,}000$ lb. per sq. in.

(7) A 10-in., 25.4-lb. I-beam 10 ft. long, without lateral supports except at its ends carries two symmetrically placed concentrated loads of 7200 lb. each, 4 ft. apart. Report on its safety. $p_f = 16{,}000 - 200\,l'/b$. $p_s = p_{dc} = 10{,}000$ lb. per sq. in.

(8) A 12-in., 31.8-lb. I-beam without lateral support between ends carries a central load of 9000 lb. over a span of 20 ft. Express an opinion as to its safety. $p_f = 16{,}000 - 150\,l'/b$ lb. per sq. in. $p_s = p_{dc} = 10{,}000$ lb. per sq. in.

(9) Recommend a size for a steel I-beam to carry a uniformly distributed superimposed load of 900 lb. per lin. ft., if the span is 15 ft. and there is no lateral support for the beam between the end bearings. $p_f = 19{,}000 - 300\,l'/b$. $p_s = p_{dc} = 10{,}000$ lb. per sq. in.

(10) An air hoist carrying, including its own weight, a load of 3000 lb. travels along the bottom flange of an 8-in., 18.4-lb. I-beam of 15-ft. span supported laterally only at the ends. Assuming the load as a concentrated one, is the beam safe? $p_f = 16{,}000 - 150\,l'/b$.

(11) A 6-in., 12.5-lb. I is cantilevered freely 5 ft. out from a wall and carries a superimposed uniformly distributed load of 600 lb. per lin. ft. Express an opinion as to its safety. $p_f = 16{,}000 - 200\,l'/b$ lb. per sq. in. $p_s = p_{dc} = 10{,}000$ lb. per sq. in.

(12) A steel I-beam built rigidly into a wall at each end and continuously supported laterally, having a span of 15 ft., carries a uniform load, including its own weight, of 2500 lb. per lin. ft. Recommend a size for the beam. $p_f = 18{,}000$ lb. per sq. in. $p_s = p_{dc} = 12{,}000$ lb. per sq. in.

(13) An 8-in., 21-lb. W.F. of 15-ft. span continuously supported laterally, is built solidly into a massive concrete pier at each end. What total uniformly distributed load (including its own weight) could the beam support? $p_f = 18{,}000$ lb. per sq. in. $p_s = p_{dc} = 12{,}000$ lb. per sq. in.

(14) A 10-in., 25.4-lb. I of 15-ft. span is built solidly into a massive concrete pier at one end and rests on an ordinary bearing plate at the other. What net uniformly distributed superimposed load could the beam support. $p_f = 18{,}000$ lb. per sq. in. $p_s = p_{dc} = 12{,}000$ lb. per sq. in.

(15) A 10-in., 25.4-lb. I-beam of 12-ft. span is rigidly fixed at the ends and restrained against lateral buckling. How much load, uniformly distributed, could be applied to the beam per running foot? $p_f = 16{,}000$ lb. per sq. in. $p_s = p_{dc} = 10{,}000$ lb. per sq. in.

(16) A 6-in., 12.5-lb. I-beam of 20-ft. span is simply supported at the two ends and is continuous over a pier at the centre. If the total uniformly distributed load borne by the beam is 1000 lb. per lin. ft., express an opinion as to the safety of the beam. $p_f = 18{,}000$ lb. per sq. in. $p_s = p_{dc} = 12{,}000$ lb. per sq. in.

(17) A girder consisting of two 15-in., 42.9-lb. I's has a span centre to centre of bearings of 15 ft. In addition to its own weight, the girder carries two columns 5 ft. apart on centres and symmetrically situated with respect to the centre of the girder. Find the maximum permissible load on each of these columns. $p_f = 16{,}000$ lb. per sq. in. $p_s = p_{dc} = 10{,}000$ lb. per sq. in.

(18) An 8-in., 18.4-lb. I of 10-ft. effective span is loaded to capacity by a central concentrated load. What is the maximum deflection? $p_f = 16{,}000$ lb. per sq. in. $E = 29{,}000{,}000$ lb. per sq. in.

(19) A 12-in., 31.8-lb. I-beam of 8-ft. span carries a total uniformly distributed load of 6000 lb. per lin. ft. If it rests on a shelf angle at each end with a bearing of $3\frac{1}{2}$ in., express an opinion as to the safety of the beam against vertical buckling of the web at the ends. $j = 0.25$. $p_{vc} = 19{,}000 - 173\ d/t$.

(20) An I-beam of 5-ft. span centre to centre is to carry a total uniformly distributed load, including its own weight, of 51,000 lb. Select a section. Length of bearings = 3.5 in. $p_f = 16{,}000$ lb. per sq. in. $p_s = 10{,}000$ lb. per sq. in. $p_{dc} = 15{,}000 - 150\ c/t$, where c = clear distance between fillets. $p_{vc} = 19{,}000 - 173\ d/t$. $j = 0.25$.

(21) The reaction at the end of a 20-in., 65.4-lb. I-beam is 80,000 lb. The length of flange bearing on the support is $5\frac{1}{2}$ in. Investigate the beam for shear, diagonal compression and vertical compression. $p_s = 10{,}000$ lb. per sq. in. $p_{dc} = p_{vc} = 15{,}000 - 120\ d/t$. $j = 0.25$.

(22) A 12-in., 31.8-lb. I-beam of 5-ft. span centre to centre carries a *total* uniformly distributed load of 16,000 lb. per lin. ft. If the end bearings are $3\frac{1}{2}$ in. long, express an opinion as to the safety of the beam in shear, diagonal web compression and vertical web compression. $p_s = 10{,}000$ lb. per sq. in. $p_{dc} = 19{,}000 - 250\ c/t$. $p_{vc} = 15{,}000 - 110\ c/t$. ($c$ = clear distance between fillets.) $j = 0.25$.

(23) An 18-in., 54.7-lb. I-beam of 5-ft. effective span carries a superimposed uniform load of 30,000 lb. per lin. ft. Express an opinion concerning the safety of the beam in shear and web compression, if the length of the support is 6 in. $p_s = 10{,}000$ lb. per sq. in. $p_{dc} = 15{,}000 - 150\ c/t$. $p_{vc} = 19{,}000 - 173\ d/t$. ($c$ = clear distance between fillets.) $j = 0.25$.

(24) A 15-in., 42.9-lb. I-beam of 6-ft. span centre to centre of bearings, carries a concentrated load of 84,000 lb. 2 ft. from the centre of one support and applied over a length of 14 in. If the length of the support is 6 in., report on the safety of the beam in shear and web compression. $p_s = 10{,}000$ lb. per sq. in. $p_{dc} = 15{,}000 - 150\ c/t$. $p_{vc} = 19{,}000 - 173\ d/t$. $j = 0.25$ for supports and 0.50 for intermediate concentrated load. (c = clear distance between fillets.)

(25) An 8-in., 21-lb. W.F. is cantilevered out 3 ft. on each side of a support 6 in. wide. It carries a uniformly distributed load of 5600 lb. per lin. ft. including its own weight. Report on the safety of the beam. $p_f = 18{,}000$ lb. per sq. in. $p_s = 10{,}000$ lb. per sq. in. $p_{dc} = 15{,}000 - 130\ c/t$. $p_{vc} = 18{,}000 - 150\ d/t$. $j = 0.50$. (c = clear distance between fillets.)

(26) A 10-in., 25.4-lb. I rests on two piers 10 ft. apart centre to centre and extends 3 ft. 3 in. beyond the centre of each. The total load borne by the beam, including its own weight, is 6100 lb. per lin. ft. The supports are 8 in. long. Report

on the sufficiency of the beam. $p_f = 16{,}000$ lb. per sq. in. $p_s = p_{dc} = 10{,}000$ lb. per sq. in. $p_{vc} = 19{,}000 - 173\, d/t$. $j = 0.50$. Neglect deflection.

(27) A steel lintel carries a $13\frac{1}{2}$-in. brick wall weighing 120 lb. per cu. ft. over a clear opening of 10 ft. Calculate the probable maximum bending moment on the lintel due to the brickwork if the wall is solid and well seasoned for some distance above and adjacent to the span, and if the bearing at each end is 9 in.

(28) A lintel composed of two steel I-beams is to carry a $13\frac{1}{2}$-in. brick wall over a clear opening of 12 ft. with a bearing of $8\frac{1}{2}$ in. at each end. The brickwork is well seasoned and not broken up by openings and the wall continues past the supports of the beam for several feet at each end. Recommend a size for the beams. $p_f = 16{,}000$ lb. per sq. in. $p_s = p_{dc} = 10{,}000$ lb. per sq. in. Weight of brickwork = 120 lb. per cu. ft. Maximum deflection = 1/320 of span. $E = 30{,}000{,}000$ lb. per sq. in.

(29) A thoroughly seasoned and well-built brick wall 18 in. thick is carried by a lintel composed of two I-beams over a clear opening of 8 ft. with a bearing of 9 in. at each end. Suggest a section. Weight of brickwork = 120 lb. per cu. ft. $p_f = 18{,}000$ lb. per sq. in. $p_s = p_{dc} = 12{,}000$ lb. per sq. in. Maximum deflection = 1/360 of span. $E = 29{,}000{,}000$ lb. per sq. in.

(30) A 20-in., 65.4-lb. I-beam of 20-ft. span has two opposite holes for $\frac{7}{8}$-in. rivets punched out of the bottom flange at mid-span. What uniform load in pounds per lineal foot might safely be applied to the beam? $p_f = 16{,}000$ lb. per sq. in. $p_s = p_{dc} = 10{,}000$ lb. per sq. in.

(31) A 12-in., 31.8-lb. I-beam of 15-ft. span contains near mid-span open rivet holes for $\frac{3}{4}$-in. rivets in each gauge line of its bottom flange, and these holes are opposite each other. Estimate the total safe load in pounds that may be applied to the beam. $p_f = 18{,}000$ lb. per sq. in. $p_s = p_{dc} = 12{,}000$ lb. per sq. in.

(32) A 10-in., 25.4-lb. I-beam of 12 ft. span continuously supported laterally, carries a single concentrated load of 8000 lb. at mid-span applied to the bottom flange by two $\frac{3}{4}$-in. bolts directly opposite to each other. Express an opinion as to the safety of the beam. $p_f = 16{,}000$ lb. per sq. in. $p_s = p_{dc} = 10{,}000$ lb. per sq. in.

(33) A 15-in., 42.9-lb. I is reinforced with a $6 \times \frac{1}{2}$-in. plate on each flange connected thereto by $\frac{3}{4}$-in. rivets driven opposite each other on two gauge lines. Find the total uniformly distributed load, including its own weight, which the reinforced beam will carry over a span, centre to centre, of 20 ft. $p_f = 16{,}000$ lb. per sq. in. $p_s = p_{dc} = 10{,}000$ lb. per sq. in.

(34) A 20-in., 65.4-lb. I is reinforced with a $7 \times \frac{1}{2}$-in. plate on each flange connected thereto by $\frac{7}{8}$-in. rivets driven staggered on two gauge lines. Find the total uniformly distributed load, *exclusive* of its own weight, which the reinforced beam will carry over a span, centre to centre, of 25 feet. $p_f = 18{,}000$ lb. per sq. in. $p_s = p_{dc} = 12{,}000$ lb. per sq. in.

(35) An 18-in., 54.7-lb. I is reinforced with a $6 \times \frac{3}{8}$-in. plate on each flange connected thereto by $\frac{7}{8}$-in. rivets driven staggered on two gauge lines. Find the net uniformly distributed load, in pounds per lineal feet, which the reinforced beam will carry over a span, centre to centre, of 22 feet. $p_f = 16{,}000$ lb. per sq. in. $p_s = p_{dc} = 11{,}000$ lb. per sq. in.

(36) A 15-in., 42.9-lb. I of 20-ft. span is reinforced by a $6 \times \frac{3}{8}$-in. plate riveted to each flange with $\frac{3}{4}$-in. rivets driven opposite each other on two gauge lines. If the beam is adequately supported laterally and carries a total uniformly distributed load of 2000 lb. per lin. ft., express an opinion concerning the safety of the beam so far as bending moment is concerned. $p_f = 16{,}000$ lb. per sq. in.

(37) A 10-in., 25.4-lb. I-beam of 15-ft. span is reinforced with one $6 \times \frac{3}{8}$-in. plate on each flange, the rivets being spaced in each flange on a staggered pitch of 6 in. Find the total safe uniformly distributed load, if $p_f = 16{,}000$ and $p_s = p_{dc} = 10{,}000$ lb. per sq. in., respectively. Rivets, $\frac{3}{4}$ in. dia.

(38) A 12-in., 31.8-lb. I, 20 ft. long, is reinforced by adding a $6 \times \frac{1}{4}$-in. plate to each flange, and is loaded to capacity. If the permissible bending stress on steel is 16,000 lb. per sq. in., and the permissible shearing and bearing stresses on rivets are 10,000 and 20,000 lb. per sq. in., respectively, find how many $\frac{3}{4}$-in. rivets would be *theoretically* necessary between the end of the plate and the centre of the girder in each flange, if near the centre of the span they be driven staggered on two gauge lines.

(39) Find the length of reinforcing plates theoretically necessary in the case of the beam of Exercise Problem (34) above.

(40) An 8-in., 18.4-lb. I heads into another beam of the same size and is connected to it by two angles $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{5}{16}$ in. If the reaction of the first beam is 10,000 lb., find the number of field rivets required through the web of the second beam to give a safe connection. Neglect tension in the rivet shafts. Rivets, $\frac{3}{4}$ in. Safe shearing and bearing stresses on field rivets = 8000 and 16,000 lb. per sq. in., respectively.

(41) Find the number of shop and field rivets necessary to connect a 10-in., 25.4-lb. I to an 18-in., 54.7-lb. I, using $\frac{3}{8}$-in. angles. The end shear in the 10-in. I is 24,500 lb. Rivets, $\frac{3}{4}$ in. Permissible shear in shop rivets 10,000 lb. per sq. in., and in field rivets 8000 lb. per sq. in. Permissible bearing in shop rivets 20,000 lb. per sq. in., and in field rivets 16,000 lb. per sq. in. Neglect turning moment on rivets in beam web.

(42) Devise a suitable connection for a 10-in., 25.4-lb. I with end shear 10,800 lb. to the web of a 15-in., 42.9-lb. I using $\frac{3}{4}$-in. rivets, and neglecting the moment on the rivets in the beam web. Permissible shear on shop rivets 12,000 lb. per sq. in.; bearing on shop rivets 24,000 lb. per sq. in.; shear on field rivets 9000 lb. per sq. in.; bearing on field rivets 18,000 lb. per sq. in.

(43) A 12-in. I-beam is attached to a girder by a pair of connection angles containing three rivets at 3-in. spacing in a vertical line on $2\frac{1}{2}$-in. gauge. If the reaction is 15,000 lb. and the restraining moment at the support is neglected, find the resultant stress on one of the extreme rivets through the web of the beam.

(44) Two 12-in., 31.8-lb. I's with a reaction of 12,000 lb. each head into a girder consisting of one 18-in., 54.7-lb. I directly opposite each other and connected to the girder by the same field rivets. Find the number of field rivets through the web of the girder. Rivets, $\frac{3}{4}$ in. Safe shearing and bearing stresses = 9000 and 18,000 lb. per sq. in., respectively. Connection angles, $\frac{3}{8}$ in. thick.

(45) Two 15-in., 42.9-lb. I's each carrying a total uniformly distributed load of 45,000 lb. frame into a 20-in., 65.4-lb. I directly opposite each other and at the same elevation. If $\frac{3}{4}$-in. rivets are used and the safe shearing and bearing stresses on field rivets are assumed at 8000 and 16,000 lb. per sq. in., respectively, find the number of rivets required through the web of the 20-in. beam and suggest an arrangement for them. Neglect tension in the rivet shafts.

(46) Calculate the flexural stress at the face of the beam web in the bearing plate of Art. 111, assuming the moment to be allocated in proportion to the section moduli of the beam flange (neglecting the fillet) and the plate.

(47) It is proposed to seat a column carrying 85,000 lb. on the top flange of an 18-in., 54.7-lb. I-beam at a point some distance from one support. Will the beam

web be able to resist the load safely? Assume $j = 0.50$. $p_{vc} = 19{,}000 - 173\ d/t$. Length of column base parallel to beam = 14 in.

(48) A beam connection delivers 15,000 lb. to a girder web. If in the beam web there are three rivets spaced 3 in. apart vertically, and the gauge for this line of rivets is $2\frac{1}{4}$ in., find the total stress on the most highly stressed rivet due to the combination of direct and turning force.

(49) Two $\frac{7}{8}$-in. holes are punched opposite each other in the tension flange of a 15-in., 42.9-lb. I-beam at mid-span. If the span is 15 ft., what total uniformly distributed load, including the weight of the beam, may be safely carried by the beam? $p_f = 18{,}000$ lb. per sq. in. $F = 1.0 - 0.25/d^{1/2}$.

(50) A roof of 45-deg. slope is supported on roof trusses 18 ft. centre to centre. The purlins are spaced 4.5 ft. centre to centre up the slope. If the specified intensity of wind pressure on a flat surface normal to the wind is 30 lb. per sq. ft., calculate the wind moment in a purlin.

$$p_n = p \cdot \frac{2 \sin \alpha}{1 + \sin^2 \alpha}$$

(51) The spacing of purlins on a roof is 4 ft. and the overhang of the roofing past the centre of the eave purlin is 18 in. If the normal loading on the roof is 30 lb. per sq. ft., what is the resulting loading per lineal foot on the eave purlin?

CHAPTER VI

BOX GIRDERS

113. Formulae for Box Girder Design.—The following formulae are applicable to the design of box girders:

Moment of inertia of the cross section

$$s = \frac{Q_h}{A_n} \tag{1}$$

$$I_n = I_g - I_h - A_n s^2 \tag{2}$$

$$I_n' = I_g - I_h \tag{3}$$

$$I_{nc}' = \left(1.0 - \frac{0.25}{d_e^{1/2}}\right) I_n' \tag{4}$$

Section modulus of a box girder

$$S_n = \frac{I_n}{y_e} \tag{5}$$

$$S_n' = \frac{I_n'}{y_e} \tag{6}$$

$$S_{nc}' = \frac{I_{nc}'}{y_e} \tag{7}$$

Theoretical length of any cover plate for uniformly loaded girder (see Appendix II)

$$x_n = l\sqrt{\frac{s_1' + s_2' + \ldots s_n'}{S}} \tag{8}$$

Existing shearing stress per lineal inch of girder at a selected level

$$f_{s\hat{v}} = \frac{QV}{I} \tag{8a}$$

Pitch of rivets in unloaded flange

$$p = \frac{nvI}{QV} \tag{9}$$

Pitch of rivets in loaded flange

$$p = \frac{nv}{\left\{\left(\frac{QV}{I}\right)^2 + w^2\right\}^{1/2}} \tag{10}$$

References

Hool and Kinne—Structural Members and Connections.
Engineering News-Record, March 10, 1927, p. 401.

The significance of the symbols employed in the above formulae, and elsewhere in this chapter, is as follows:

A_g = gross area of section;
A_n = net area of section;
d = depth of beam;
d_e = extreme depth of composite section;
f_s = existing shearing stress;
I = moment of inertia of cross section (general symbol);
I_g = moment of inertia of gross section taken about its own gravity axis;
I_h = moment of inertia of the holes in a section about the gravity axis of the gross section;
I_n = moment of inertia of net cross section taken about neutral axis of net section;
I_n' = moment of inertia (uncorrected) of net cross section taken about neutral axis of gross section;
I_{nc}' = moment of inertia of net section taken about gravity axis of gross section and corrected to compensate for error in assumption of the position of the neutral axis;
l = span of girder, centre to centre of bearings;
n = number of rows of rivets that may be allocated to one web;
p = pitch of flange rivets;
Q = statical moment of area of all parts on one side of neutral axis being served by all webs through the medium of the full group of similarly situated rivets under consideration;
Q_h = statical moment of holes taken about gravity axis of gross section;
S = total required net section modulus of girder;
S_n = section modulus of net section with respect to gravity axis of net section;
S_n' = uncorrected section modulus of net section with respect to gravity axis of gross section;
S_{nc}' = section modulus of net section taken about gravity axis of gross section and corrected to compensate for error in assumption of the position of the neutral axis;
s = shift of the neutral axis of a section brought about by perforation;
s_1' = section modulus required of two outside flange plates in an assemblage;
$s_2', s_3' \ldots s_n'$ = section moduli contributed to the total required section modulus of the girder by successive pairs of flange plates from the outside, beginning with the second pair;
x_n = theoretical length of the nth cover plate from the outside;
y_e = distance of the extreme fibre from the neutral axis of the net or the gross section, as the expression may indicate;
V = total shear resisted by one web;

v = least value, or safe resistance, of a rivet;
w = superimposed load on a girder per inch of length, applicable to one web.

114. Design of Uncompensated Box Girder.—A box girder of 22 ft. 8 in. clear span to carry a superimposed load of 8650 lb. per lin. ft. is to consist of two I-beams with flange plates on the two flanges of the same gross area, as shown in Fig. 70. The completed section must not be over 23 in. deep nor 14 in. wide because of certain necessary clearances. The girder is stayed continuously laterally. It is supported on concrete piers. Prepare a design for the girder.

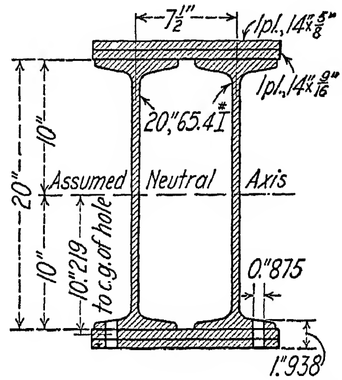


Fig. 70.—Section of Uncompensated Box Girder.

Permissible stresses: Flexure, p_f = 16,000 lb. per sq. in., the holes in the tension flange only to be deducted; shearing, actual, at the neutral axis, p_s = 12,000 lb. per sq. in.; diagonal compression, $p_{dc} = 12{,}000/(1 + c^2/1500\,t^2)$ (c = clear distance between fillets, t = web thickness); vertical compressive stress, $p_{vc} = 19{,}000 - 173\,d/t$, counting as effective column section at the support the area $(a + d/4)t$, the length of bearing being a, and the depth of beam d; shearing on shop rivets, p_s = 10,000 lb. per sq. in; bearing on shop rivets, p_b = 20,000 lb. per sq. in.; bearing on concrete supports, 400 lb. per sq. in.

Rivets, $\frac{3}{4}$ in.; holes for deduction, $\frac{7}{8}$ in. Maximum allowable deflection = 1/360 of the centre to centre span. E = 29,000,000 lb. per sq. in.

Assuming the distance from the face of a support to the centre of a bearing as 8 in., the centre to centre span = 24 ft.

Assuming the weight of the girder as 250 lb. per lin. ft., the total uniformly distributed load = 8650 + 250 = 8900 lb. per lin. ft.

Moment.—Maximum moment = $Wl/8$ = (8900 × 24) × 288/8 = 7,680,000 in.-lb.

Required section modulus at the centre is

$$S = \frac{M}{p_f} = \frac{7{,}680{,}000}{16{,}000} = 481 \text{ in.}^3$$

Assume a girder, as shown in Fig. 70, with a maximum section of two 20-in., 65.4-lb. I's, two 14 × $\frac{5}{8}$-in. and two 14 × $\frac{9}{16}$-in. flange plates, with the $\frac{9}{16}$-in. plates next the beam flanges. The section will be reduced at the proper points by cutting off the outer cover plates.

This section will be sufficient to keep the deflection within the limit of 1/360 of span. To realize this limit, for a symmetrical section, the extreme depth d_e must be not less than $75\,p_fl/E$, for a simply supported, uniformly loaded beam. For p_f = 16,000 lb. per sq. in., and E = 29,000,000 lb. per sq. in. d_e must be

not less than $l/24.1$ or $288/24.1 = 11.95$ in. The adopted section is very much deeper than this.

Gross moment of inertia of two 20-in., 65.4-lb. I's about the neutral axis of the gross section, $I_1 = 2 \times 1169.5 = 2339$ in.^{4}

Gross moment of inertia of two sets of flange plates, $I_2 = 2(I_0 + Ay_0^2)$, where I_0 = moment of inertia of plates on *one* flange about their own gravity axis; A = area of plates on one flange; and y_o = distance of gravity axis of plates on one flange about neutral axis of gross section of girder.

$$I_2 = 2\{(\tfrac{1}{12} \times 14 \times 1.1875^3) + (14 \times 1.1875 \times 10.594^2)\}$$
$$= 2(2 + 1860) = 3724 \text{ in.}^4$$

Evidently, the quantity I_o might have been omitted without material error.

Total gross moment of inertia, I_g, of section $= I_1 + I_2 = 2339 + 3724 = 6063$ in.^{4}

Moment of inertia, I_h, of two $\frac{7}{8}$-in. holes through a total of $\frac{3}{4} + 1\frac{3}{16} = 1\frac{15}{16}$ in. of metal is approximately $2(0.875 \times 1.938) \times 10.219^2 = 354$ in.^{4}

Net uncorrected moment of inertia of girder, $I_n' = 6063 - 354 = 5709$ in.^{4}

From Eq. (4), Art. 113, the net corrected moment of inertia is

$$I_{nc}' = \left(1.0 - \frac{0.25}{d_e^{1/2}}\right)I_n' = \left(1.0 - \frac{0.25}{22.375^{1/2}}\right) \times 5709 = 5400 \text{ in.}^4$$

From Eq. (7), Art. 113, net corrected section modulus is

$$S_{nc}' = \frac{I_{nc}'}{y_e} = 5400/11.1875 = 483 \text{ in.}^3$$

This is adequate.

Length of Cover Plates.—Since the moment decreases at a high rate near the ends of the girder, the outer, or $\frac{5}{8}$-in. plates, will be required for only a fraction of the girder length. The inner, or $\frac{9}{16}$-in., plates will be carried full length. For the section where inner cover plates only are employed, the uncorrected net moment of inertia, 5709, will be diminished by the moment of inertia of the outer plates, the gross area being considered for the plate on the compression flange and the net area for the plate on the tension flange. The reduction is approximately $(14 \times 0.625 \times 10.875^2) + \{(14 - 1.75) \times 0.625 \times 10.875^2\} = 1941$ in.^{4} The net moment of inertia of the two beams and the two inner plates, is, therefore, $5709 - 1941 = 3768$ in.^{4}, or when corrected

$$I_{nc}' = \left(1.0 - \frac{0.25}{21.125^{1/2}}\right) \times 3768 = 3562 \text{ in.}^4$$

The corresponding section modulus is

$$S_{nc}' = \frac{3562}{10.5625} = 337.5 \text{ in.}^3$$

The theoretical point of cut-off of the outer $\frac{5}{8}$-in. plates may be determined from Eq. (8) of Art. 113.

Corrected section modulus required at centre is 481 in.^{3} and that provided by beams and inner pair of plates is 337.5. Difference, or $s_1' = 481 - 337.5 = 143.5$.

Hence, theoretical length of outer flange plates, $x_1 = 24(143.5/481)^{1/2} = 13.1$ ft. To this add sufficient length at each end to accommodate two rows of rivets, or say 9 in., to ensure action of plates where first needed. Practical length is then 14 ft. 8 in.

Shear.—End reaction $= 8900 \times 24/2 = 106{,}800$ lb.

For flexural economy, the deepest beams permitted should be used, that is 20-in. The two 20-in., 65.4-lb. I's, give a total shear area of $2 \times 20 \times 0.5 = 20$ sq. in.

Average shearing stress on web $= 106{,}800/20 = 5340$ lb. per sq. in.

As the maximum shearing stress for typical beams is usually from 10 to 20% in excess of the average stress, it is evident that the section chosen is adequate. For beams of this size the excess is about 17% and the maximum stress is, therefore, about $5340 \times 1.17 = 6250$ lb. per sq. in.

To calculate the exact shearing stress, it is necessary to use Eq. (10), Art. 72.

Web Buckling.—Existing diagonal compressive stress on beam webs at end, being equal to average shearing stress, is 5340 lb. per sq. in.

Permissible diagonal compressive stress, $p_{dc} = 12{,}000/(1 + 17^2/1500 \times 0.5^2) = 6780$ lb. per sq. in. Hence, the section is adequate.

Existing vertical compressive stress on web of one beam over support $=$ reaction $\div 2(a + d/4)t = 106{,}800/2(14 + 20/4)0.5 = 5620$ lb. per sq. in., the length of bearing being assumed as 14 in.

Permissible vertical compressive stress, $p_{vc} = 19{,}000 - 173\, d/t = 19{,}000 - 173 \times 20/0.5 = 12{,}080$ lb. per sq. in. Section is amply strong for vertical buckling.

Bearing on Supports.—End reaction $= 106{,}800$ lb.

Required bearing area on concrete support $= 106{,}800 = 267$ sq. in.

Use a 14×20-in. bearing plate $= 280$ sq. in.

Thickness of bearing plate must be sufficient to transmit existing pressure of approximately 400 lb. per sq. in. to extreme edges of plate without exceeding bending stress of 16,000 lb. per sq. in. on plate.

Bearing plate acts as cantilever beam, Fig. 71, subjected to upward load of 400 lb. per sq. in.

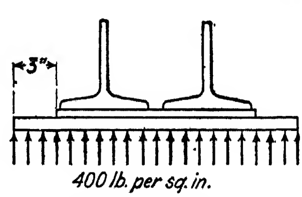


FIG. 71.—Cantilever Beam Action of Bearing Plate.

Moment at edge of cover plate on strip 1 in. wide $= \frac{1}{2} \times 400 \times (3)^2 = 1800$ in.-lb.

Section modulus required, $S = 1800/16{,}000 = 0.113$ in.^{3}

But $S = \frac{1}{6}\, bt^2$, where b is breadth and t thickness of rectangular section. Letting $\frac{1}{6}\, bt^2 = 0.113$, it follows that $t = 0.82$. Adopt a thickness of $\frac{7}{8}$ in. As the overhang is only $3/0.875 = 3.43$ times the bearing plate thickness, the usual limit for deflection, viz. 4, is not exceeded.

Details.—For details, see Fig. 72.

Number of rivets required in each outer, $\frac{5}{8}$-in., flange or cover plate from its end to the point of maximum moment in the girder is the number necessary to transmit to this plate the total stress that it should carry at mid-span.

For the outer $\frac{5}{8}$-in. plate alone, the average working stress is $16{,}000 \times 10.87/11.19 = 15{,}540$ lb. per sq. in.

Total stress borne by one $14 \times \frac{5}{8}$-in. tension plate with two $\frac{7}{8}$-in. holes out is $(14 - 2 \times 0.875) \times 0.625 \times 15{,}540 = 119{,}200$ lb.

Safe resistance of a $\frac{3}{4}$-in. rivet in this situation is its value in single shear $= 0.442 \times 10{,}000 = 4420$ lb.

Number of rivets required through outer plate from its end to centre $= 119{,}200/4420 = 27$, or say, 14 in each line.

Number of rivets required through the inner cover plate from centre of bearing to end of outer cover plate must develop the strength of the inner plate. Average stress in plate $= 16{,}000 \times 10.28/10.56 = 15{,}560$ lb. per sq. in.

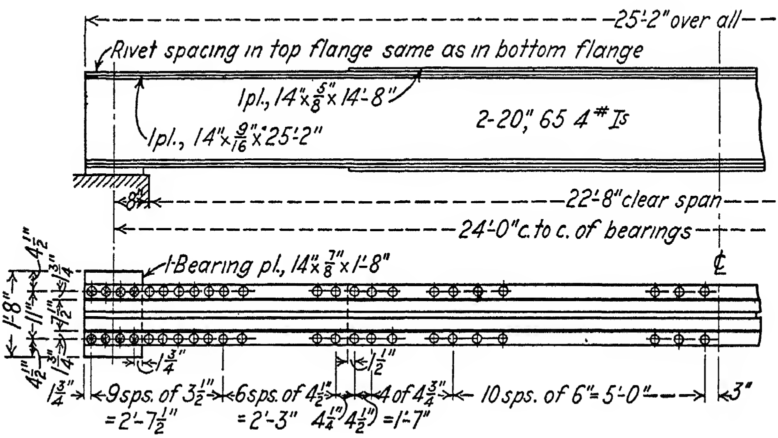


FIG. 72.—Detail of Box Girder.

Capacity of plate $= (14 - 2 \times 0.875) \times 0.5625 \times 15{,}560 = 107{,}400$ lb. Number of rivets required in plate $= 107{,}400/4420 = 24$, or 12 in each line.

Since maximum allowable spacing is 6 in., the rivets are spaced at 6 in. centres from the centre to a point 5 ft. 3 in. from the centre of the girder, beyond which the spacing diminishes gradually to $3\frac{1}{2}$ in. to allow for the rapid increment in flange stress near the ends. The numbers of rivets provided in the outer and inner cover plates, namely 32 and 28 respectively, are in excess of requirements.

It would be possible to calculate the exact theoretical spacing required by the method of Art. 116, but the method employed above is satisfactory for the case in hand.

The rivets connecting the bearing plate to the girder must be countersunk on the under side.

The same riveting arrangement is adopted for the top flange as for the bottom one.

115. Capacity of a Compensated Built-Up Box Girder.—A compensated, built-up, 3-web box girder of the section shown in Fig. 73, and of 40-ft. span centre to centre is to be loaded uniformly to its safe capacity. The girder is stayed laterally at short intervals. Rivet holes are to be deducted only on the tension side. Find the total safe superimposed uniform load.

Rivets, $\frac{7}{8}$ in. Permissible flexural stress, p_f = 18,000 lb. per sq. in. Permissible average shearing stress in web (including the effect of diagonal buckling, as in the specification of the A.I.S.C.), p_s = 12,000 lb. per sq. in.

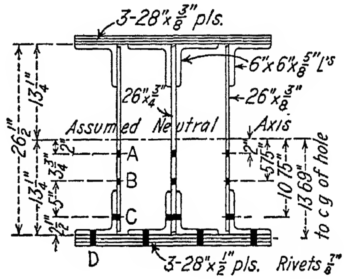


FIG. 73.—Compensated Built-Up Box Girder.

It will be necessary, first, to find the net section modulus. If the aggregate thickness of cover plates on the tension flange were equal to that of the plates on the compression flange, the loss of section by rivet holes below the centre line of the webs would throw the neutral axis substantially above the line mentioned. By adding a compensating $\frac{1}{8}$ in. to the thickness of each of the tension flange plates, the moment of inertia of the rivet holes is more than offset and the neutral axis is kept in the desirable position of the centre line of the webs. No correction of the net section modulus is, therefore, needed when calculating this quantity with respect to the centre line of the webs.

The computation of the net moment of inertia of the compensated section is set forth in Table 2. In this table, distances above the centre line of the webs, which is assumed to be the position of the neutral axis, are designated as positive, and distances measured below it are negative. Areas of metal are marked plus, and areas of holes minus. The moment of inertia of the holes about their own gravity axes is neglected, since it is too small to affect the result appreciably. The summations of the second, fourth, fifth and sixth columns are algebraic. The summations of the fourth and fifth columns when added should equal the summation of the sixth.

Net section modulus of girder is net moment of inertia divided by distance to extreme (tensile) fibre, or

$$S = \frac{I}{y_e} = \frac{22{,}090.9}{14.75} = 1493 \text{ in.}^3$$

As the section has been so compensated as to bring the neutral axis of the net section as nearly as possible to the centre line of the webs, the axis about which the net moment of inertia has been taken, no correction of S need be made.

Now total safe uniformly distributed load that may be carried by girder, including its own weight, so far as flexure is concerned, is, from Eq. (5), of Art. 72

$$W_f = \frac{8\,Sp_f}{l} = \frac{8 \times 1493 \times 18{,}000}{480} = 447{,}900 \text{ lb.}$$

Weight of main material of girder, assuming for convenience that all cover plates run full length, and taking the gross area (169.38 sq. in.) from the table, is $169.38 \times 3.4 \times 40 = 23{,}000$ lb. Adding 6% for diaphragms and rivet heads, the weight of the girder itself, between centres of end bearings = 24,380 lb.

TABLE 2

NET SECTION MODULUS OF COMPENSATED BUILT-UP BOX GIRDER

(1)	(2)	(3)	(4)	(5)	(6)
Part of Section	Area of Part, Square Inches	Distance (y_0) from Assumed Neutral Axis to Gravity Axis of Part, Inches	Moment of Inertia (I_0) of Part about Its Own Gravity Axis, Inches^4	Ay_0^2, Inches^4	$I=I_0+Ay_0^2$, Inches^4
3 webs.........	+39.00	0.0	+2197.0	0.0	+2,197.0
4 top angles....	+28.44	+11.52	+96.8	+3,780.0	+3,876.8
4 bottom angles	+28.44	−11.52	+96.8	+3,780.0	+3,876.8
3 top covers....	+31.50	+13.81	+3.3	+6,007.7	+6,011.0
3 bottom covers	+42.00	−14.00	+7.9	+8,232.0	+8,239.9
3 holes A......	−1.50	−2.00	....	−6.0	−6.0
3 holes B.....	−1.50	−5.75	....	−49.6	−49.6
3 holes C......	−4.00	−10.75	....	−462.0	−462.0
4 holes D.....	−8.50	−13.69	....	−1,593.0	−1,593.0
	+153.88		+2401.8	+19,689.0	+22,090.9

Safe superimposed uniformly distributed load for moment = 447,900 − 24,380 = 423,520 lb.

Total safe load on the basis of shear resistance, $W_s = 2\,p_sA_w$ where p_s = safe shearing stress considered as an average stress on the webs only and A_w = gross area of the webs. From the table, $A_w = 39.00$ sq. in., hence

$$W_s = 2 \times 12{,}000 \times 39 = 936{,}000 \text{ lb.}$$

Hence safe superimposed uniformly distributed load, so far as shear is concerned, is 936,000 − 24,380 = 911,620 lb. The capacity is, therefore, determined by moment rather than shear.

116. Flange Riveting of Plate and Angle, Three-Web Box Girder.—Find the theoretical rivet spacing required in the flanges of the box girder of Art. 115 at a point 4 ft. from one support under the maximum safe load.

Rivets, $\frac{7}{8}$ in. p_s = 13,500 lb. per sq. in. p_b = 24,000 lb. per sq. in. if in

single shear and 30,000 lb. per sq. in. if in double shear. Assume that at the section considered there is only one cover plate on each flange.

As the total safe uniformly distributed load, including the weight of the girder, is 447,900 lb., the load per lineal foot is 447,900/40 = 11,198 lb. and the total shear at a section 4 ft. from one support is 11,198 × (20 − 4) = 179,000 lb.

Unloaded Flange.—According to Eq. (9) of Art. 113, the safe pitch of rivets attaching a part of the unloaded flange of a box girder to the remainder of the section is

$$p = \frac{nvI}{QV}$$

the gross, rather than the net, moment of inertia being used.

For the rivets through the bottom flange angles and the web plates, Q is the statical moment of these flange angles and the tension cover plate taken about the neutral axis of the whole section. For the rivets through the bottom flange angles and the cover plate, Q is the statical moment of the tension cover plate alone. Table 3, which is readily prepared from the quantities in the table of Art. 115, shows the computation of Q. All distances are taken as positive, as in this calculation Q is an arithmetical quantity.

TABLE 3

COMPUTATION OF Q FOR GIRDER FLANGE

Part of Section	Area of Part, Square Inches	Distance (y_0) from Assumed Neutral Axis to Gravity Axis of Part, Inches	Statical Moment $Q = Ay_0$ of Part, Inches^3
1 bottom cover.....	+14.00	+13.50	+189.0
4 bottom angles....	+28.44	+11.52	+328.0
			+517.0

The least value, v, of a rivet through an outer web is its single shearing value = 0.601 × 13,500 = 8110 lb., while for one through the $\frac{3}{4}$-in. inner web it is its double shearing value, or 16,220 lb. The *gross* moment of inertia of the section at this point is found, with the aid of Table 2, to be 14,398 in.^{4}

Since each of the outer webs is one-half the thickness of the middle one, the amount of total shear at the section under consideration resisted by an outer web = 179,000/4 = 44,750 lb. and by the inner web it is 89,500 lb.

Hence, for the rivets through an outer web ($n = 1$)

$$p = \frac{8110 \times 14{,}398}{517 \times 44{,}750} = 5.04 \text{ in.}$$

For the rivets through the inner web ($n = 1$)

$$p = \frac{16{,}220 \times 14{,}398}{517 \times 89{,}500} = 5.04 \text{ in.}$$

Rivets connecting a cover plate to the flange angles have as their safe resistance the single shearing value, or 8110 lb. Q, from the table, is 189, I is as above, and V is as employed for the corresponding ribs in the calculation above. Hence the theoretical pitch of the rivets through an outer flange angle and a cover plate ($n = 1$) is

$$p = \frac{8110 \times 14{,}398}{189 \times 44{,}750} = 13.8 \text{ in.}$$

Two angles being attached to the centre rib, which takes double the shear of an outer rib, and two rows of rivets being driven through these angles into the cover plate, p will work out also as 13.8 in. Practical limitations respecting rivet spacing would require this theoretical pitch to be reduced in all cases so as not to exceed 6 in.

Loaded Flange.—Rivets through the top flange angles and the webs must not only resist the increment of horizontal shear per lineal inch, but must transmit to the webs the superimposed load per lineal inch. Hence, according to Eq. (10) of Art. 113, the theoretical pitch should be for loaded flanges

$$p = \frac{nv}{\left\{\left(\frac{QV}{I}\right)^2 + w^2\right\}^{1/2}}$$

In this case w, for an outer rib $= 423{,}520/480 \times 4 = 220$ lb. per lin. in., assuming $\frac{1}{4}$ of the vertical superimposed load as going to an outer rib. Hence for an outer rib ($n = 1$)

$$p = \frac{8110}{\left\{\left(\frac{517 \times 44{,}750}{14{,}398}\right)^2 + (220)^2\right\}^{1/2}} = 5.01 \text{ in.}$$

For an inner rib ($n = 1$)

$$p = \frac{16{,}220}{\left\{\left(\frac{517 \times 89{,}500}{14{,}398}\right)^2 + (440)^2\right\}^{1/2}} = 5.01 \text{ in.}$$

So little difference arises in the spacing of the rivets in the top and bottom flanges that the pitch would be made the same for both, say, 5 in. at this section.

For the cover plate rivets on the top flange, the pitch would be the same as for those on the bottom flange. The direct vertical loading does not influence these rivets.

117. Length of Flange Plates for Girder with Irregular Loading.—Let the superimposed load for the girder of Art. 114 be, instead of uniform, two unequal

concentrated loads, as shown in Fig. 74. Determine by graphical means the theoretical and practical length of the outer cover plates.

The bending moment and the required section modulus at the points of concentrated loading and at certain intermediate points is set forth in Table 4. The section modulus required is plotted for the whole span in Fig. 74, it being assumed as fairly represented by a straight line between the points mentioned in the table at the end of this article.

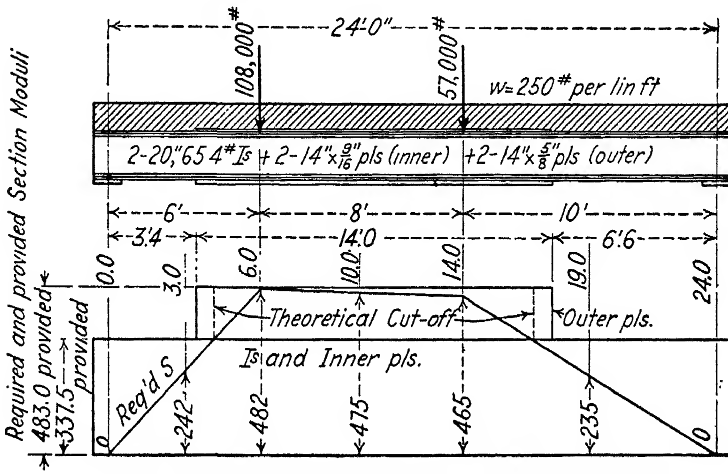


FIG. 74.—Graphical Determination of Length of Flange Plates.

In the problem of Art. 114 the net corrected section modulus of the girder at its maximum section is 483 in.^{3}, and at a section where only the beams and the inner cover plates exist, it is 337.5 in.^{3} Plotting the latter quantity vertically on the section modulus diagram of Fig. 74, and drawing a horizontal line across the diagram, the points of theoretical cut-off of the outer plates are found

TABLE 4

REQUIRED SECTION MODULUS FOR BOX GIRDER

Distance of Point from Left Support, Feet	Uniform Load Moment, Foot-Pounds	Moment from Concentrated Load, Foot-Pounds	Combined Moment, Foot-Pounds	Required Section Modulus S, Inches^3
3.0	7,875	314,250	322,125	242
6.0	13,500	628,500	642,000	482
10.0	17,500	615,500	633,000	475
14.0	17,500	602,500	620,000	465
19.0	11,875	301,250	313,125	235

where this line cuts the section modulus graph. Theoretically these plates should be 12.55 ft. long.

Adding about 9 in. at each end to ensure that they are available for service where first needed, the plates would be made about 14 ft. long in practice. The left-hand end should be located about 3.4 ft. from the centre of the left-hand support.

118. Exercise Problems on the Design of Box Girders.—The following exercise problems are based on the principles employed in the solution of the problems of this chapter. See Appendix I for the answers.

(1) Calculate the section modulus of the girder of Art. 114, assuming the neutral axis as at the gravity axis of the net, rather than of the gross section. Where does the former axis lie?

(2) A box girder of 20 ft. centre to centre consists of two 15-in., 42.9-lb. I's with a 14 × $\frac{1}{2}$-in. plate riveted to each flange by $\frac{3}{4}$-in. rivets. Find the greatest allowable uniformly distributed load which the girder can carry in addition to its own weight. $p_f = 16{,}000$ lb. per sq. in.; $p_s = p_{dc} = 10{,}000$ lb. per sq. in.

(3) A box girder of 24 ft. span consists of two 15-in., 42.9-lb. I's and four flange plates 14 × $\frac{1}{2}$-in. Assuming that the girder has been designed for a maximum flexural stress of 16,000 lb. per sq. in., find the number and distribution of $\frac{3}{4}$-in. rivets required in the flange plates. Safe shearing and bearing stresses on rivets = 12,000 and 24,000 lb. per sq. in. respectively.

(4) A box girder of 20-ft. effective span consists of two 12-in., 31.8-lb. I's with one 14 × $\frac{1}{2}$-in. plate on each flange. What is its total capacity in pounds per lineal foot? Rivets, $\frac{3}{4}$ in. $p_f = 16{,}000$ lb. per sq. in. $p_s = p_{dc} = 10{,}000$ lb. per sq. in. Maximum allowable deflection = 1/360 of span. $E = 29{,}000{,}000$ lbs. per sq. in.

(5) Find the theoretical lengths of the outer and middle cover plates of the box girder of Art. 115, Fig. 73.

CHAPTER VII

PLATE GIRDERS

119. Formulae for Plate Girders.—The following formulae are specially applicable to the design of plate girders:

Shearing stress at any point on a cross section

$$f_s = \frac{QV}{It} \tag{1}$$

Proportioning for shear, web absorbing uniformly all the shearing force V

$$f_{as} = \frac{V}{ht} \tag{2}$$

$$A_w = ht = \frac{V}{p_s} \tag{3}$$

Proportioning for moment by the exact, or moment of inertia, method

$$f_e = \frac{My_e}{I} = \frac{M}{S} \tag{4}$$

$$M = M_r = \frac{Ip_f}{y_e} = Sp_f \tag{5}$$

Proportioning for moment by the approximate (truss-chord) method

$$f_t = \frac{M}{(A_f' + eA_w)d} \tag{6}$$

$$f_c = \frac{M}{(A_f + eA_w)d} \tag{7}$$

$$M_{rt} = (A_f' + eA_w)p_t d \tag{8}$$

$$M_{rc} = (A_f + eA_w)p_c d \tag{9}$$

$$A_t = A_f' + eA_w = \frac{M}{p_t d} \tag{10}$$

$$A_f' = \frac{M}{p_t d} - eA_w \tag{11}$$

$$A_c = A_f + eA_w = \frac{M}{p_c d} \tag{12}$$

$$A_f = \frac{M}{p_c d} - eA_w \tag{13}$$

Proportioning girders with inclined flanges for shear and moment;
Top flange horizontal, bottom flange sloping downward to right at angle α

$$V_w = V - \frac{M \tan \alpha}{d} \tag{14}$$

$$f_{as} = \frac{V - \dfrac{M \tan \alpha}{d}}{ht} \tag{15}$$

$$f_t = \frac{M}{(A_f' + eA_w)d \cos \alpha} \tag{16}$$

$$f_c = \frac{M}{(A_f + eA_w)d} \tag{17}$$

Top flange sloping upward to right at angle β, bottom flange sloping downward to right at angle α

$$V_w = V - \frac{M}{d}(\tan \alpha + \tan \beta) \tag{18}$$

$$f_{as} = \frac{V - \dfrac{M}{d}(\tan \alpha + \tan \beta)}{ht} \tag{19}$$

$$f_t = \frac{M}{(A_f' + eA_w)d \cos \alpha} \tag{20}$$

$$f_c = \frac{M}{(A_f + eA_w)d \cos \beta} \tag{21}$$

Top flange sloping downward to right at angle β, bottom flange sloping downward to right at angle α

$$V_w = V - \frac{M}{d}(\tan \alpha - \tan \beta) \tag{22}$$

$$f_{as} = \frac{V - \dfrac{M}{d}(\tan \alpha - \tan \beta)}{ht} \tag{23}$$

$$f_t = \frac{M}{(A_f' + eA_w)d \cos \alpha} \tag{24}$$

$$f_c = \frac{M}{(A_f + eA_w)d \cos \beta} \tag{25}$$

Theoretical length of any cover plate for uniformly loaded girder

$$x_n = l\left(\frac{a_1 + a_2 + \ldots a_n}{A}\right)^{1/2} \tag{26}$$

Maximum deflection of a girder of variable section

$$= \frac{l}{E} \Sigma \frac{M}{I} m \cdot dx \tag{27}$$

Rivet pitch in flanges;
In unloaded flanges and in cover plates

$$p = \frac{nvd}{KV} \tag{28}$$

In loaded flanges

$$p = \frac{nv}{\left\{\left(\frac{KV}{d}\right)^2 + w^2\right\}^{1/2}} \tag{29}$$

Flange stress apportionment factor;
For the angles and plates of a flange, exclusive of web equivalent

$$K = \frac{A_f}{A_f + eA_w} \tag{30}$$

For the cover plates only

$$K = \frac{A_p}{A_f + eA_w} \tag{31}$$

Rivet pitch in unloaded sloping flanges, measured along the slope of the flange, and in the cover plates on these flanges

$$p = \frac{nvd}{KV_w} \tag{32}$$

where V_w is given by Eqs. (14), (18) or (22), whichever applies.

Approximate stress in a girder flange due to a combination of vertical and lateral moment,

Tension flange

$$f_x + f_y = \frac{M_x}{(A_f' + eA_w)d} + \frac{M_y x_e}{I_y} \tag{33}$$

Compression flange

$$f_x + f_y = \frac{M_x}{(A_f + eA_w)d} + \frac{M_y x_e}{I_y} \tag{34}$$

Permissible spacing of intermediate stiffeners, American Institute of Steel Construction—Buildings (1934):

$$s_c = 85\, t\sqrt{(18{,}000/f_{as}) - 1} \tag{35}$$

but not over 72 in.

American Association of State Highway Officials—Highway Bridges (1931):

$$s_c = 100\, t\sqrt{(24{,}000/f_{as}) - 1} \tag{36}$$

but not over 72 in., nor the depth of the web.

American Railway Engineering Association—Railway Bridges (1935):

$$s_c = \frac{255{,}000\, t}{f_{as}} \sqrt[3]{\frac{f_{as} t}{c}} \tag{37}$$

but in no case is s, the spacing centre to centre, to be over 72 in.

REFERENCES

Hool and Kinne—Structural Members and Connections.
Johnson, Bryan and Turneaure—Modern Framed Structures, Pt. III.
Shedd—Structural Design in Steel.
Spofford—Theory of Structures.

The significance of the symbols employed in the above formulae, and elsewhere in this chapter, is as follows:

A = total area of a flange, including the web equivalent;
A_c = area required for the compression flange, including the web equivalent;
A_f = gross area of the particular flange to which the expression applies, exclusive of web equivalent;
A_f' = net area of the tension flange, exclusive of web equivalent;
A_p = area of cover plate or plates considered;
A_t = effective area required for the tension flange, including the web equivalent;
$A_w = ht$ = gross area of a girder web;
a_1 = area required of outside cover plate of a flange;
$a_2, a_3 \ldots a_n$ = areas of successive cover plates of a flange counting from the outside, beginning with the second plate;
α = angle of inclination to the horizontal of the bottom flange of a girder;
b = breadth of a girder flange;
β = angle of inclination to the horizontal of the top flange of a girder;
Δ = maximum deflection of a girder;
c = clear depth of web between flange angles, or side plates, in inches;
d = effective depth of a girder, that is, the distance between centres of gravity of the flanges;
E = modulus of elasticity of the material;
e = web equivalent fraction, that is, the fraction of the gross area of the web that may be considered as available flange material concentrated at the centre of gravity of a flange;
f_{as} = existing average shearing stress on a girder web;
f_c = existing average compressive stress in the compression flange;
f_e = existing fibre stress due to flexure at an extreme fibre distant y_e from the neutral axis;
f_s = true shearing stress existing at any fibre distant y from the neutral axis;
f_t = existing average tensile stress in the tension flange;
f_x = existing maximum fibre stress due to flexure about the x-axis;
f_y = existing maximum fibre stress due to flexure about the y-axis;

h = depth of girder web plate;

I = moment of inertia of the cross section about its gravity axis normal to the plane of bending;

I_y = moment of inertia of the cross section about the y-axis;

K = a flange stress apportionment factor, representing the ratio of the total stress borne by a part, or all, of the flange material proper to the total stress borne by the full flange area, including the web equivalent; see Eqs. (30) and (31) of this article;

l = span, centre to centre of bearings;

l' = spacing of lateral supports to a compression flange;

M = bending moment;

M_r = moment of resistance of a cross section;

M_{rc} = moment of resistance of a girder based on the strength of the compression flange;

M_{rt} = moment of resistance of a girder based on the strength of the tension flange;

M_x = bending moment about the x-axis;

M_y = bending moment about the y-axis;

m = bending moment at the centre of any segment of a beam due to a load of 1 lb. acting at the point where the deflection is required;

n = number of rivets in a row transverse to the axis of a part being connected;

p = pitch of rivets;

p_c = permissible stress in compression;

p_f = permissible flexural stress at extreme fibre;

p_s = permissible shearing stress;

p_t = permissible tensile stress;

Q = statical moment of the area above or below a given point on a beam cross section, taken about the neutral axis;

S = effective section modulus of a girder subjected to symmetrical bending, required or provided, as the expression may indicate;

s = spacing of stiffeners, centre to centre, in inches;

s_c = clear spacing of stiffeners, in inches;

t = thickness of the web of a girder;

V = total vertical shear at a cross section;

V_w = total vertical shear resisted by the web of a girder, one or both of the flanges of which are inclined to the horizontal;

v = least value or safe resistance of a rivet;

w = vertical load per inch of length applied directly to a flange of a girder;

x_e = normal distance from the y-axis to the extreme fibre of a flange;

x_n = theoretical length of the nth cover plate on a flange, counting from the outside;

y_e = distance of an extreme fibre of a beam from the neutral axis.

120. Design of Plate Girder Web for Shear.—The maximum end shear in a plate girder is 140,000 lb. and the web must be 42 in. deep by reason of the

bending moment. Determine the necessary section of the web according to two specifications: (1) $p_s = 10{,}000$ lb. per sq. in. of gross area, and (2) $p_s = 12{,}000$ lb. per sq. in. of net area. In both cases it is to be assumed that the shear is uniformly distributed over the web plate and wholly absorbed by it. Rivets, $\frac{3}{4}$ in. Nominal rivet pitch in end stiffeners, $3\frac{1}{2}$ in.

Case 1.—Gross area required, from Eq. (3) of Art. 119 $= A_w = V/p_s = 140{,}000/10{,}000 = 14.00$ sq. in. Use a $42 \times \frac{3}{8}$-in. web plate for which $A_w = 42 \times \frac{3}{8} = 15.75$ sq. in. A $42 \times \frac{5}{16}$-in. plate is insufficient.

Case 2.—Net area required $= 140{,}000/12{,}000 = 11.67$ sq. in. Since there may be $\frac{7}{8}$-in. holes spaced $3\frac{1}{2}$ in. apart in the end stiffeners, the loss of area at the section of maximum shear may be $0.875/3.5 = 25\%$. Hence, the gross area required in web $= 1.33 \times 11.67 = 15.56$ sq. in. A $42 \times \frac{3}{8}$-in. web plate with $A_w = 15.75$ sq. in. will be adequate.

121. Design of Web for Diagonal Compression.—A plate girder $32\frac{1}{2}$ in. back to back of flange angles with 6×6-in. flange angles and an unstiffened $32 \times \frac{5}{16}$-in. web plate is subjected to a maximum shear of 120,000 lb. Report on the safety of the web in diagonal compression on the basis of the A.I.S.C. specification, whereby $p_s = p_{dc} = 18{,}000/(1 + c^2/7200\,t^2)$ for the gross area of unstiffened webs where c, the clear distance between flanges, exceeds 60 times the web thickness t, and $p_s = p_{dc}$ in no case exceeds 12,000 lb. per sq. in.

Average existing stress on web, from Eq. (2) of Art. 119 is

$$f_{as} = \frac{V}{ht} = \frac{120{,}000}{32 \times 0.3125} = 12{,}000 \text{ lb. per sq. in.}$$

As

$$c = 32.5 - 2 \times 6 = 20.5 \text{ in., and } t = 0.3125$$

therefore

$$c/t = 20.5/0.3125 = 65.7$$

Hence

$$p_s = \frac{18{,}000}{1 + \dfrac{(65.7)^2}{7200}} = 11{,}250 \text{ lb. per sq. in.}$$

The web is, therefore, overstressed and should either be thickened or stiffened.

122. Moment of Resistance of Plate Girder by Exact and Approximate Methods.—Calculate (1) by the exact, or moment of inertia, method, and (2) by the approximate, or truss-chord, method, the safe moment of resistance of the plate girder section shown in Fig. 75.

Permissible stress in tension flange, $p_t = 16{,}000$ lb. per sq. in. of net area. Permissible stress in compression flange, $p_c = 14{,}500$ lb. per sq. in. Web equivalent, $e = \frac{1}{7}$. Rivets, $\frac{7}{8}$ in.

Exact Method.—The net section modulus of the girder will be found by obtaining the net section modulus with respect to the gravity axis of the gross section and then applying an appropriate correction for the error in the assumption of the position of the neutral axis.

By taking statical moments of all gross areas about the horizontal axis through the centre of the web, designating moment arms above and below this axis as respectively positive and negative, the distance of the centre of gravity of the gross section from the axis of reference is found. The distance is 1.31 in. *below* the centre of the web.

The moment of inertia of the gross section about its own gravity axis is then found to be 35,417 in.^{4}

The moment of inertia of the holes below the gravity axis of the gross section (see Fig. 75) taken about this axis is found to be 2996 in.^{4}

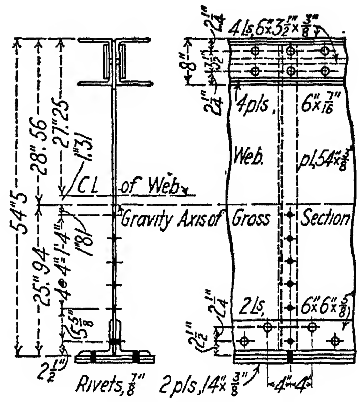


FIG. 75.—Moment of Resistance of Plate Girder.

Uncorrected net moment of inertia = 35,417 − 2996 = 32,421 in.^{4} and uncorrected section modulus = 32,421/28.56 = 1136 in.^{3}, the extreme fibre being at the upper compressive surface.

Applying the correction factor $1.0 - 0.25/d_e^{1/2}$, of Eq. (23), Art. 72, and Eq. (4), Art. 113, the corrected net section modulus = $\{1.0 - 0.25/(55.25)^{1/2}\} \times 1136 =$ 1098 in.^{3}

Moment of resistance of section on the basis of a maximum extreme fibre stress of 14,500 lb. per sq. in., which is the limiting stress at the most highly stressed (compressive) fibre, is from Eq. (5) of Art. 119

$$M_{rc} = 1098 \times 14{,}500 = 15{,}920{,}000 \text{ in.-lb.}$$

Approximate Method.—As the two flanges may not be equally strong, the moment of resistance with respect to each must be calculated and the lesser adopted as the governing one.

The critical right section is that containing the stiffener rivet holes, as this is the worst for the web and as weak a section as can be selected for the tension flange.

Considering the method of calculating deductions from tension members or flanges explained in Art. 5, it is seen that the maximum deduction is two holes from each angle and two from each flange plate. The net area of the tension flange would then be as follows:

Two $6 \times 6 \times \frac{5}{8}$-in. angles = $14.22 - 4 \times 1 \times 0.625$	= 11.72 sq. in.
Two $14 \times \frac{3}{8}$-in. plates = $10.50 - 4 \times 1 \times 0.375$	= 9.00 sq. in.
Web equivalent = $\frac{1}{7} \times 54 \times 0.375$	= 2.89 sq. in.
Total net area	= 23.61 sq. in.

The effective depth is found by subtracting from the distance back to back of flange angles, the estimated distance from these surfaces to the centres of

gravity of the flanges. For the top flange this is obviously 4 in., and for the bottom flange it may, since cover plates are used, be taken safely as the distance to the edge of the web plate (Art. 123), or 0.25 in. Hence $d = 54.5 - (4 + 0.25) = 50.25$ in.

Moment of resistance of girder with respect to tension flange, from Eq. (8) of Art. 119, is

$$M_{rt} = 23.61 \times 16{,}000 \times 50.25 = 19{,}000{,}000 \text{ in.-lb.}$$

The gross area of the compression flange, including $\frac{1}{7}$ of gross area of web = 27.07 sq. in., and hence moment of resistance of girder with respect to compression flange is, from Eq. (9) of Art. 119

$$M_{rc} = 27.07 \times 14{,}500 \times 50.25 = 19{,}700{,}000 \text{ in.-lb.}$$

The governing moment of resistance is, therefore, that of the tension flange. It exceeds the moment of resistance by the exact, or moment of inertia, method by 19,000,000 − 15,920,000 = 3,080,000 in.-lb., or 19.4%.

The approximate method is not sufficiently accurate for shallow girders with heavy, and particularly four-angle, flanges. Had the web equivalent e been $\frac{1}{8}$ instead of $\frac{1}{7}$ the excess would have been 17.5%, which is also too wide of the mark.

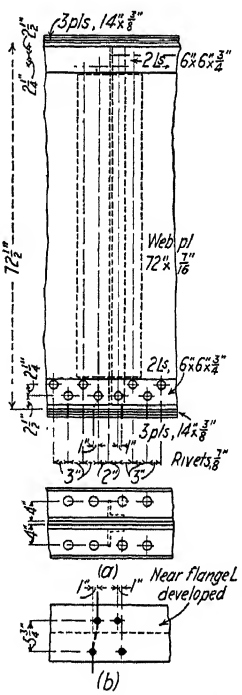


FIG. 76.—Flexural Strength of Plate Girder.

123. Flexural Sufficiency of Girder with T-Flanges.—A plate girder of 60-ft. span, centre to centre of bearings, carries a total uniformly distributed static load, including its own weight, of 6800 lb. per lin. ft. Its composition at the centre is as shown in Fig. 76. The top flange is supported transversely at intervals of 12 ft. 6 in. Report on its flexural sufficiency according to the approximate, or truss-chord, method of proportioning.

Rivets, $\frac{7}{8}$ in. Permissible stress in compression flange, $p_c = 16{,}000 - 150\,l'/b$ lb. per sq. in., where l' = spacing of lateral supports to the top flange and b = breadth of the flange. Permissible stress in tension flange, $p_t = 16{,}000$ lb. per sq. in. of net area. Web equivalent, $e = \frac{1}{8}$.

Maximum bending moment $M = wl^2/8 = 6800 \times (60)^2 \times \frac{12}{8} = 36{,}720{,}000$ in.-lb.

The gross area of either flange is as follows:

2 angles, $6 \times 6 \times \frac{3}{4}$ in.	$= 2 \times 8.44$	= 16.88 sq. in.
3 cover plates, $14 \times \frac{3}{8}$ in.	$= 3 \times 5.25$	= 15.75 sq. in.
Web equivalent, $\frac{1}{8} \times 72 \times 0.4375$		= 3.94 sq. in.
		= 36.57 sq. in.

The critical section of the tension flange obviously passes through one of the inner lines of rivets in the web splice plate and through the corresponding flange

angle rivets nearest the centre line of the girder, as shown in Fig. 76. The effect of the rivet holes on the contribution of the web to the net area of the tension flange is looked after in the value of the web equivalent specified, viz. $\frac{1}{8}$. From each angle a deduction in accordance with the method of Art. 5 should be made. If the gauge of the outstanding leg be 4 in., and the angle be developed as in Fig. 76(*b*), the distance between the gauge lines containing the two rivets in question is $5\frac{3}{4}$ in. and the stagger is 1 in. From Fig. 1(*c*), the deduction for one angle is, therefore, $1 + 0.98 = 1.98$ holes, and for two angles it is 3.96 holes. For each plate the deduction is 2 holes. Hence the net section of the tension flange is as follows:

2 angles, $16.88 - 3.96 \times 1 \times 0.75$	= 13.91 sq. in.
3 cover plates, $15.75 - 6 \times 1 \times 0.375$	= 13.50 sq. in.
Web equivalent	= 3.94 sq. in.
	31.35 sq. in. net

The effective depth may be taken as approximately equal to the depth of the web plate. To verify the soundness of this assumption, let the position of the centre of gravity of a flange be calculated. Gross areas are manifestly to be used for the compression flange, and so slight is the effect of the holes in altering the position of the centre of gravity of the tension flange, that they may be disregarded and the same results used for the two flanges.

Taking the statical moment of the areas of the angles and cover plates (but no part of the web) about the gravity axis of the angles as a reference axis, Fig. 77, we have $16.88 \times 0 + (15.75 \times 2.34) = 36.9$ in.^{3} Dividing by 32.63, the gross area of the angles and cover plates, the distance of the centre of gravity of the flange from the gravity axis of the angles is $36.9/32.63 = 1.13$ in. and from the back of the angles it is $1.78 - 1.13 = 0.65$ in. The calculated effective depth is, therefore, $72.50 - 2 \times 0.65 = 71.2$ in. or only 1.1% less than the depth of the web.

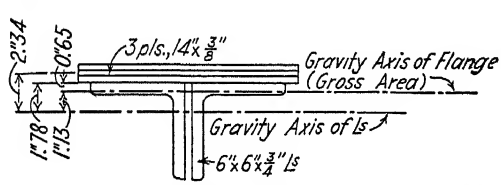


FIG. 77.—Determination of Gravity Axis of Flange.

Average stress in the compression flange under maximum loading is from Eq. (7) of Art. 119.

$$f_c = \frac{M}{\left(A_f + \frac{A_w}{8}\right)d} = \frac{36{,}720{,}000}{36.57 \times 71.2} = 14{,}100 \text{ lb. per sq. in.}$$

Corresponding stress in the tension flange is, from Eq. (6), of Art. 119

$$f_t = \frac{M}{\left(A_f' + \frac{A_w}{8}\right)d} = \frac{36{,}720{,}000}{31.35 \times 71.2} = 16{,}450 \text{ lb. per sq. in.}$$

For the tension flange the permissible stress is $p_t = 16{,}000$ lb. per sq. in. of net area, but for the compression flange it is only $p_c = 16{,}000 - 150 \times 150/14 = 14{,}390$ lb. per sq. in. of gross area. The girder is, therefore, overstressed in the tension flange and understressed in the compression flange.

124. Shear and Flexural Sufficiency of Girder with Sloping Tension Flange.—At a section where the maximum total vertical shear is 100,000 lb. and the maximum simultaneous bending moment is 1,000,000 ft.-lb. a girder has the composition indicated in Fig. 78, the bottom flange being inclined to the horizontal at an angle of 20 deg. Report on the sufficiency of the girder for shear and moment.

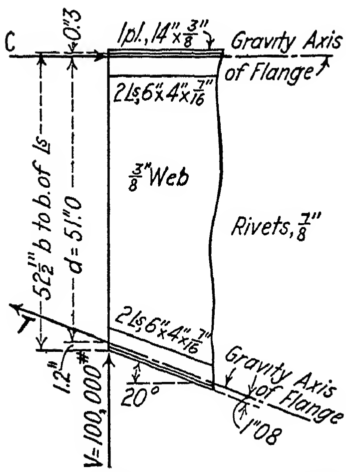


FIG. 78.—Shearing and Flexural Sufficiency of Girder with Sloping Tension Flange.

Web equivalent, $e = \frac{1}{8}$. The top flange is supported laterally at intervals of 11 ft. $p_s = 10{,}000$ lb. per sq. in. of gross area. $p_c = 15{,}000 - 150\, l'/b$ lb. per sq. in. of gross area, where l' = spacing of lateral supports to the flange and b = breadth of the flange. $p_t = 16{,}000$ lb. per sq. in. of net area. Rivets, $\frac{7}{8}$ in.

Shear.—At the section under consideration the depth of the web plate, h, is 52 in.

With one cover plate only on the top flange, the centre of gravity of the flange is somewhat lower than the edge of the web plate, or say 0.3 in. below the back of the angles. For the bottom flange it is, allowing for the slope of the flange, about 1.2 in. vertically above the back of the angles. Hence, the effective depth, d, of the girder $= 52.5 - (0.3 + 1.2) = 51.0$ in.

From Eq. (14) of Art. 119, the shear borne by the web, apart from that absorbed by the inclined bottom flange with its web equivalent, is

$$V_w = 100{,}000 - \frac{1{,}000{,}000 \times 12 \times 0.3639}{51} = 14{,}500 \text{ lb.}$$

Hence from Eq. (15) of Art. 119, the existing average shearing stress on the web is

$$f_{as} = \frac{14{,}500}{52 \times 0.375} = 750 \text{ lb. per sq. in.}$$

This is much below the allowable 10,000 lb. per sq. in.

Moment.—The net area of the tension flange, including the web equivalent, is as follows:

Two angles, $6 \times 4 \times \frac{3}{4}$ in. = 13.88	
$- 2 \times 1 \times 0.75$	= 12.38 sq. in. net
Web equivalent, $\frac{1}{8} \times 52 \times 0.375$	= 2.44 sq. in. net
$A_f' + eA_w$	= 14.82 sq. in. net

Hence, from Eq. (16) of Art. 119

$$f_t = \frac{1,000,000 \times 12}{14.82 \times 51 \times 0.9397} = 16,900 \text{ lb. per sq. in.}$$

As the permissible tensile stress is $p_t = 16,000$ lb. per sq. in., this flange should be increased in section.

The gross area of the compression flange, including the web equivalent, is as follows:

Two angles, $6 \times 4 \times \frac{7}{16}$ in.	= 8.36 sq. in.
One plate, $14 \times \frac{3}{8}$ in.	= 5.25 sq. in.
Web equivalent, $\frac{1}{8} \times 52 \times 0.375$	= 2.44 sq. in.
$A_f + eA_w$	= 16.05 sq. in.

Hence, from Eq. (17) of Art. 119

$$f_c = \frac{12,000,000}{16.05 \times 51} = 14,650 \text{ lb. per sq. in.}$$

As $p_c = 16,000 - 150 \times 132/14 = 14,590$ lb. per sq. in., the flange is slightly overstressed in flexure, but not enough to warrant an increase in section.

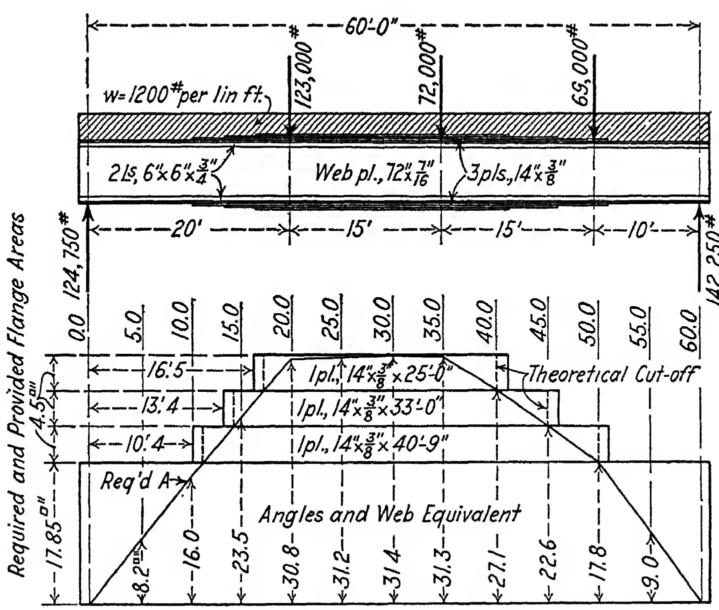


FIG. 79.—Graphical Determination of Length of Flange Plates.

125. Graphical Determination of Length of Flange Plates.—A girder of 60-ft. span, centre to centre of bearings is loaded with a uniformly distributed load of 1200 lb. per lin. ft., including its own weight, and in addition with three concentrated loads as shown in Fig. 79. The composition of the flanges

at the point of maximum moment is as set forth in the problem of Art. 123. Find the points of theoretical and practical cut-off for the flange plates on the tension flange. Web equivalent, $e = \frac{1}{8}$. $p_t = 16{,}000$ lb. per sq. in. of net area.

The moments at intervals of 5 ft. and the corresponding required areas of the tension flange are set forth in Table 5. In determining the latter the effective depth of the girder has been assumed as the depth of the web, or 72 in., at all sections. For an exact solution this would not be sufficiently accurate, being most in error near the ends of the span. Strictly, it should be computed from the position of the centres of gravity of the flanges and should be determined for each change of section.

On Fig. 79 the required net areas of the tension flange at the successive sections are plotted vertically, and the areas of flange elements assigned to meet these requirements are plotted to the same scale.

The points of theoretical cut-off are fixed in the same manner as for the problem of Art. 117. To ensure that the plates are available for stress at the points where first needed, they are extended about a foot past each point of theoretical cut-off. Scaling from Fig. 79, the lengths of the three plates should be, counting from the outside, 25 ft., 33 ft., and 40 ft. 9 in. respectively, and the location of these plates with respect to the supports may also be determined by scaling.

A similar diagram should properly be made for the compression flange and the lengths of corresponding plates on the two flanges made the same.

TABLE 5

REQUIRED AREA OF TENSION FLANGE AT VARIOUS SECTIONS

Distance of Point from Left Support, Feet	Moment from Uniform Load, Foot-Pounds	Moment from Concentrated Load, Foot-Pounds	Combined Moment, Foot-Pounds	Required Net Area of Tension Flange, $A_f' + A_w/8$, Square Inch
5	165,000	617,500	782,500	8.2
10	300,000	1,235,000	1,535,000	16.0
15	405,000	1,852,500	2,257,500	23.5
20	480,000	2,470,000	2,950,000	30.8
25	525,000	2,472,500	2,997,500	31.2
30	540,000	2,475,000	3,015,000	31.4
35	525,000	2,477,500	3,002,750	31.3
40	480,000	2,120,000	2,600,000	27.1
45	405,000	1,762,500	2,167,500	22.6
50	300,000	1,405,000	1,705,000	17.8
55	165,000	702,500	867,500	9.0

126. Rivet Pitch in Plate Girder Flanges.—At a certain section of the plate girder shown in Fig. 75 the total vertical shear is 78,000 lb., the make-up

of the cross section being as indicated in that illustration. The top flange carries a directly applied vertical load of 6720 lb. per lin. ft. Calculate the theoretical rivet spacing in the vertical legs of the flange angles and in the cover plates on the bottom flange.

Web equivalent, $e = \frac{1}{7}$. Rivets, $\frac{7}{8}$ in. $p_s = 10{,}000$ lb. per sq. in. $p_b = 20{,}000$ lb. per sq. in.

Loaded Flange.—From Eq. (29) of Art. 119 the pitch of the rivets in the vertical legs of the angles of a loaded flange, n being equal to two, is

$$p = \frac{2\,v}{\left\{\left(\frac{KV}{d}\right)^2 + w^2\right\}^{\frac{1}{2}}}$$

The least value or safe resistance, v, of one rivet in this situation is its bearing value on the $\frac{3}{8}$-in. web, which is $0.875 \times 0.375 \times 20{,}000 = 6560$ lb.

For the top flange the value of K, as defined in Eq. (30) of Art. 119, is the gross area of the angles and flange plates of this flange divided by the sum of this area and $\frac{1}{7}$ of the gross area of the web.

The area of the angles and plates of this flange, A_f, is

4 angles, $6 \times 3\frac{1}{2} \times \frac{3}{8}$ in. $= 4 \times 3.42$	= 13.68 sq. in.
4 plates, $6 \times \frac{7}{16}$ in. $= 4 \times 6 \times 0.4375$	= 10.50 sq. in.
Total value of A_f	= 24.18 sq. in.
Web equivalent $= eA_w = \frac{1}{7} \times 54 \times 0.375$	= 2.89 sq. in.
$A_f + eA_w$	= 27.07 sq. in.

Hence

$$K = \frac{24.18}{24.18 + 2.89} = 0.89$$

In the data, V is given as 78,000 lb.

The effective depth, d, has already been determined in Art. 122, as 50.25 in.

The direct vertical loading on the top flange must be based on the inch as the unit of length. It is, therefore, $6720/12 = 560$ lb. per lin. in.

Hence, the theoretical pitch required is

$$p = \frac{2 \times 6560}{\left\{\left(\frac{0.89 \times 78{,}000}{50.25}\right)^2 + (560)^2\right\}^{\frac{1}{2}}} = 8.83 \text{ in.}$$

The maximum allowable pitch is, of course, 6 in.

Unloaded Flange.—From Eq. (28) of Art. 119, the staggered pitch of rivets in the vertical legs of the angles of an unloaded flange ($n = 1$) is

$$p = \frac{vd}{KV}$$

The values of v, d and V have already been established in connection with the calculation of the top flange spacing. Applying Eq. (30) of Art. 119, the gross area of the tension flange being 24.72 sq. in., exclusive of web equivalent,

$$K = \frac{24.72}{24.72 + 2.89} = 0.89$$

Since the gross area of the two flanges will always be very nearly, if not exactly, equal, the value of K determined for one flange will apply with sufficient exactness to the other, as is evident from the above computations of K.

The theoretical staggered pitch is, therefore

$$p = \frac{6560 \times 50.25}{0.89 \times 78{,}000} = 4.74 \text{ in.}$$

Tension Flange Cover Plates.—From Eq. (28) of Art. 119, the pitch of the cover plate rivets driven in pairs is

$$p = \frac{2\,vd}{KV}$$

The safe resistance, v, of a rivet in this situation is its single shearing value $= 0.601 \times 10{,}000 = 6010$ lb.

The quantities d and V are already known. The value of K is to be determined from Eq. (31) of Art. 119.

For the bottom flange, on which these cover plates occur, the gross areas are: $A_p = 10.50$ sq. in.; $A_f = 24.72$ sq. in. The value of A_w is 20.25 sq. in. Hence

$$K = \frac{10.50}{24.72 + 0.143 \times 20.25} = 0.38$$

The theoretical pitch of the cover plate rivets is, therefore

$$p = \frac{2 \times 6010 \times 50.25}{0.38 \times 78{,}000} = 20.4 \text{ in.}$$

The maximum permissible pitch is 6 in. in the line of stress.

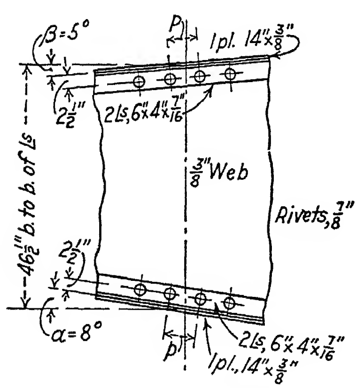


Fig. 80.—Riveting of Sloping Flanges.

127. Riveting of Sloping Flanges.—A plate girder with sloping flanges, composed as indicated in Fig. 80, is subjected to a total vertical shear of 80,000 lb. at a certain section and a simultaneous moment of 850,000 ft.-lb. Neither flange carries any directly applied vertical load. Find the theoretical rivet pitch in both flanges and in the cover plates.

Web equivalent, $e = \frac{1}{8}$. Rivets, $\frac{7}{8}$ in. $p_s = 10{,}000$ lb. per sq. in. $p_b = 20{,}000$ lb. per sq. in.

From Eqs. (32) and (18) of Art. 119, the pitch along the slope of either flange for the rivets through the web is, where only a single row is employed

$$p = \frac{vd}{K\left\{V - \frac{M}{d}(\tan\alpha + \tan\beta)\right\}}$$

The safe resistance, v, of one rivet in this situation is its bearing value on the $\frac{3}{8}$-in. web, or from tables, 6560 lb.

The effective depth, d, at this point may safely be taken as the depth of the web plates, or 46 in.

The maximum shear V at the section is given as 80,000 lb., and the simultaneous moment is $850{,}000 \times 12 = 10{,}200{,}000$ in.-lb.

Values of K for both the compression and the tension flanges may be calculated from Eq. (30) of Art. 119.

The gross area of the two angles and the plate of the compression flange is 13.61 sq. in., and the corresponding gross area for the tension flange is 14.49 sq. in.

The web equivalent at this section is $\frac{1}{8} \times 46 \times 0.375 = 2.16$ sq. in.

Hence, for the compression flange

$$K = \frac{13.61}{13.61 + 2.16} = 0.86$$

For the tension flange

$$K = \frac{14.49}{14.49 + 2.16} = 0.87$$

As has been pointed out in Art. 126, K, in general, for parallel flanges, is practically the same for the two flanges. It will be taken as 0.865 for both flanges in this case.

Hence, the theoretical pitch for either flange, measured along the slope, is

$$p = \frac{6560 \times 46}{0.865\left\{80{,}000 - \frac{10{,}200{,}000(0.1405 + 0.0875)}{46}\right\}}$$

$$= \frac{6560 \times 46}{0.865 \times 29{,}400} = 11.85 \text{ in.}$$

The maximum allowable pitch is, however, 6 in.

Cover Plates.—From Eqs. (28) and (18) of Art. 119, the pitch of rivets in the cover plates, they being in pairs, is

$$p = \frac{2\,vd}{K\left\{V - \frac{M}{d}(\tan\alpha + \tan\beta)\right\}}$$

The safe resistance, v, of a rivet is its single shearing value = 6010 lb.

From Eq. (31) of Art. 119, for the cover plate on the top flange

$$K = \frac{5.25}{13.61 + 2.16} = 0.33$$

For the cover plates of the bottom flange

$$K = \frac{6.13}{14.49 + 2.16} = 0.37$$

Adopting a mean value of 0.35

$$p = \frac{2 \times 6010 \times 46}{0.35 \times 29{,}400} = 53.7 \text{ in.}$$

For practical reasons the pitch must not exceed 6 in.

128. Light-Duty Plate Girder Web Splice.—Design a web splice for calculated moment and simultaneous shear existing at a point 20 ft. from one support of the girder of Art. 123, which has a span of 60 ft. centre to centre of bearings and carries a total uniformly distributed static load of 6800 lb. per lin. ft., considered for simplicity, as all applied to the top flange. The composition of the girder at the section is as shown in Fig. 81.

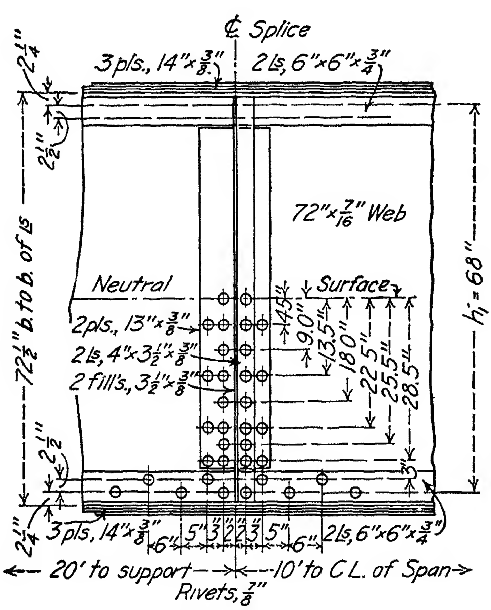


Fig. 81.—Light-Duty Web Splice.

Web equivalent, $e = \frac{1}{8}$. Permissible stress in tension flange, $p_t = 16{,}000$ lb. per sq. in. of net area. Permissible stress in compression flange, $p_c = 16{,}000 - 150\ l'/b$ lb. per sq. in. $p_s = 12{,}000$ lb. per sq. in. $p_b = 24{,}000$ lb. per sq. in. Rivets, $\frac{7}{8}$ in.

This splice is located at a favorable point, that is, just inside the ends of the outer cover plates. The latter plates might be cut off theoretically at 1.35 ft. on the abutment side of the splice and are actually discontinued about 2.5 ft. from the splice. The excess flange area existing at the splice may be considered as relieving the web of some of its bending moment. The web need be spliced, therefore, for only that amount of bending moment which it is called upon to carry.

Moment.—At the splice the bending moment in the girder is (l = span; x = distance from near support),

$$\begin{aligned} M &= \tfrac{1}{2}\, wlx - \tfrac{1}{2}\, wx^2 \\ &= \tfrac{1}{2} \times 6800 \times 60 \times 20 - \tfrac{1}{2} \times 6800 \times (20)^2 \\ &= 2{,}720{,}000 \text{ ft.-lb.} = 32{,}640{,}000 \text{ in.-lb.} \end{aligned}$$

Net area of tension flange, exclusive of web equivalent (see Art. 123), is 27.41 sq. in. Letting the effective depth of the girder equal the depth of the web plate, or 72 in., the moment of resistance of the net area of the angles and cover plates of the tension flange is, from Eq. (8) of Art. 119

$$M_{rt} = 27.41 \times 16{,}000 \times 72 = 31{,}600{,}000 \text{ in.-lb.}$$

Gross area of compression flange, exclusive of web equivalent (see Art. 123) is 32.63 sq. in. As $p_c = 16{,}000 - 150 \times 150/14 = 14{,}390$ lb. per sq. in., the moment of resistance of the angles and cover plates of the compression flange is, from Eq. (9) of Art. 119

$$M_{rc} = 32.63 \times 14{,}390 \times 72 = 33{,}800{,}000 \text{ in.-lb.}$$

The maximum moment of resistance that can safely be developed is, therefore, that based upon the capacity of the tension flange. The web need consequently be spliced for a moment of only 32,640,000 − 31,600,000 = 1,040,000 in.-lb.

In accordance with a method commonly employed in practice, the splice will be designed primarily for moment and then tested for shear.

Two 13 × $\frac{3}{8}$-in. vertical splice plates between the flanges, with two vertical lines of rivets on each side of the splice, as shown in Fig. 81, will be adopted. Without detailed calculation, it is apparent that both the shear and the flexural value of these two punched plates is equal to the corresponding values of the punched web. The vertical spacing of the rivets should, for convenience in fabrication, be based upon that used in the end and intermediate stiffeners, and this will be assumed as $4\frac{1}{2}$ in. for the inner rows, except two spaces of 3 in. at each end. In the outer rows every other rivet will be omitted. This will probably give a sufficiently strong splice in this location.

The resisting moment of the rivets in the vertical splice plates on one side of the splice may be expressed as

$$M_{rr} = \frac{v}{h_1/2}\, \Sigma y^2 \qquad (1)$$

where v = least value of one flange rivet through the web, h_1 = distance between the extreme line of rivets in one flange and the corresponding line in the other flange, and Σy^2 = sum of the squares of the vertical distances of rivets on one side of the splice and on both sides of the neutral surface measured from the neutral surface.

The safe resistance of one $\frac{7}{8}$-in. rivet is its bearing value on the web = 0.875 × 0.4375 × 24,000 = 9190 lb. From Fig. 81, h_1 = 68 in. For the rivets in the

vertical splice plates, $\Sigma y^2 = 1 \times (0)^2 + 4 \times (4.5)^2 + 2 \times (9.0)^2 + 4 \times (13.5)^2 + 2 \times (18.0)^2 + 4 \times (22.5)^2 + 2 \times (25.5)^2 + 4 \times (28.5)^2 = 8194.5$. Hence

$$M_{rr} = \frac{9190}{34} \times 8194.5 = 2{,}215{,}000 \text{ in.-lb.}$$

This is considerably greater than the requirement, but to preserve uniformity of vertical spacing with the stiffener rivets the number of rivets will not be decreased. Besides there is the effect of shear yet to consider.

Narrowing the splice plates so as to include only one line of rivets on each side of the splice would not be permitted under most specifications, which require at least two rows of rivets on each side.

While the splice provided more than develops the moment required of the web at this section, although it is 12 in. shallower than the web, it is well to put two additional rivets in the flange angles through the web on each side of the splice, in both flanges. This can be done by decreasing the required pitch of the flange rivets for a short distance on each side of the splice. The required pitch of flange rivets in the unloaded flange at this point, Eq. (28), Art. 119, is 8.9 in. If in 16 in. from the splice 4 rivets are used, as shown in Fig. 81, 2.2 of them are available for web splice purposes and the other 1.8 will meet flange riveting requirements. The excess flange material provided by reason of the cover-plate extensions at this point may be utilized as splice material for the parts of the web plate between the vertical legs of the flange angles. The web splice may then be regarded as covering the full depth of the web.

If two rivets in each flange on one side of the splice be considered as wholly available for the web splice, they will contribute to the moment of resistance of the splice the amount

$$\frac{9190}{34} \times \{2 \times (31.5)^2 + 2 \times (34.0)^2\} = 1{,}160{,}000 \text{ in.-lb.}$$

The total moment of resistance provided in the splice would, therefore, be $2{,}215{,}000 + 1{,}160{,}000 = 3{,}375{,}000$ in.-lb.

Shear.—Consider now the resultant stress on an extreme rivet due to the simultaneous action of shear and moment at the splice. The shear at the section is $6800\ (30 - 20) = 68{,}000$ lb.

Due to the excess capacity of the splice for moment the moment stress on an extreme rivet in the flange will not need to be so much as the safe resistance of a rivet, or 9190 lb., but will be thus reduced in the ratio of the required moment of resistance of the splice to the provided moment of resistance, that is, $9190 \times 1{,}040{,}000/3{,}375{,}000 = 2830$ lb.

Regarding the shear as uniformly distributed over all the rivets in the two rows on one side of the splice from top to bottom of the girder, the stess produced thereby on any rivet is $68{,}000/27 = 2520$ lb.

The maximum resultant on an extreme rivet in the flange angles is, therefore, $\{(2830)^2 + (2520)^2\}^{1/2} = 3780$ lb., or much less than the safe resistance of a rivet in this situation.

The splice is, therefore, adequate for the combined effect of moment and shear.

129. Heavy-Duty Plate Girder Web Splice.—Design a web splice for the full moment of resistance of the web and the simultaneous maximum calculated shear existing at a point 17 ft. from one support of the girder of Arts. 123 and 128. Assume that the centre cover plates are terminated a short distance on the side of the splice next to the centre of the span, and that there is no appreciable excess flange area to lessen the flexural duty of the web. The number of rivets in splice plates not in direct contact with the part spliced is to be increased by 10% for each intervening plate. Other data are as in Art. 128 and Fig. 82.

Moment.—The web must be spliced for its full moment of resistance, or, from Eq. (8) of Art. 119, for

$$M_{rw} = \tfrac{1}{8} \times (72 \times 0.4375) \times 16{,}000 \times 72 = 4{,}536{,}000 \text{ in.-lb.}$$

As in the problem of Art. 128, two 13 × $\frac{3}{8}$-in. vertical splice plates will be employed. The vertical spacing of rivets will be the same as in that problem, but all four vertical lines will contain the same number of rivets. Hence, for the rivets in the vertical splice plates, on one side of the splice, $\Sigma y^2 = 2 \times (0)^2 + 4\{(4.5)^2 + (9.0)^2 + \ldots + (28.5)^2\} = 10{,}305$. Therefore, the moment of resistance of the rivets thus provided is, from Eq. (1) of Art. 128

$$M_{rr} = \tfrac{9190}{34} \times 10{,}305 = 2{,}785{,}000 \text{ in.-lb.}$$

The deficiency in required moment of resistance of the rivets, or 4,536,000 − 2,785,000 = 1,751,000 in.-lb., will be supplied by providing additional rivets through the flange angles and the web, horizontal splice plates being placed on the vertical legs of these angles.

It has been assumed that a rivet in the outer gauge line of the flange angle has a value of 9190 lb. A rivet on the inner gauge line will at the same time, on the basis of relative distances from the neutral surface resist 9190 × 31.5/34 = 8510 lb. The moments of resistance of these two rivets will be respectively 9190 × 34 and 8510 × 31.5 in.-lb. If m rivets be required on the outer gauge line and n rivets on the inner gauge line of each flange for web splice purposes, it follows that $(2m \times 9190 \times 34) + (2n \times 8510 \times 31.5) = 1{,}751{,}000$ in.-lb. By trial, it is seen that this relation will be satisfied if $m = 2$ and $n = 1$, these rivets being for the present regarded as wholly available for the web splice. As the flange angles intervene between the horizontal splice plates and the web, the total number should be increased by 2 × 10 = 20%. Two rivets will, therefore, be placed in each gauge line but the moment of resistance developed thereby will be taken as only $\frac{5}{6}$ of the nominal.

Shear.—The effect of the shear at the joint in augmenting the stress on the exteme splice rivets in the flange should next be investigated. This shear is 6800 (30 − 17) = 88,400 lb.

The moment stress on an extreme rivet will be less than the safe resistance of the rivet in the ratio of the required moment of resistance of the splice rivets, 4,536,000 in.-lb., to the provided moment of resistance, which is 2,785,000 + $(\frac{5}{6} \times 2 \times 2 \times 9190 \times 34) + (\frac{5}{6} \times 2 \times 2 \times 8510 \times 31.5)$ = 2,785,000 + 1,935,000 = 4,720,000 in.-lb. The rivet stress, is, therefore, 9190 × 4,536,000/4,720,000 = 8830 lb.

Due to shear, regarding all rivets in the two vertical rows on each side of the splice as resisting equally, the stress on each rivet is 88,400/34 = 2600 lb.

The maximum resultant stress on an extreme rivet in the horizontal splice plates, is, therefore $\{(8830)^2 + (2600)^2\}^{1/2} = 9210$ lb. As this is practically the same as the safe resistance of a rivet, no additional rivets will be required because of shear effect.

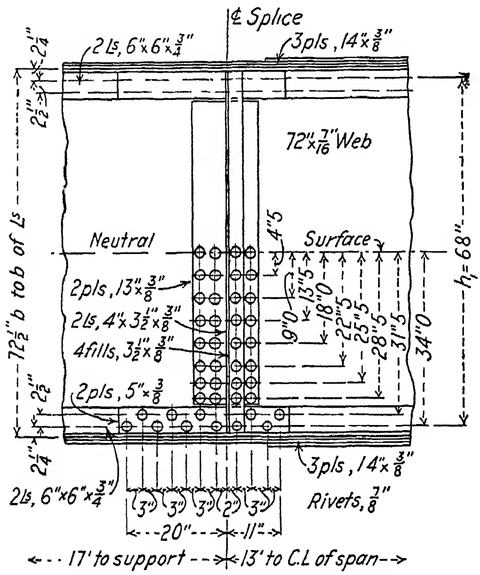


FIG. 82.—Heavy-Duty Web Splice.

The horizontal splice plates must be proportioned to develop a moment of resistance equal to that developed by the web splice rivets in the flange, that is 1,935,000 in.-lb. As the arm of the resistance to be developed by them, assuming them to contain the two gauge lines of the flange, is 65.5 in., their safe capacity must be 1,935,000/65.5 = 29,600 lb.

At the centre line of the plates the permissible stress in the tension flange is 16,000 × 32.75/36 = 14,550 lb. per sq. in., and the net area required of the plates is 29,600/14,500 = 2.04 sq. in. Two 5 × $\frac{3}{8}$-in. plates, having a net area of 3.00 sq. in., and slightly ground at the edges next to the fillets of the angles, will be used.

On the side of the splice next the centre of the girder these horizontal splice plates need extend only far enough to engage the 4 rivets already decided upon. No additional rivets are required by reason of the fact that these rivets serve also to deliver increment of flange stress from the web into the flange.

On the abutment side of a splice that is not at the centre of a span, the number of rivets must be increased by reason of the fact that the rivets through the horizontal splice plates must serve also for flange riveting purposes. In the vicinity of this splice the staggered pitch of the flange rivets should be, for the loaded flange, from Eq. (29) of Art. 119

$$p = \frac{9190}{\left\{\left(\dfrac{0.87 \times 88{,}400}{72}\right)^2 + (567)^2\right\}} = 7.59 \text{ in.}$$

If the distance from the splice to the extreme splice rivet in the horizontal plates, on the side next the support be fixed by trial at 20 in., the number of rivets required in this length for flange riveting purposes is 20/7.59 = 2.6,

or 3. If to these are added the 4 rivets required for the web splice we have 7 rivets on the abutment side of the splice.

Fixing a 3-in. staggered spacing of the rivets, as shown in Fig. 82, the horizontal splice plates are kept short.

130. Design of a Building Plate Girder.—Prepare a complete design with engineer's details of a building plate girder of 40-ft. span centre to centre of bearings to carry a uniformly distributed superimposed load of 6000 lb. per lin. ft.

The web is to be spliced at the centre and T-flanges with cover plates are to be employed. Consider $\frac{1}{8}$ of the gross area of the web as effective flange area, and assume that the top flange is laterally supported at intervals not exceeding 12 times its width. The web thickness is not to be less than $\frac{1}{160}$ of the clear vertical distance between flange angles. Minimum thickness of material $\frac{5}{16}$ in.

Permissible stresses are to be as follows: Shear on web, p_s = 10,000 lb. per sq. in. of gross area, uniformly distributed; diagonal compression, p_{dc} = 1,024,000 t/s (t = web thickness and s = spacing of stiffeners, centre to centre of rivet lines); flexure, tension flange, p_t = 16,000 lb. per sq. in. of net area; flexure, compression flange, $p_c = 19{,}000 - 300\, l'/b$, but not over 16,000 lb. per sq. in. (l' = spacing of lateral supports, and b = breadth of flange); compression on end stiffeners, $p_c = 19{,}000 - 100\, l/r$, with a maximum of 13,000 lb. per sq. in., l being taken as one-half the depth of the web; bearing on ends of stiffeners, p_b = 24,000 lb. per sq. in.

Rivets, $\frac{3}{4}$ in.; shear on shop rivets, p_s = 12,000 lb. per sq. in.; bearing on shop rivets, p_b = 24,000 lb. per sq. in.

Bearing pressure on concrete supports, p_b = 600 lb. per sq. in.

Section for Shear.—Assuming the weight of the girder as 210 lb. per lin. ft., the total uniformly distributed load = 6210 lb. per lin. ft. Hence, maximum end shear $= 6210 \times 40 \times \frac{1}{2} = 124{,}200$ lb.

The required web section, Eq. (3), Art. 119, is $V/p_s = 124{,}200/10{,}000 = 12.42$ sq. in., gross.

Adopting a depth for the girder of one-tenth the span, a $48 \times \frac{5}{16}$-in. web plate (gross area = 15.0 sq. in.) would suffice for shear. A $\frac{5}{16}$-in. web is thicker than $\frac{1}{160}$ of clear distance between flange angles. Should the stiffener spacing required near the ends be prohibitively close, the web thickness would need to be increased. The diagonal compressive stress in the web at, say, 2 ft. from the support is represented by the average shearing stress and $= 6210\,(20 - 2)/15 = 7450$ lb. per sq. in. The stiffener spacing, s, must not be greater than 1,024,000 t/f_{as}, where f_{as} = average existing shearing stress; or $1{,}024{,}000 \times 0.3125/7450 = 43$ in. This is a practicable spacing.

Should the required flange rivet spacing at the ends of the girder work out to be closer than is practicable, it may be best to overcome the difficulty by thickening the web to increase the bearing value of the rivets.

Section for Moment.—The maximum moment is $M = \frac{1}{8}\,wl^2 = \frac{1}{8} \times 6210 \times (40)^2 = 1{,}242{,}000$ ft.-lb. $= 1{,}242{,}000 \times 12 = 14{,}904{,}000$ in.-lb.

The required tension flange section is, from Eq. (10) of Art. 119, $A_f' + \frac{1}{8} A_w = \frac{M}{p_t d} = 14{,}904{,}000/16{,}000 \times 48 = 19.45$ sq. in., d being assumed as equal to the depth of the web plate.

The tension flange section provided is as follows:

2 angles, $6 \times 4 \times \frac{5}{8}$ in.; less four $\frac{7}{8}$-in. holes	=	9.53 sq. in. net
2 plates, $13 \times \frac{3}{8}$ in.; less four $\frac{7}{8}$-in. holes	=	8.44 sq. in. net
$\frac{1}{8}$ of web $= \frac{1}{8} \times 48 \times \frac{5}{16}$	=	1.88 sq. in.
		19.85 sq. in. net

The section chosen conforms to the customary requirement that at least 50% of the flange area, in excess of the web equivalent, should be made up of angles. Since the exact rivet stagger that will be adopted in detailing is unknown, the most severe arrangement possible has been assumed.

The required compression flange section is based on the buckling tendency of this flange. Since the unsupported length of flange is 12 ft. and its width is 13 in., $l'/b = 12 \times 12/13 = 11$. Hence permissible stress is $p_c = 19{,}000 - 300 \times 11 = 15{,}700$ lb. per sq. in. Required *gross* area of flange, Eq. (12), Art. 119, $= 14{,}904{,}000/15{,}700 \times 48 = 19.80$ sq. in. As the provided net area of the tension flange is greater than this, the top flange may be made the same as the bottom flange.

Length of Cover Plates.—Length of outer cover plate on the tension flange, Eq. (26), Art. 119, $= x_1 = l(a_1/A)^{1/2}$, where l = length of span, centre to centre, a_1 = net area required of outer cover, and A = total required net flange area, including web equivalent. For the present case,

$$x_1 = 40\ (3.82/19.45)^{1/2} = 40 \times 0.443 = 17.72 \text{ ft.}$$

Adding about 6 in. at each end to provide for two lines of rivets, the adopted length becomes 18 ft. 9 in.

Length of second cover plate, from the outside, $= x_2 = l\{(a_1 + a_2)/A\}^{1/2} = 40\ (8.04/19.45)^{1/2} = 40 \times 0.642 = 25.68$ ft. Adding about 12 in., adopted length becomes 26 ft. 9 in.

These lengths are also on the safe side for the top flange, for which gross areas would be used in the computation of lengths. As this is a building girder, it is not necessary to protect the upper flange by carrying the inner cover the full length of the girder.

End Stiffeners.—Total end reaction = 124,200 lb. Of this it will be assumed that, due to the deflection of the girder, 60%, or $0.60 \times 124{,}200 = 74{,}500$ lb. must be borne by the inner pair of angles.

Assuming the permissible bearing stress at 24,000 lb. per sq. in., required bearing area of two angles on the flange angles = 74,500/24,000 = 3.10 sq. in. Since the outstanding legs of the stiffener angles must extend at least to the rounded corners of the flange angles, they will be 5 in. wide and the net bearing length will be approximately $4\frac{3}{8}$ in.

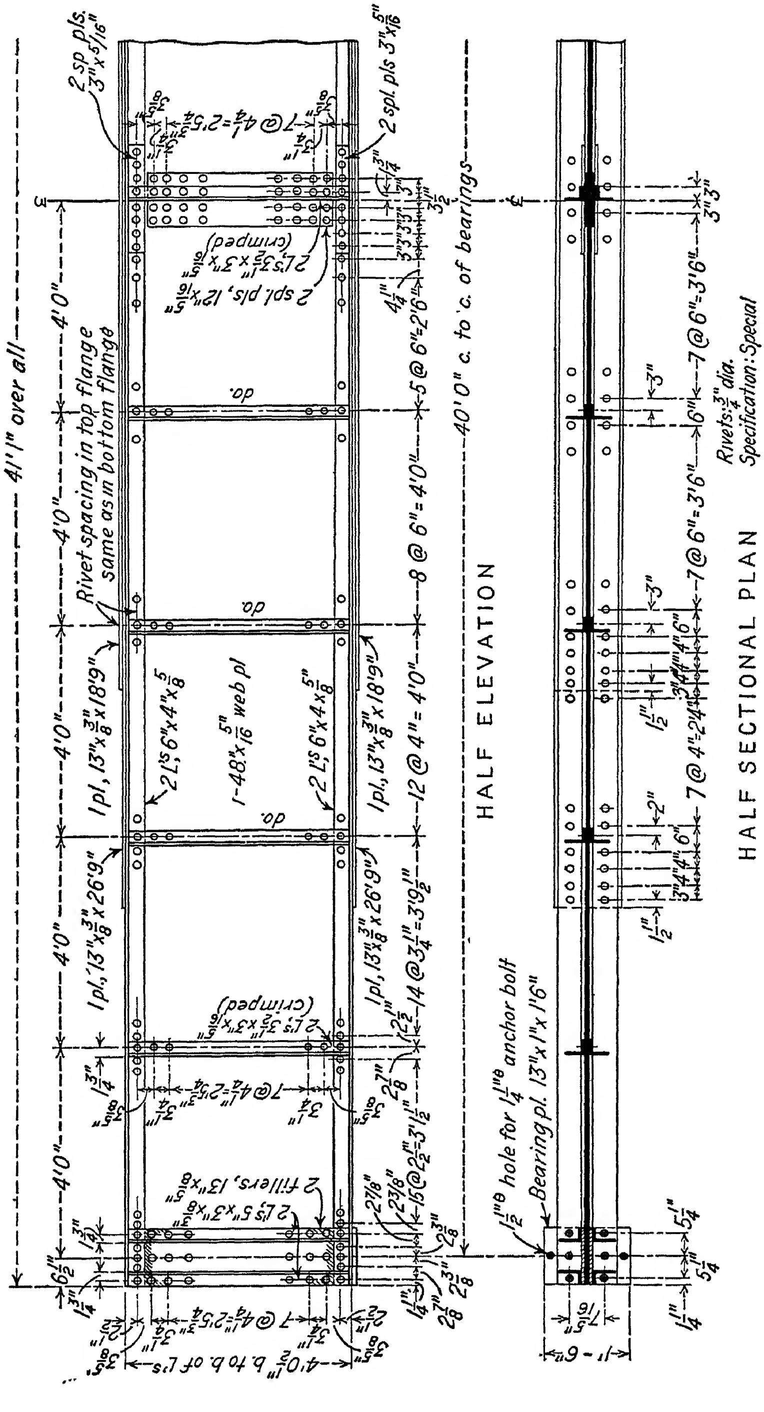


FIG. 83.—Details of Building Plate Girder.

Required thickness of each stiffener angle = $3.10/(2 \times 4.38) = 0.35$ in., or say $\frac{3}{8}$ in. With 3-in. legs next the web, the angles will be $5 \times 3 \times \frac{3}{8}$ in., and the same size will be used for the outer pair. They will be arranged with fillers, as shown in Fig. 83.

Radius of gyration of two angles separated by a space of $1\frac{9}{16}$ in. about an axis in the plane of the web is

$$r = \{2(7.4 + 2.86 \times 2.48^2)/5.72\}^{1/2} = 2.96 \text{ in.}$$

If the effective length of the stiffener column be taken as $48/2 = 24$ in., the permissible compressive stress is $p_c = 19{,}000 - 100 \times 24/2.96 = 18{,}190$ lb. per sq. in. The stress, according to the specification, must be limited to 13,000 lb. per sq. in. and hence the required area of the two inner angles is $74{,}500/13{,}000 = 5.73$ sq. in. As the area of the angles provided to satisfy end bearing requirements is 5.72 sq. in., the size already adopted is adequate.

The number of rivets required in the inner pair of angles will, if loose fillers are used, be increased 1% for each $\frac{1}{16}$ in. of thickness of fillers, or 20%.

Safe resistance of one $\frac{3}{4}$-in. rivet in bearing on $\frac{5}{16}$-in. web $= 0.75 \times 0.3125 \times 24{,}000 = 5625$ lb.

Number of connecting rivets for the inner pair of angles required through tight fillers $= 74{,}500/5625 = 14$, or 17, if loose fillers are used, these numbers being exclusive of those in the flange angles.

Since it is impracticable to place this number in one pair of angles, two $13 \times \frac{5}{8}$-in. fillers ground to bear against the bottom flange angles will be used, serving both pairs of stiffener angles. The riveting arrangement shown in Fig. 83 provides for the inner pair of stiffener angles the 10 rivets in the angles plus one-half those on the centre line of the fillers, or 15 in all. The vertical pitch is made suitable for the intermediate stiffeners and the web splice.

Bearing Plates.—Required area of bearing plates = end reaction divided by permissible bearing pressure on concrete $= 124{,}200/600 = 208$ sq. in. In order to secure close rivet spacing in the flanges at the ends, use a plate 13×18 in. = 234 sq. in. This area is not excessive in view of the concentration of load on the inner pair of stiffener angles.

Net thickness of bearing plate after planing should be at least $t = l(3\,w/p_f)^{1/2}$ = overhang of plate in inches, w = actual upward pressure on plate in pounds per square inch, and p_f = permissible bending stress on plate. Since the overhang, l, is 2.85 in., and $w = 530$ lb. per sq. in., $t = 2.85(3 \times 530/16{,}000)^{1/2} = 0.90$ in. To allow for planing, a plate 1 in. thick will be provided.

Intermediate Stiffeners.—Applying the rule that the spacing of stiffener should not exceed $s = 1{,}024{,}000\ t/f_{as}$, where t = thickness of web, and f_{as} = average intensity of shearing stress in pounds per square inch on the web at the section considered, the safe spacing at a point 2 ft. from the centre of the girder bearing has already been found to be 43 in. For a section 6 ft. from the end the spacing is found to be 55 in. However, practical considerations make a spacing greater than the depth of web undesirable. A spacing of 4 ft. each way from the centre line will therefore be adopted as shown in Fig. 83.

The outstanding legs should be about 1/30 depth of girder + 2 in. = 48/30 + 2 = 3.6, say $3\frac{1}{2}$ in. Use two $3\frac{1}{2} \times 3 \times \frac{5}{16}$-in. angles, crimped. For intermediate stiffeners, the rivet spacing will be the same as for the end stiffeners, by reason of economic considerations in fabrication.

Flange Riveting.—It will be sufficient to compute the rivet spacing at the centre of each panel. For the bottom, or unloaded, flange the pitch, according to Eq. (28) of Art. 119, is ($n = 1$)

$$p = \frac{vd}{KV}$$

and for the top, or loaded flange, according to Eq. (29) of Art. 119, it is

$$p = \frac{v}{\left\{\left(\frac{KV}{d}\right)^2 + w^2\right\}^{\frac{1}{2}}}$$

Pitch at Support.—Least value, v, of a $\frac{3}{4}$-in. rivet is bearing on $\frac{5}{16}$-in. web $= \frac{3}{4} \times \frac{5}{16} \times 24{,}000 = 5630$ lb.

The effective depth, d, at this section $= 48.5 - 2 \times 1.03 = 46.44$ in.

K for the tension flange = area of two $6 \times 4 \times \frac{5}{8}$-in. angles, with four $\frac{7}{8}$-in. holes deducted $\div$ (net area of the two angles $+ \frac{1}{8}$ of gross area of web). This $= 9.53/(9.53 + 1.88) = 0.83$. Four holes are deducted to allow for those through the bearing plate opposite the flange rivets.

K for the compression flange, being based on gross area, $= 11.72/(11.72 + 1.88) = 0.86$. See Eq. (30), Art. 119.

$V = 124{,}200$ lb.

$w = 6000/12 = 500$ lb. per lin. in.

Hence, pitch for bottom flange $= p = 5630 \times 46.44/0.83 \times 124{,}200 = 2.54$ in.

For top flange, $p = 5630/\{(0.86 \times 124{,}200/46.44)^2 + (500)^2\}^{\frac{1}{2}} = 2.40$ in.

Although, properly, the value of K used for tension flange and that used for the compression flange should be based on net and gross areas respectively, no material error would result from applying the value found for the compression flange to both flanges.

A pitch of $2\frac{3}{8}$ in. in each flange is adopted to facilitate the details, as shown in Fig. 83. As it is not prohibitively small, the $\frac{5}{16}$-in. web plate does not require to be thickened.

Pitch at 2 ft. from End.—Here, $V = 124{,}200 - 6210 \times 2 = 111{,}800$ lb.

v, d and w are same as for end. K for bottom flange, computed for two holes out of angles, $= 0.85$. K for top flange $= 0.86$.

p, for bottom flange $= 2.75$ in.

p, for top flange $= 2.65$ in.

Adopt a pitch of $2\frac{1}{2}$ in. for both flanges.

Pitch at 6 ft. from End.—$V = 124{,}200 - 6210 \times 6 = 86{,}900$ lb.

p, for bottom flange $= 3.54$ in.

p, for top flange $= 3.35$ in.

Adopt a pitch of $3\frac{1}{4}$ in. for both flanges in this panel.

Pitch at 10 ft. from End.—$V = 124{,}200 - 6210 \times 10 = 62{,}100$ lb.

Assume the effective depth at this section as equal to the depth of the web plate, or 48 in.

K for bottom flange = Net area of two angles and one cover plate ÷ total net area of one flange at section.

Deducting four $\frac{7}{8}$-in. holes from the two angles and two holes from the cover plate this $= (9.53 + 4.22)/(9.53 + 4.22 + 1.88) = 0.88$.

K, for the top flange $= 16.60/(16.60 + 1.88) = 0.90$.

p, for bottom flange = 4.95 in.

p, for top flange = 4.37 in.

Adopt a pitch of 4 in. for both flanges.

Pitch at 14 ft. from End.—$V = 124{,}200 - 14 \times 6210 = 37{,}300$ lb.

$d = 48$ in.

K for bottom flange = net area of two angles and two cover plates ÷ total net area of one flange at section $= (9.53 + 8.44)/(9.53 + 8.44 + 1.88) = 0.91$.

K for top flange $= 21.47/(21.47 + 1.88) = 0.92$.

p, for bottom flange = 8.0 in.

p, for top flange = 6.5 in.

Since 6 in. is maximum allowable pitch, adopt this for both flanges.

Pitch at 18 ft. from End.—Adopt 6 in.

Riveting of Cover Plates.—Pitch of rivets in cover plates, rivets being in pairs, opposite to each other is, from Eq. (28), Art. 119

$$p = \frac{2\,vd}{KV}$$

Pitch will be computed at theoretical ends of each cover plate, that is at points 7 and 10.7 ft. from centre of end bearings, for the inner and outer covers respectively.

Pitch at Theoretical End of Inner Plate.—v=single shearing value of $\frac{3}{4}$-in. rivet = 5300 lb.

d = effective depth of girder = 48 in.

K for bottom flange = net area of one cover plate ÷ total net area of flange at section $= 4.22/(9.53 + 4.22 + 1.88) = 0.27$.

K for top flange $= 4.88/(16.60 + 1.88) = 0.26$.

$V = 124{,}200 - 6210 \times 7.0 = 80{,}700$ lb.

Hence, p for bottom flange plate $= (2 \times 5300 \times 48)/(0.27 \times 80{,}700) = 23.4$ in.

Actual pitch in the line of stress must not exceed 6 in. At the ends of cover plates it should be comparatively close for a foot at least. In this case it is made 4 in. for both flanges (see Fig. 83).

Pitch at Theoretical End of Outer Plate.—v and d are as before.

K for bottom flange = net area of two cover plates ÷ total net area of flange at section $= 8.44/(9.53 + 8.44 + 1.88) = 0.43$.

K for top flange $= 9.75/(21.47 + 1.88) = 0.42$.

$V = 124{,}200 - 6210 \times 10.7 = 57{,}700$ lb.

Hence, p for bottom flange $= (2 \times 5300 \times 48)/(0.43 \times 57{,}700) = 20.5$ in.

Actual spacing at ends is made about 4 in., as shown in Fig. 83.

Pitch at Other Points.—Except at the ends of cover plates, the pitch is made the maximum permissible, that is 6 in.

Web Splice.—Shear at centre of girder is zero, hence web is spliced only for bending.

Moment of resistance of web, Eq. (8), Art. 119, $= \frac{1}{8} A_w p_t d = \frac{1}{8} \times 48 \times \frac{5}{16} \times 16{,}000 \times 48 = 1{,}440{,}000$ in.-lb.

Use two vertical splice plates $12 \times \frac{5}{16}$ in. and two horizontal splice plates 3 in. wide on the flange angles.

Assume arrangement of rivets in vertical splice plates shown in Figs. 83 and 84. Since safe stress on flange rivets, 21.75 in. from the neutral axis is 5630 lb., safe stress on a rivet 1 in. from neutral axis is $5630/21.75 = 260$ lb.

Moment of resistance of rivets in vertical splice plates on one side of splice $= 260\, \Sigma y^2$, where y = vertical distance to any rivet from the neutral axis. This $= 260[4\{(2.125)^2 + (6.375)^2 + (10.625)^2 + (14.875)^2 + (18.125)^2\}] = 260 \times 2832 = 736{,}000$.

Difference in moment of resistance to be developed by horizontal splice plates $= 1{,}440{,}000 - 736{,}000 = 704{,}000$ in.-lb.

Assume n rivets in the horizontal splice plates on each flange on each side of the centre line. Then

$$2n \times 5630 \times 21.75 = 704{,}000 \text{ in.-lb., and } n = 2.88 \text{ rivets.}$$

The flange rivets must transfer the horizontal increment of flange stress from web to the flanges in addition to developing the moment of resistance for web splice. The average theoretical rivet spacing in the top flange at quarter-point of the horizontal splice plates, say 6 in. from the centre line of the girder, taking $K = 0.92$, is found to be 11.2 in. Hence, assuming a total of four rivets on each side of the centre line in the horizontal splice plates, spaced as shown in Fig. 84, the number required for the purpose of transmitting the increment of flange stress from the web out to the flange angles in the length from mid-span to the end rivet of the splice plates would be $10.75/11.2 = 0.96$. Thus, $2.88 + 0.96 = 3.84$ rivets are required, and the four assumed are sufficient.

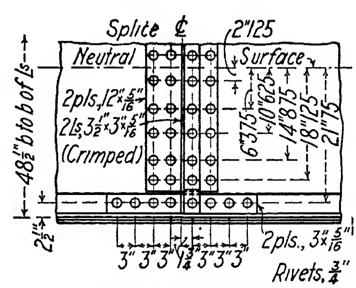


FIG. 84.—Web Splice.

The horizontal splice plates must transmit the stress carried by 2.88 rivets $= 2.88 \times 5630 = 16{,}200$ lb. Required net area of two plates $= 16{,}200/16{,}000 = 1.02$ sq. in., or 0.51 sq. in. for one plate. Use $3 \times \frac{5}{16}$-in. plates having net area of 0.67 sq. in., each.

Details.—Engineer's details are given in Fig. 83.

131. Exercise Problems on the Design of Plate Girders.—The following exercise problems are based on the principles employed in the solution of the problems of this chapter. See Appendix I for the answers.

(1) A plate girder of 50-ft. span centre to centre of bearings carries a total uniformly distributed load of 6000 lb. per lin. ft. The web is $60 \times \frac{3}{8}$ in., the flange angles are 6×6 in. and the depth back to back of flange angles is $60\frac{1}{2}$ in. The clear distance between stiffeners near the end is 40 in. Is the girder safe in shear and in diagonal compression? $p_s = p_{dc} = 18{,}000/(1 + c^2/7200\ t^2)$ lb. per sq. in. of gross area, where c = clear distance between flange angles or between stiffeners, whichever is the greater, and t = web thickness.

(2) A plate girder section consists of a $48 \times \frac{3}{8}$-in. web plate and four $6 \times 4 \times \frac{1}{2}$-in. angles with the 6-in. legs turned out. The depth of the girder is $48\frac{1}{2}$ in. back to back of angles. If $\frac{7}{8}$-in. rivets are used and the safe bending stress on steel is 16,000 lb. per sq. in., find the moment which the girder can safely resist, based on the strength of the tension flange. Web equivalent, $\frac{1}{8}$.

(3) A plate girder consists of a web plate $34 \times \frac{3}{8}$ in., and four flange angles $5 \times 3\frac{1}{2} \times \frac{3}{8}$ in. with the long legs turned out. If the depth of the girder back to back of angles is $34\frac{1}{2}$ in. find its safe moment of resistance. Rivets, $\frac{3}{4}$ in. p_t = 16,000 lb. per sq. in. $p_c = 16{,}000 - 150\ l'/b$. Web equivalent, $\frac{1}{7}$. Lateral supports to top flange, 8 ft. apart.

(4) A plate girder has a span of 30 ft. centre to centre of bearings and the web plate has been fixed at $36 \times \frac{5}{16}$ in. Each flange is to consist of two angles without cover plates, the distance back to back of flange angles being 36 in. Find the size of the angles. Rivets, $\frac{3}{4}$ in. $p_t = p_c$ = 16,000 lb. per sq. in. Total uniformly distributed load = 4000 lb. per lin. ft. Top flange is supported continuously. Web equivalent, $\frac{1}{8}$.

(5) A 40-ft. plate girder $40\frac{1}{2}$ in. deep, back to back of flange angles, is to be made up of a $40 \times \frac{3}{8}$-in. web plate and two flanges, each consisting of a pair of equal-leg angles, without cover plates. If the total uniformly distributed load is 4000 lb. per in. ft., including the weight of the girder, suggest a make-up for the flanges. Rivets, $\frac{7}{8}$ in. Web equivalent, $\frac{1}{8}$. $p_t = p_c$ = 16,000 lb. per sq. in.

(6) A plate girder of 40-ft. span consists of a $48 \times \frac{5}{16}$-in. web plate and four angles $6 \times 3\frac{1}{2} \times \frac{1}{2}$ in. The depth back to back of angles is $48\frac{1}{2}$ in. and the 6-in. legs are turned out. The total uniformly distributed load carried by the girder including its own weight is 125,000 lb. Report on the safety of the structure. Web equivalent, $\frac{1}{8}$. $p_s = p_{dc}$ = 8000 lb. per sq. in. of gross area. $p_t = p_c$ = 16,000 lb. per sq. in. Rivets, $\frac{3}{4}$ in.

(7) A plate girder of 36-ft. span centre to centre of bearings consists of a $40 \times \frac{3}{8}$-in. web plate without intermediate stiffeners and four angles $6 \times 4 \times \frac{5}{8}$ in. The depth back to back of angles is 40 in. and the 6-in. legs are turned out. The *total* uniformly distributed load carried by the girder *including* its own weight is 4200 lb. per lin. ft. Express an opinion as to the safety of the structure. Web equivalent, $\frac{1}{7}$. $p_s = p_{dc} = 18{,}000 - 100\ c/t$, with a maximum of 11,000 lb. per sq. in. (c = clear distance between flange angles and t = web thickness.) p_t = 17,000 lb. per sq. in. p_c = 16,000 lb. per sq. in. Rivets, $\frac{7}{8}$ in.

(8) A plate girder of 35-ft. span centre to centre of bearings consists of a $40 \times \frac{3}{8}$-in. web plate and four angles $6 \times 4 \times \frac{1}{2}$ in. The depth back to back of angles is $40\frac{1}{2}$ in. and the 6-in. legs are turned out. Find the safe load per lineal foot for the

girder, including its own weight. Web equivalent, $\frac{1}{8}$. $p_s = p_{dc} = 10{,}000$ lb. per sq. in. of gross area. $p_t = p_c = 16{,}000$ lb. per sq. in. Rivets, $\frac{7}{8}$ in.

(9) A plate girder of 36-ft. span centre to centre of bearings carries a total uniformly distributed load, *including* its own weight, of 200,000 lb. Assuming cover plates to be used on the flanges, find the least net area that must be made up by angles and cover plates for the tension flange, if the girder be $42\frac{1}{2}$ in. deep, back to back of flange angles. Web equivalent, $\frac{1}{8}$. $p_s = p_{dc} = 10{,}000$ lb. per sq. in. of gross area. $p_t = 16{,}000$ lb. per sq. in. Minimum thickness of web $= \frac{5}{16}$ in. Rivets, $\frac{3}{4}$ in.

(10) A plate girder of 30-ft. span continuously supported laterally at the top flange carries a total uniformly distributed load, including its own weight, of 8000 lb. per lin. ft. If the maximum permissible depth of web is 36 in., suggest sections for both the web and the flanges. Web equivalent, $\frac{1}{8}$; $p_s = p_{dc} = 10{,}000$ lb. per sq. in. of gross area. $p_t = p_c = 16{,}000$ lb. per sq. in. Minimum thickness of metal allowed $= \frac{3}{8}$ in. Rivets, $\frac{3}{4}$ in.

(11) If the total area of the flange of a plate girder of 50-ft. span is 30 sq. in., including the web equivalent, and there are three cover plates each of 5.0 sq. in. area forming part of this area, find the theoretical length of the second cover plate, if the girder loading is uniformly distributed.

(12) The load on a plate girder of 30-ft. span may be assumed as increasing uniformly from each end to mid-span. Each flange has an effective area of 12 sq. in., made up of the web equivalent, two angles and one cover plate. If the area of the latter is 3.5 sq. in., how long should it be, theoretically and practically.

(13) The web of a uniformly loaded plate girder of 50-ft. span centre to centre is $60 \times \frac{3}{8}$ in. and each flange consists of two $6 \times 6 \times \frac{5}{8}$-in. angles and three $13 \times \frac{3}{8}$-in. cover plates. Find the theoretical length of each cover plate, assuming one-eighth of the gross area of the web to act as flange area. Use gross areas.

(14) Find the theoretical length of a $13 \times \frac{1}{2}$-in. outside cover plate on the tension flange of a uniformly loaded plate girder if there are two holes for $\frac{7}{8}$-in. rivets through it and the total net area of one flange, including web equivalent, is 25 sq. in. Span of girder centre is 50 ft. About how long would the plate be made in practice?

(15) Each flange of a plate girder of 42-ft. span is made up of two $6 \times 4 \times \frac{1}{2}$-in. angles and one $13 \times \frac{1}{2}$-in. cover plate and the web consists of a $40 \times \frac{3}{8}$-in. web plate. If the live load is a moving electric car, find the theoretical length of the cover plate on the tension flange, assuming that at the point of cut-off the rivets in the two legs of the flange angles are approximately opposite. Suggest, also, a practical length for it. Web equivalent, $\frac{1}{8}$. Rivets, $\frac{7}{8}$.

(16) The end reaction of a plate girder is 90,000 lb. and the end section consists of a $36 \times \frac{5}{16}$-in. web plate and four angles $5 \times 3\frac{1}{2} \times \frac{3}{8}$ in. Assuming $\frac{3}{4}$-in. rivets and the safe shearing and bearing stresses on rivets as 10,000 and 20,000 lb. per sq. in., respectively, find how many rivets would be necessary to secure two pairs of $4 \times 3 \times \frac{5}{16}$-in. stiffener angles to the web if between the flange angles they be connected to the web through loose fillers? Rivet numbers are to be increased 1% for each $\frac{1}{16}$-in. thickness of loose fillers.

(17) At a certain section a plate girder consists of a $48 \times \frac{3}{8}$-in. web plate and four angles $6 \times 3\frac{1}{2} \times \frac{1}{2}$ in. The depth back to back of angles is $48\frac{1}{2}$ in., and the 6-in. legs are turned out. If the total vertical shear at the section is 80,000 lb., and if there is no directly applied load on either flange, find the rivet spacing in each flange. Rivets, $\frac{7}{8}$ in. Web equivalent, $\frac{1}{8}$. $p_s = 10{,}000$ and $p_b = 20{,}000$ lb. per sq. in.

(18) A plate girder consists at a certain section of a $40 \times \frac{3}{8}$-in. web plate and four angles $6 \times 4 \times \frac{1}{2}$ in. with the 6-in. legs turned out, and with a distance of $40\frac{1}{2}$ in. back to back of angles. If the shear at the section be 40,000 lb., find the theoretical rivet spacing of the rivets in the unloaded flange. Rivets, $\frac{7}{8}$ in. $p_s =$ 10,000 and $p_b =$ 20,000 lb. per sq. in. Web equivalent, $\frac{1}{7}$.

(19) At a certain section where the vertical shear is 50,000 lb., a plate girder consists of a $44 \times \frac{3}{8}$-in. web plate, four angles $6 \times 4 \times \frac{3}{8}$ in., with the 6-in. legs turned out, and two $14 \times \frac{3}{8}$-in. cover plates. The depth, back to back of angles, is $44\frac{1}{2}$ in. Find the theoretical spacing of the rivets in the bottom (unloaded) flange, assuming the stagger between the rivets in the two legs of a flange angle is 2 in. Rivets, $\frac{3}{4}$ in. $p_s =$ 12,000 and $p_b =$ 24,000 lb. per sq. in. Web, equivalent, $\frac{1}{8}$.

(20) At a certain section a plate girder consists of a $56 \times \frac{3}{8}$-in. web plate and four angles $6 \times 4 \times \frac{9}{16}$ in. The depth back to back of angles is $56\frac{1}{2}$ in. and the 6-in. legs are turned out. If the total vertical shear at the section is 100,000 lb. and the load directly applied to the top flange is 4800 lb. per lin. ft., find the theoretical rivet spacing in the top flange, assuming that the web takes no share of the bending moment. Rivets, $\frac{7}{8}$ in. $p_s =$ 10,000 and $p_b =$ 20,000 lb. per sq. in.

(21) A plate girder consists of a $48 \times \frac{5}{16}$ in. web plate and four angles $6 \times 3\frac{1}{2} \times \frac{7}{16}$ in. The depth, back to back of angles is 48 in. and the 6-in. legs are turned out. If the total vertical shear at a certain cross section is 50,000 lb. and if a load of 1200 lb. per lin. ft. is borne directly by the top flange, find the rivet spacing in the top flange. Web equivalent, $\frac{1}{9}$. Rivets, $\frac{7}{8}$ in. $p_s =$ 12,000 and $p_b =$ 24,000 lb. per sq. in.

(22) A plate girder consists of a $36 \times \frac{5}{16}$ in. web plate and four angles $6 \times 3\frac{1}{2} \times \frac{3}{8}$ in. The depth back to back of angles is $36\frac{1}{2}$ in. and the 6-in. legs are turned out. Find the rivet spacing for the top flange at a section where the vertical shear is 40,000 lb., assuming that there is a directly applied load of 2400 lb. per lin. ft. on the top flange. Web equivalent, $\frac{1}{8}$. Rivets, $\frac{3}{4}$ in. $p_s =$ 10,000 and $p_b =$ 20,000 lb. per sq. in.

(23) At a certain section where the vertical shear is 90,000 lb., a plate girder is composed of a $42 \times \frac{3}{8}$-in. web plate, four angles $6 \times 6 \times \frac{7}{16}$ in. with the 6-in. legs turned out, and four $13 \times \frac{3}{8}$-in. cover plates. The depth of the girder, back to back of angles, is $42\frac{1}{2}$ in. If the load applied to the top flange is 3000 lb. per lin. ft., find the theoretical staggered rivet spacing of the rivets in the top flange. Web equivalent, $\frac{1}{8}$. Rivets, $\frac{7}{8}$ in. $p_s =$ 13,500 and $p_b =$ 30,000 lb. per sq. in.

(24) A plate girder of 50-ft. span carries a total uniformly distributed load of 8000 lb. per lin. ft., including its own weight. The section at the end consists of a $60 \times \frac{3}{8}$-in. web plate and four $6 \times 6 \times \frac{1}{2}$-in. angles. If the staggered pitch of the rivets in the top flange at the end is 2 in., report on their adequacy, assuming the entire load to be directly applied to the top flange. Web equivalent, $\frac{1}{8}$. Rivets, $\frac{7}{8}$ in. $p_s =$ 10,000 and $p_b =$ 20,000 lb. per sq. in. Depth of girder, back to back of angles, $= 60\frac{1}{2}$ in.

*

CHAPTER VIII

STEEL TRUSSES AND BENT

132. Warren Truss on Masonry Walls.—Warren trusses at 15-ft. centres are to be used for the support of a reinforced concrete roof of a building 50 ft. wide in the clear between brick side walls. The roof covering is to be tar and gravel, weighing 6 lb. per sq. ft., applied to a reinforced concrete slab laid on the tops of steel purlins which rest on the trusses at panel points only. Gutter drainage will be provided by sloping the top surface of the slab $\frac{3}{8}$ in. to the foot towards scuppers through the parapet wall at the middle of all panels. The brick walls are 14 in. thick between piers, which are 18 in. thick (Toronto brick standards). Design and prepare engineer's details for an intermediate truss.

The snow load will be 30 lb. per sq. ft. of horizontal projection, but the wind force will be assumed as absorbed wholly by the walls.

Specification.—Except as otherwise indicated the Standard Specification of the A.I.S.C., 1928, will apply. The following provisions are of particular importance in the present design:

(1) Permissible stresses in pounds per square inch are

Tension on net section............................ 18,000

Compression on gross area of struts..................

$$\cdots \frac{18{,}000}{1 + \dfrac{l^2}{18{,}000\, r^2}}$$

with a maximum of............................. 15,000

In this l = unsupported length of column, and r = corresponding radius of gyration. For main compression members, l/r must not exceed 120, and for bracing and other secondary members, 200.

Flexure, net section, lateral deflection being prevented... 18,000

Shear on gross section of beam webs, where c, the clear height between flanges, does not exceed 60 times the web thickness, t 12,000

Shear on power-driven rivets......................... 13,500

Bearing on power-driven rivets in double shear......... 30,000

Bearing on power-driven rivets in single shear.......... 24,000

(2) Sections shall preferably be symmetrical.

(3) Full provision shall be made for stresses caused by eccentric loads.

(4) Stitch rivets in tension members composed of two angles shall not be over 3 ft. 6 in. apart, and in similar compression members not over 2 ft. apart. The ratio l/r for each angle of a compression member shall not exceed $\frac{3}{4}$ of that for the whole member.

(5) Connections carrying calculated stresses except for lacing, sag bars or angles, hand rails, or beam connections, shall have not less than 2 rivets; or for field connections not less than 3 rivets.

(6) At a joint the gravity lines of members shall meet at a point if practicable; if not, provision shall be made for any eccentricity.

(7) The centre of gravity of a group of rivets transmitting stress into a member shall lie on the gravity axis of the member; if not, provision shall be made for the resulting eccentricity.

(8) Material for interior construction shall not be thinner than $\frac{1}{4}$ in. except for linings or fillers and rolled structural shapes.

General Dimensions and Form of Truss.—Assuming the distance from the face of a side wall to the centre of the truss bearing as $7\frac{1}{2}$ in., the centre to centre span of the truss will be $50 + 2 \times 0.625 = 51.25$ ft.

The simplest and most efficient form of Warren truss for the situation is that shown in Figs. 85 and 86. It is not necessary to use a vertical at the end support, as the brick wall will carry the edge of the roof slab.

For economy, the mean depth of a Warren roof truss should be about $\frac{1}{10}$ of the span, or in this case, say 5 ft.

To give adequate slope for drainage to the gutters, the depth at the centre will be 5.75 ft. and at the hip U_1, 4.55 ft. This gives a roof slope of 0.70 in. per ft., which is adequate.

As the truss diagonals should preferably not be inclined at a greater slope than 45 deg. with the vertical, and as the thickness of the roof slab should be kept down in order to save weight, the top chord panels will be made 5.125 ft. long and the bottom chord panels 10.25 ft. long, as shown in Fig. 85.

Roof Slab.—Assuming a $2\frac{1}{2}$-in. slab, weighing 150 lb. per cu. ft., the load borne by it will be, in pounds per square foot

Tar and gravel roofing	6
Slab, $150 \times 2.5/12$	30
Snow	30
Total	66

Neglecting restraint at the slab supports, the maximum moment is

$$M = \tfrac{1}{8} \times 66 \times (5.125)^2 \times 12 = 2600 \text{ in.-lb.}$$

On the basis of permissible stresses in the concrete and the steel of $p_c = 650$ and $p_t = 16{,}000$ lb. per sq. in., respectively, and a modular ratio of $n = 15$, the required depth of slab to the centre of steel is

$$d = \left(\frac{M}{1296}\right)^{\frac{1}{2}} = \left(\frac{2600}{1296}\right)^{\frac{1}{2}} = 1.42 \text{ in.}$$

Adding 1 in. for protection of reinforcement, the total required depth would be 2.42, or say 2.5 in.

The area of reinforcement per foot width of slab, if j be assumed as $\frac{7}{8}$, and the actual value of d, that is 1.5 in., be used, is

$$A_t = \frac{M}{p_t jd} = \frac{2600}{16{,}000 \times 0.875 \times 1.5} = 0.124 \text{ sq. in.}$$

This may be supplied in the form of mesh material or rods. For temperature and shrinkage stresses an area of metal about 25% of the above should run parallel to the purlins.

Purlins.—Assuming 7-in., 15.3-lb. I's for the purlins, the load per lineal foot borne by one purlin is $5.125 \times 66 + 15.3 = 353.3$ lb.

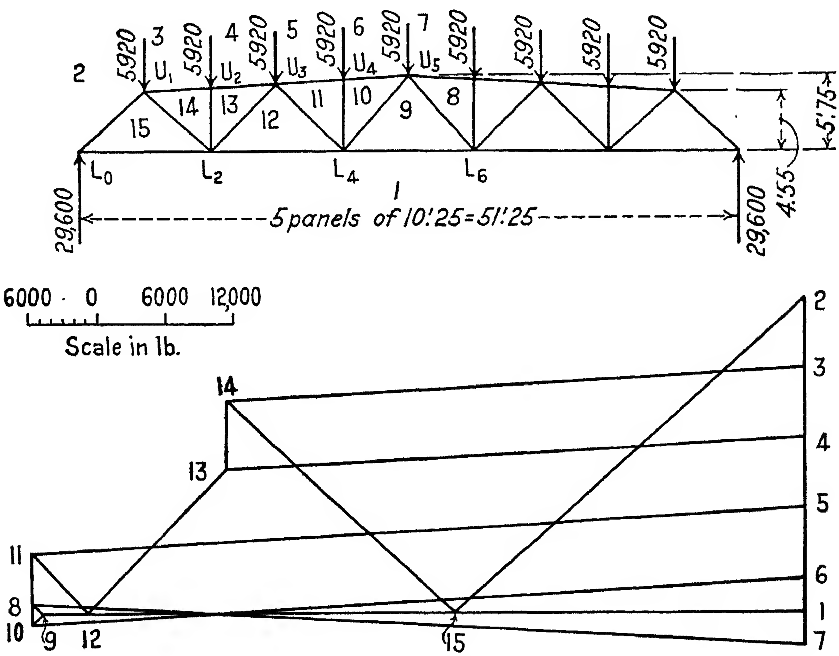


FIG. 85.—Stress Sheet for Truss.

Moment on a purlin is

$$M = \tfrac{1}{8} \times 353.3 \times (15)^2 \times 12 = 119{,}300 \text{ in.-lb.}$$

As the roof slab will be let down $\frac{1}{2}$ in. on the top flanges of the purlins, lateral support will be provided. Hence, $p_t = 18{,}000$ lb. per sq. in., and the required section modulus is $S = 119{,}300/18{,}000 = 6.6$ in.^{3}

A 6-in., 12.5-lb. I with $S = 7.3$ would be sufficient for flexure. The deflection, however, applying Eq. (17) of Art. 72, is

$$\Delta = \frac{5}{384} \cdot \frac{(353.3 \times 15) \times (180)^3}{29{,}000{,}000 \times 21.8} = 0.64 \text{ in.}$$

As this is in excess of the usual amount permitted, that is $180/360 = 0.5$ in., it will be necessary to use a 7-in., 15.3-lb. I. For it, $\Delta = 0.38$ in.

Truss Panel Load.—The weight of steel in the trusses will be pre-estimated by application of the Jones and Laughlin formula

$$w_s = \frac{P}{40}\left(\frac{l}{20} + \frac{12}{s}\right) \tag{1}$$

in which w_s = weight of steel in trusses per square foot of horizontal projection served by a truss; P = total capacity of truss in pounds per square foot; l = center to centre span of truss in feet, and s = spacing of trusses, centre to centre, in feet. This, if P be assumed as 75, gives

$$w_s = \frac{75}{40}\left(\frac{51.25}{20} + \frac{12}{15}\right) = 6.3 \text{ lb.}$$

Allowing 1.7 lb. per sq. ft. for lateral bracing, the total load in pounds per square foot of horizontal projection supported by a truss is

Snow load	30.0
Tar and gravel roof covering	6.0
Slab	30.0
Purlins 15.3/5.125	3.0
Bracing	1.7
Trusses	6.3
Total	77.0

The truss panel load will, therefore, be $5.125 \times 15 \times 77 = 5920$ lb.

Stress in Truss Members.—The stresses in the truss members are readily found by the stress diagram of Fig. 85. The applied loads are laid off on a load line 2–7, and proceeding from the left-hand support the stresses in the members of the half-truss are found without complication. As the half panel load next the support goes directly into the wall, it does not affect the stresses in the truss. Stresses scaled from the lower diagram of Fig. 85 are listed in column 2 of Table 6.

Proportioning of Members.—Table 6 contains the essential facts relating to the proportioning of the truss members. Permissible stresses are as set forth under *Specification* at the beginning of this article.

To conform with paragraph (2) of the specification above, all truss member sections will be made of two angles. Adopting $\frac{3}{4}$-in. rivets, it follows that the minimum practicable angle, having regard also to paragraph (8) of the specification, is a $2\frac{1}{2} \times 1\frac{1}{2} \times \frac{1}{4}$-in. angle.

In order to obviate the computation of stresses caused by the eccentric application of loads to members, as a means of satisfying paragraphs (3) and (7) of the specification, allowances will be made for such effects either by connecting angle members to gussets on gauge lines as close as practicable to the gravity lines of the members, or by providing appropriate excess areas in the

members, or by both methods. In view of the further fact that bending due to eccentricity of application of axial loading is largely prevented in double-angle, stitch-riveted members connected back to back on opposite sides of a single gusset plate, as in Fig. 86, the allowances for eccentricity mentioned above will be adequate.

The required areas given in column 7 of Table 6, do not contain, therefore,

TABLE 6

STRESSES IN AND SECTIONS OF TRUSS MEMBERS

Member	Maximum Stress, Pounds	Length, Centre to Centre, l, Inches	Radius of Gyration, r, Inches	$\frac{l}{r}$	Allowable Stress, Pounds per Square Inch	Area Required, Square Inches	Section	Outstanding Leg, Inches	Area Provided, Square Inches
1	2	3	4	5	6	7	8	9	10
U_1U_3	−50,300	61.5	1.26	49	15,000	3.35 G	2 L's, $4\times3\times\frac{3}{8}$	3	4.96 G.
U_3U_5	−67,200	61.5	1 26	49	15,000	4.48 G.	2 L's, $4\times3\times\frac{3}{8}$	3	4.96 G.
L_0L_2	+30,200	.....	.. ...	...	18,000	1.68 N.	2 L's $3\frac{1}{2}\times3\times\frac{3}{8}$	3	3.94 N.
L_2L_4	+62,100	.....	.....	.....	18,000	3.45 N.	2 L's, $3\frac{1}{2}\times3\times\frac{3}{8}$	3	3.45 N.
L_4L_6	+66,300	......	. ...	....	18,000	3.69 N.	2 L's, $4\times3\times\frac{3}{8}$	3	3.81 N.
L_0U_1	−40,200	83.0	1.11	75	13,710	2.93 G.	2 L's, $3\frac{1}{2}\times3\times\frac{1}{4}$	3	3.12 G.
U_1L_2	+26,700	. ..	......	..	18,000	1.49 N.	2 L's, $3\times2\times\frac{1}{4}$	2	1.94 N.
U_2L_2	− 5,920	58 2	0 70	83	13,020	0.45 G.	2 L's, $2\frac{1}{2}\times1\frac{1}{2}\times\frac{1}{4}$	$1\frac{1}{2}$	1.88 G.
L_2U_3	−16,800	87 0	0.78	112	10,610	1.58 G.	2 L's, $2\frac{1}{2}\times2\times\frac{1}{4}$	2	2.12 G.
U_3L_4	+ 7,100	....	....	..	18,000	0.40 N.	2 L's, $2\frac{1}{2}\times1\frac{1}{2}\times\frac{1}{4}$	$1\frac{1}{2}$	1.44 N.
U_4L_4	− 5,920	65.4	0.70	93	12,160	0.49 G.	2 L's, $2\frac{1}{2}\times1\frac{1}{2}\times\frac{1}{4}$	$1\frac{1}{2}$	1.88 G.
L_4U_5	+ 1,200	.....	.. ...	....	18,000	0.07 N.	2 L's, $2\frac{1}{2}\times1\frac{1}{2}\times\frac{1}{4}$	$1\frac{1}{2}$	1.44 N.

NOTE. + = tension. − = compression.

any extra amounts for stresses due to eccentricity, or for secondary stresses generally. As eccentricity is to be neglected in the double-angle members adopted, the full net area of tension members and the full gross area of compression members is assumed to be effective.

In computing the radii of gyration of compression members, it is assumed that $\frac{3}{8}$-in. gusset plates are used at all panel points.

As there are no reversals of stress, each member is to be designed as a tension or a compression member, but not as both. The "N" opposite an area in Table 6 denotes net area and applies only to tension members; "G" denotes gross area and applies only to compression members.

In computing net areas of tension members, where both legs are connected, the curves of Fig. 1(b) are employed. A forecast of details is necessary in order to make a correct allowance for deductions. Consulting Fig. 86, it is seen, for example, that the rivet stagger at the beginning of the attachment of either L_2L_4 or L_4L_6 to the connection plates at L_4 will be, for a compact joint, about $1\frac{1}{2}$ in. The distance between the gauge lines in one of the angles is, for the

quently 7 rivets are used. The same principle is observed at U_3 and U_5. In testing the sufficiency of the provided rivets, the resultant of the axial force and the vertical purlin reaction should be considered as applied to the rivets.

The axial force to be provided for at U_3 and L_2, where no splices occur, is the difference between the stresses in the two chord members meeting at the joint.

The double-angle members are strich-riveted together as required by paragraph (4) of the specification. In the compression members, the ratio of the spacing of stitch rivets to the radius of gyration of one of the angles in a direction normal to the plane of the truss is considerably less than $\frac{3}{4}$ of the l/r for the whole member.

Bearings.—At one end of the truss, expansion and contraction are provided for by allowing the shoe plate to slide on a bearing plate, slotted holes for the anchor bolts being provided through the shoe plate.

If the allowable bearing pressure on the brickwork be 150 lb. per sq. in., the required area of bearing is $4\frac{1}{2} \times 5920/150 = 178$ sq. in. Shoe and bearing plates will be made 12×15 in., giving 180 sq. in. The 15-in. dimension will be run parallel to the wall.

As the maximum existing upward pressure on the bearing plates is $26{,}600/180 = 148$ lb. per sq. in., and the projection past the edge of the flange angles is $\frac{1}{2}\{15 - (2 \times 3 + 0.375)\} = 4.31$ in., the maximum moment on the shoe and bed plates combined, per inch of width, is $\frac{1}{2} \times 148 \times (4.31)^2 = 1375$ in.-lb.

Required section modulus $= 1375/18{,}000 = 0.076$ in.^{3}

As shoe and bed plates resist the flexure jointly, but not as a unit, since they are not riveted together, two $\frac{1}{2}$-in. plates are required. They give $S = 2 \times \frac{1}{6} \times 1 \times (\frac{1}{2})^2 = 0.83$ in.^{3} A bed plate will, of course, be used at the fixed end.

Two $\frac{3}{4}$-in. anchor bolts will be used at each end.

A sliding bearing of the roof slab on the wall is required at the end of the truss where provision is made for expansion. The wall is recessed at the sliding truss bearing, but may be built solidly around the fixed bearing.

Although the present problem does not require the design of the lateral system, the connection of laterals to the bottom chord of the truss has to be allowed for in detailing the truss itself, as has been done in Fig. 86.

133. Cantilever Shelter Truss.—A driveway along the side of a building is to be covered by a roof cantilevered out from the wall columns, which are spaced 18 ft. centre to centre. The cantilever span of the trusses, measured from the face of the wall, will be 9 ft., and the No. 20 corrugated steel roofing will project 9 in. past the theoretical ends of the trusses. The roofing material rests on four lines of 6-in., 8.2-lb. channel purlins, and a line of $\frac{1}{2}$-in. sag rods at the centre of each bay is used, as shown in Fig. 88. A $5 \times 3 \times \frac{5}{16}$-in. angle is riveted to the upper purlin to reinforce it against the pull of the sag rods. No. 22 corrugated steel will be used to curtain the trusses at each end of the shelter.

Design one of the trusses to the loading and for the specifications which follow.

Loading.—The snow load will be 15 lb. per sq. ft. of horizontal projection to be considered as acting simultaneously with a downward normal component

(estimated by Duchemin's formula) of a wind force of 20 lb. per sq. ft. on a vertical surface. The structure must also withstand, without snow load, an *upward* normal wind force of the same magnitude as the downward one already mentioned.

Specification.—The following brief specification will govern:

(1) Permissible stresses in pounds per square inch

Tension on net section	16,000
Tension on bolts at root of thread	15,000
Compression on gross area of struts,	
(*a*) Where l/r does not exceed 120	$19{,}000 - 100\ l/r$
with a maximum of	13,000
(*b*) Where l/r lies between 120 and 200	$13{,}000 - 50\ l/r$
In these formulae l = unsupported length of strut, and r = corresponding radius of gyration.	
Shear on power-driven rivets	12,000
Shear on unfinished bolts	10,000
Bearing on power-driven rivets	24,000

(2) The minimum thickness of material will be $\frac{3}{16}$ in.

(3) Neglect secondary stresses, whether due to the usual eccentricity of connection of angle members, or to joint rigidity.

(4) Calculate net section of a tension member containing more than one line of rivets by the rule of Eq. (1), Art. 5, considering

$$x = 1.30 - s/g$$

General Dimensions and Form of Truss.—Consider the horizontal distance from the wall to the intersection of the two chords as 9 ft. Adopting a roof slope of 1 in 2, or 26° 34′, which is the best where corrugated steel roofing is used, the depth of the truss at the wall line will be 4 ft. 6 in.

The form of truss adopted will be that shown in Figs. 87 and 88.

Having regard to the necessary position of the upper purlin, the first two horizontal panels of the truss will be made 2 ft. 11 in. each, and the third 3 ft. 2 in., measured to the outer face of the wall.

Truss Panel Loads.—Allowing for lap, the No. 20 corrugated steel weighs 2.23 lb. per sq. ft. of sloping area. The weight of trusses in pounds per square foot of horizontal projection will be assumed, in the absence of weight formulae for cantilever trusses, as 2.0 lb. per sq. ft. Hence the dead load borne by the trusses in pounds per square foot of horizontal projection is as follows:

Corrugated steel, No. 20, = 2.23 sec 26° 34′	= 2.5
Purlins = 8.2/2.92	= 2.8
Sag rods, say	= 0.2
Bracing, say	= 0.6
Trusses, say	= 2.0
	8.1

The panel dead load is, therefore, $2.92 \times 18 \times 8.1 = 430$ lb.

Since the snow load is 15 lb. per sq. ft. of horizontal projection, the panel snow load is $2.92 \times 18 \times 15 = 790$ lb.

The normal wind force on the roof, from Duchemin's formula, $p = 20$, being the wind intensity on a vertical square foot, and the roof slope α being $26° 34'$, is

$$p_n = p \cdot \frac{2 \sin \alpha}{1 + \sin^2 \alpha} = 20 \times \frac{2 \times 0.4472}{1 + (0.4472)^2} = 14.9 \text{ lb. per sq. ft.}$$

Hence the normal panel load due to wind, whether it act downward or upward $= 2.92 \times 18 \times 14.9 = 880$ lb.

In accordance with the usual custom, half panel loads will be assumed to act at the eave purlin and at the purlin next to the wall.

Stresses in Truss Members.—The stresses in truss members due to the dead load are determined graphically in Fig. 87(*a*). After laying off the load line, the stresses in the members are determined successively without complication,

TABLE 7

MAXIMUM STRESSES IN POUNDS IN MEMBERS OF TRUSS

Member	Dead Load, Pounds	Snow Load, Pounds	Wind Load Down, Pounds	Wind Load Up, Pounds	Maximum Stress			
					Tension		Compression	
					Cols.	Stress	Cols.	Stress
1	2	3	4	5	6	7	8	9
L_0U_1	+ 480	+ 880	+ 880	− 880	2+3+4	2240	2+5	400
U_1U_2	+ 960	+1760	+1550	−1550	2+3+4	4270	2+5	590
U_2U_3	+1480	+2710	+2290	−2290	2+3+4	6480	2+5	810
L_0L_1	− 430	− 790	− 980	+ 980	2+5	550	2+3+4	2200
L_1L_2	− 430	− 790	− 980	+ 980	2+5	550	2+3+4	2200
L_2L_3	− 860	−1580	−1980	+1980	2+5	1120	2+3+4	4420
U_1L_1	0	0	0	0	....	0	0	0
U_1L_2	− 480	− 880	−1120	+1120	2+5	640	2+3+4	2480
U_2L_2	+ 220	+ 400	+ 490	− 490	2+3+4	1110	2+5	270
U_2L_3	− 610	−1120	−1410	+1410	2+5	800	2+3+4	3140

NOTE. + = tension. − = compression.

beginning at the free end of the cantilever. The ascertained stresses are listed in column 2 of Table 7.

The stresses due to snow load may be determined by multiplying the stresses due to dead load by the ratio of the respective panel loads, $790/430 = 1.839$. The results are listed in column 3 of Table 7.

Wind load stresses due to the wind acting downward are determined graph-

ically in Fig. 87(*b*). The results are listed in column 4 of Table 7, and the corresponding stresses for wind acting upward are listed in column 5.

Maximum stresses, tensile and compressive, are entered in columns 7 and 9 of Table 7.

Proportioning of Members.—Table 8 contains the essential data respecting the proportioning of truss members. Maximum stresses are transferred from Table 7. For a very light secondary structure such as this, $\frac{3}{16}$-in. material is allowable, if kept well painted. Because of the very small stresses, $\frac{1}{2}$-in. rivets will be employed.

The effective area of a single tension angle connected by one leg is found by multiplying the net section by the efficiency factor, of Art. 9

$$e = 1.0 - 0.18\, u/c$$

The same factor will be applied to the gross area of a single compression angle attached by one leg only.

In the case of double-angle tension members connected back to back on opposite sides of a single gusset plate, the support at connections is assumed to be

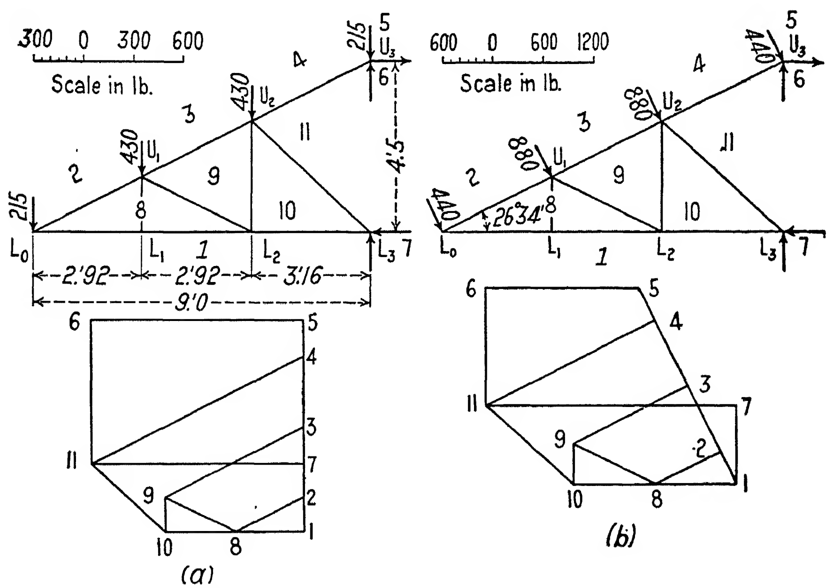


FIG. 87.—Stress Sheet for Cantilever Shelter Truss.

such that the full net section may be considered as effective. The full gross section of double-angle compression members connected in a similar manner is assumed to be effective.

With the conservative working stresses adopted, it is permissible to neglect all secondary stresses, even though standard gauges of angles be employed.

Double-angle sections for the chords are desirable, but single angles will be

permissible for the web members. The minimum section of angle to accommodate $\frac{1}{2}$-in. rivets and meet the thickness regulation is a $1\frac{3}{4} \times 1\frac{3}{4} \times \frac{3}{16}$-in. angle.

The material assigned is indicated in column 11 of Table 8. Two holes only are deducted from U_2U_3. For L_2L_3 it is assumed that the rivet stagger at L_3 is 1 in. Applying the deduction rule of paragraph (4) of the specification, s being 1 in., and g being $2 \times 1 - 0.1875 = 1.8125$ in., the number of holes to be deducted is 3.5. One hole only is taken from the single angle members in considering their sufficiency for tension but this net area is multiplied by the factor $e = 1.0 - 0.18\, u/c$. The same factor is applied to the gross area of the same members when computing their sufficiency for compression.

TABLE 8

SECTIONS OF TRUSS MEMBERS

Member	Maximum Stress		Length, Centre to Centre, l, Inches	Radius of Gyration, r, Inches	$\frac{l}{r}$	Allowable Stress, Pounds per Square Inch		Area Required, Square Inches		Section	Area Provided, Square Inches	
	Tension	Compression				Tension	Compression	Tension	Compression		Gross Effective	Net Effective
1	2	3	4	5	6	7	8	9	10	11	12	13
L_0U_1	2240	400	39	0.54	72	.. .	....	...	.. .	2 L's, $1\frac{3}{4}\times1\frac{3}{4}\times\frac{3}{16}$		
U_1U_2	4270	590	39	0.54	72	.....		. ..	.	2 L's, $1\frac{3}{4}\times1\frac{3}{4}\times\frac{3}{16}$		
U_2U_3	6480	810	42	0.54	78	16,000	11,200	0.41	0.07	2 L's, $1\frac{3}{4}\times1\frac{3}{4}\times\frac{3}{16}$	1 24	1.01
L_0L_1	550	2200	108*	0.81	133	. .	..	. ..	.. .	2 L's, $1\frac{3}{4}\times1\frac{3}{4}\times\frac{3}{16}$		
L_1L_2	550	2200	108*	0.81	133	...	. .	..	...	2 L's, $1\frac{3}{4}\times1\frac{3}{4}\times\frac{3}{16}$		
L_2L_3	1120	4420	108*	0.81	133	16,000	6,350	0.07	0.70	2 L's, $1\frac{3}{4}\times1\frac{3}{4}\times\frac{3}{16}$	1.24	0.83
U_1L_1	0	0	18	....	..	.. .	......	....	.	1 L, $1\frac{3}{4}\times1\frac{3}{4}\times\frac{3}{16}$	0.51	
U_1L_2	640	2480	39	0.35	111	16,000	7,900	0.04	0.31	1 L, $1\frac{3}{4}\times1\frac{3}{4}\times\frac{3}{16}$	0.51	0.41
U_2L_2	1110	270	35	0.35	100	16,000	9,000	0.07	0.03	1 L, $1\frac{3}{4}\times1\frac{3}{4}\times\frac{3}{16}$	0.51	0 41
U_2L_3	800	3140	52	0.35	149	16,000	5,550	0.05	0.57	1 L, $2\times2\times\frac{3}{16}$	0.58	0 41

* No support normal to plane of truss between L_0 and L_3; hence $l = 108$.

Details.—Truss details are shown in Fig. 88. According to the allowable stresses in the specification the single shearing value of a $\frac{1}{2}$-in. rivet is 2360 lb., and its bearing value on a $\frac{3}{16}$-in. gusset plate is 2250 lb. The numbers of rivets required in the ends of the various members can be determined by mentally dividing the governing rivet value into the maximum stress in column 2 or 3 of Table 8. Not less than two rivets will be allowed in a connection. Gusset plates $\frac{3}{16}$ in. thick are adequate.

Since the gusset plate may not bear truly on the column flange at L_3, enough rivets through the gusset and the connection angles will be required to transfer the resultant of the maximum downward shear and the thrust to the right. Under dead load, snow load and downward load, this is $\{(2140)^2 + (6755)^2\}^{1/2}$

= 7130 lb. The number of rivets required in bearing on the $\frac{3}{16}$-in. gusset is 7130/2250 = 4.

Because of the greater certainty of bolts in tension, as compared with rivets, the former will be used for attaching the trusses to the column flanges.

Resolving the maximum stress in U_2U_3, that is 6480-lb. tension, into vertical and horizontal components, it is found that the bolts at U_3 will be subject to a downward shear of 2900 lb. and a tension of 5800 lb. Two $\frac{1}{2}$-in. bolts would be adequate in shear, since $2 \times 0.196 \times 10{,}000 = 3920$ lb. The net area of a

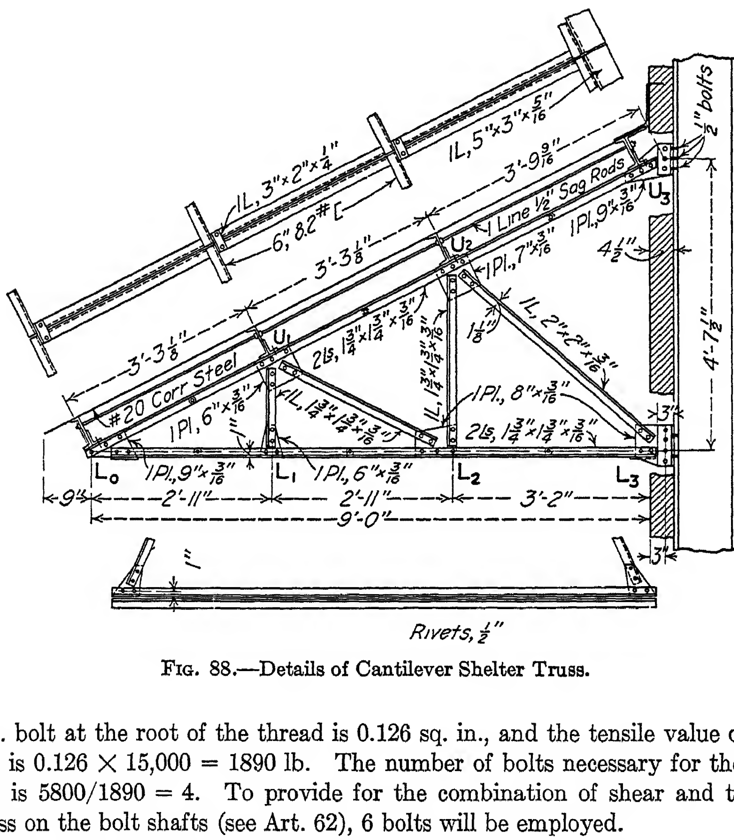


FIG. 88.—Details of Cantilever Shelter Truss.

$\frac{1}{2}$-in. bolt at the root of the thread is 0.126 sq. in., and the tensile value of one bolt is $0.126 \times 15{,}000 = 1890$ lb. The number of bolts necessary for the tension is 5800/1890 = 4. To provide for the combination of shear and tensile stress on the bolt shafts (see Art. 62), 6 bolts will be employed.

At L_3 there will be under dead load and upward wind load an upward shear of 520 lb. at the face of the column flange and a tension of 1705 lb. By reason of the considerable length of the connection angles required to connect the gusset to the column, 4 bolts will be used, as shown in Fig. 88.

The conditions of the problem do not require the design of the lateral bracing. However, as the bracing connections to the bottom chord affect the details of the truss, the bracing has been indicated in Fig. 88.

134. Stresses in Bent Due to Wind Acting as Pressure and Suction.—The bents of a steel mill building are spaced 16 ft. centres, the columns are 29 ft.

high and the trusses have a span of 43 ft., centre to centre of columns. The trusses, which carry a monitor frame, are of the type and proportions indicated in Fig. 89.

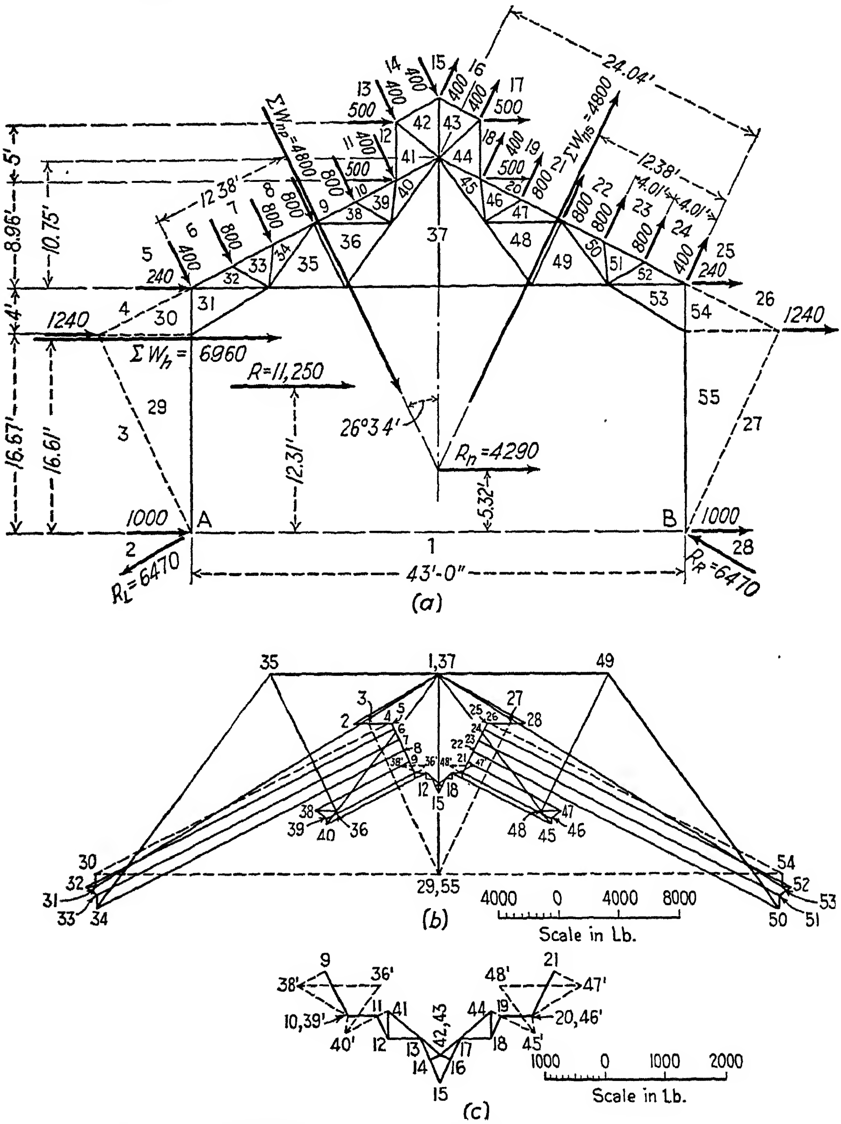


FIG. 89.—Wind Stresses in Bent Subjected to Pressure and Suction.

Assuming that the columns are only partially fixed at the base, determine by a combination of analytical and graphical means the wind stresses in the bent due to the following wind force:

A normal pressure of 7.5 lb. per sq. ft. on the windward wall, plus a normal suction of 7.5 lb. per sq. ft. on the leeward wall, plus a normal pressure of 12.5 lb. per sq. ft. on the windward wall of the monitor, plus a normal suction of 12.5 lb. per sq. ft. on the leeward wall of the monitor, plus a normal pressure on the windward slope of the roof and of the monitor and a normal suction on the leeward slope of the roof and of the monitor each equal to 12.5 lb. per sq. ft.

Assuming the plane of contra-flexure to be located above the actual column bases a distance of one-third the height to the knee brace connection, or 25/3 = 8.33 ft.—a reasonable assumption where columns are only partially fixed at the base—the side walls are virtually only 16.67 + 4.0 = 20.67 ft. high, so far as wind stresses in the bent are concerned.

Considering the wind force on the side columns as applied only at the plane of contra-flexure, at the knee brace connection and at the column top, as indicated in Fig. 89, the concentrations at these points, for both windward and leeward columns, are as follows:

Plane of contra-flexure = 7.5 × 16 × 16.67/2 = 1000 lb.

Knee brace connection = 7.5 × 16 × (16.67 + 4.0)/2 = 1240 lb.

Top of column = 7.5 × 16 × 4/2 = 240 lb.

At the top and bottom of the monitor posts, the wind concentrations are 12.5 × 16 × 5/2 = 500 lb.

The normal pressure on the windward slope being 12.5 lb. per sq. ft., the intermediate panel loads are 12.5 × 16 × 4.01 = 800 lb. Half panel loads occur at the monitor ridge and at the base of the monitor post, and will also be assumed at the main eaves and at the monitor eaves.

The suction concentrations on the leeward slopes are similar to the pressure loads on the windward slopes.

In this problem a combination of analytical and graphical methods is preferable for obtaining the wind reactions at A and B.

The resultant of all horizontal forces, ΣW_h, on the bent is 6960 lb. and it acts at 16.61 ft. above the plane of contra-flexure, AB.

The resultant of all pressure forces normal to the roof and monitor slopes is

$$\Sigma W_{np} = (4 \times 800) + (4 \times 400) = 4800 \text{ lb.}$$

This acts on a line 12.38 ft. up from the windward toe of the truss.

The resultant of all suction forces normal to the slopes, ΣW_{ns}, is likewise 4800 lb., acting on a line 12.38 ft. up from the leeward toe.

ΣW_{np} and ΣW_{ns} intersect on the centre line of the building and produce a horizontal resultant, R_n, which is obviously $2 \times 4800 \sin 26° 34' = 4290$ lb. This force acts at a point located vertically down from the ridge of the main truss a distance of $(24.04 - 12.38) \operatorname{cosec} 26° 34' = 26.10$ ft., or $(20.67 + 10.75) - 26.10 = 5.32$ ft. above the plane of contra-flexure, as shown in Fig. 89(a).

The total wind force on the bent thus resolves itself into a horizontal force

$R = \Sigma W_h + R_n = 6960 + 4290 = 11{,}250$ lb., acting at the centre of gravity of ΣW_b and R_n, or 12.31 ft. above AB.

Where the wind force is applied in equal parts as a pressure and a suction it is reasonable to assume the horizontal components of the wind reactions at A and B as equal, that is each amounting to $11{,}250/2 = 5625$ lb.

The vertical components, V_L and V_R are found, by taking moments about either support, to be $11{,}250 \times 12.31/43.0 = 3220$ lb. The former acts downward and the latter upward.

To make possible the use of a purely graphical method of stress determination for the bent, the dotted auxiliary members 3–29, 4–30, 29–30, 27–55, 26–54, and 54–55 are introduced. The inclusion of these in no wise alters the tresses in the truss members and knee braces.

In the stress diagram, Fig. 89(*b*), the load line 2–3–4 . . . 28 is first laid off and the two reactions 1–2 and 1–28 drawn in making equal angles with the vertical. These may be checked by utilizing the fact that the horizontal and vertical components of each are respectively 5625 and 3220 lb.

Beginning at A the determination of stresses in the members of the bent, real and auxiliary, is easily made in regular sequence until the joints 8–9–38–36–35–34 and 1–35–36–37 are reached, where three unknowns appear.

To remove the indeterminacy, proceed to the monitor ridge. Here there are apparently also three unknowns, but the stress in the vertical 42–43 under a resultant horizontal load at the ridge is zero, as may be shown to be correct by the theory applicable to structures with redundant members. The diagram 14–15–16–43, 42, shown enlarged in Fig. 89(*c*), may then be drawn. The joint 12–13–14–42–41 follows readily, but at joint 10–11–12–41–40–39 the stresses in 10–39 and 40–41 cannot as yet be determined. Their difference, however, represented by the dotted line 40′–41, can be found, as also the actual stress in 39–40, represented by the dotted line 10, 39′–40′ in the enlarged diagram. In the latter diagram there is also found 38′–10, 39′, the actual stress in 38–39, and 9–38′, the difference between the stresses in 9–38 and 10–39. Proceeding to the joint 36–38–39–40–37, this diagram gives 36′–38′, the actual stress in 36–38 and 36′–40′, the difference between the stresses in 36–77 and 37–40.

Coming back to 8–9–38–36–35–34, final solution is made by drawing in the main diagram, Fig. 89(*b*), an intercept 36–38 parallel to the truss member 36–38, and having a length equal to that of the dotted line 36′–38′ in the same diagram, between the line 35–36 and 9–38. The remainder of the diagram follows readily.

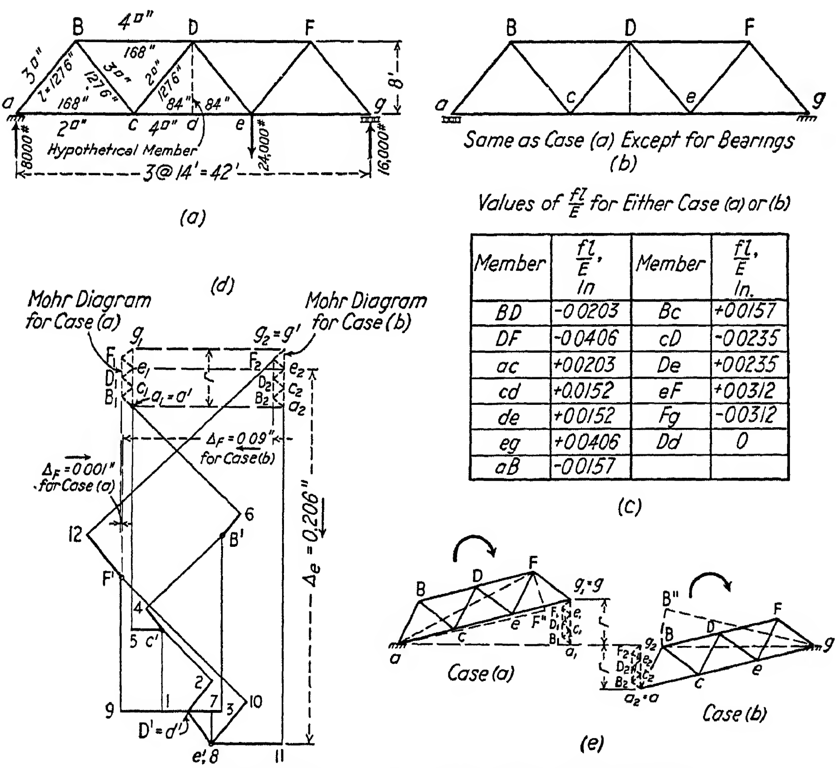
In the columns the true axial stresses will not be the *apparent* stresses for these members taken from the stress sheet, but the algebraic sum of the vertical components of stress in the column and in the inclined auxiliary member at the horizontal section under consideration. For an example of the determination of the true axial stresses in columns of this kind, see Art. 139.

135. Deflection of a Truss by the Analytical Method.—By means of the analytical method find the vertical displacement of the panel point e and the horizontal displacement of the panel point F of the truss of Fig. 90(*a*) due to a vertical downward load of 24,000 lb. at panel point e. $E = 29{,}000{,}000$ lb. per sq. in. Consider the right end of the truss as free to slide horizontally.

The displacement of any joint of a frame in any selected direction is

$$\Delta = \Sigma \frac{Pul}{AE} = \Sigma \frac{ful}{E} \tag{1}$$

in which P = total stress in any member;
A = gross area of member;
f = stress in member per unit of area;
l = length of member;
E = modulus of elasticity of member;
u = stress in the member due to a 1-lb. load applied at the joint the displacement of which is sought and in the direction in which it is desired.



Values of $\frac{fl}{E}$ for Either Case (a) or (b)

Member	$\frac{fl}{E}$, In	Member	$\frac{fl}{E}$, In.
BD	-0 0203	Bc	+0 0157
DF	-0 0406	cD	-0 0235
ac	+0 0203	De	+0 0235
cd	+0.0152	eF	+0 0312
de	+0 0152	Fg	-0 0312
eg	+0 0406	Dd	0
aB	-0 0157		

Fig. 90.—Displacement of Truss Joints by Williot-Mohr Diagram.

Where the modulus of elasticity is the same for all members, it is convenient to employ Eq. (1) in the form

$$\Delta = \frac{1}{E} \Sigma ful \tag{2}$$

Table 9 sets forth the calculation of the displacement of each of the two points. The first six columns of the table are common to both determinations,

but a set of values of u must be computed for each new panel point investigated. For calculating the displacement of the point e the 1-lb. load is assumed as acting vertically downward, and for the point F it is assumed as acting horizontally to the left.

TABLE 9

DEFLECTION OF PANEL POINTS c AND e OF TRUSS OF FIG. 90(a) BY ANALYTICAL METHOD

Member	Length, l In.	Area A, Sq. In.	Stress, P, Kips	Stress, f, Kips per In.^{2}	fl	u_e	u_F	fu_el	fu_Fl
BD	168	4	−14.00	−3.50	−588	−0.584	−0 333	+344	+196
DF	168	4	−28.00	−7.00	−1176	−1.167	−0.666	+1373	+784
ac	168	2	+7.00	+3.50	+588	+0.292	−0.830	+172	−488
ce	168	4	+21.00	+5.25	+882	+0.875	−0 498	+772	−439
eg	168	2	+14.00	+7.00	+1176	+0.583	−0.166	+687	−196
aB	127.6	3	−10.65	−3.55	−454	−0.444	−0.253	+202	+115
Bc	127.6	3	+10.65	+3.55	+454	+0.444	+0.253	+202	+115
cD	127.6	2	−10.65	−5.33	−680	−0.444	−0.253	+302	+172
De	127.6	2	+10.65	+5.33	+680	+0.444	+0.253	+302	+172
eF	127.6	3	+21.30	+7.10	+906	+0.888	−0.253	+805	−229
Fg	127 6	3	−21.30	−7.10	−906	−0.888	+0.253	+805	−229
							$\Sigma ful =$	+5956	−27

$$\Delta_e = \Sigma fu_el/E = +5956 \times 1000/29{,}000{,}000 = +\mathbf{0.206\ in.}$$
$$\Delta_F = \Sigma fu_Fl/E = -27 \times 1000/29{,}000{,}000 = -\mathbf{0.001\ in.}$$

The plus sign for the computed displacement of e indicates that it is in the same sense as that of the assumed 1-lb. load, that is downward. The minus sign for the displacement of F shows that the actual movement of 0.001 in. is to the right, that is opposite in sense to that of the applied 1-lb. load.

136. Deflection of a Truss by the Williot-Mohr Diagram.—Determine the vertical displacement of the point e and the horizontal displacement of F of the truss shown in Fig. 90 employing the Williot-Mohr diagram, assuming that (a) the right end slides and (b) the left end slides. $E = 29{,}000{,}000$ lb. per sq. in.

Member length changes to be connected according to the arrangement required by the Williot diagram may be derived from Table 9, Art. 135, by dividing the values of fl in the sixth column by E. The resulting quantities are indicated in Table (c) of Fig. 90. Because of the introduction of the hypothetical member Dd, it is convenient to consider the bottom chord ce as two members, cd and de.

With a view to making the scale of the deformations as large as possible, the reference member will be taken as the hypothetical centre vertical Dd and the reference point as d. Taking the reference member near one end of the truss would necessitate a much smaller scale in order to keep the diagram within reasonable bounds.

Williot Diagram.—The first stage of the solution is effected by constructing

an ordinary Williot diagram, Fig. 90(d), considering the ends of the truss to curl up, the point d (and also D) to remain fixed in space and the direction Dd to be fixed.

Consider the movement of c with respect to the reference point and reference member. Owing to the lengthening of cd it moves horizontally and to the left, according to the Table (c) in Fig. 90, a distance of 0.0152 in. The horizontal line $d'1$ indicates the amount of this movement. Owing to the shortening of cD the point c moves 0.0235 in. upward and to the right. The line $d'2$ drawn parallel to cD indicates this. Normals $1c'$ and $2c'$ drawn to these lines at their ends 1 and 2, in reality representing very short arcs of circles with the ends d and D of the members as centres, give an intersection c' which represents the resultant movement of c with respect to the reference point.

The point B moves horizontally 0.0203 in. to the right with respect to the fixed point D, or over the scaled length $D'3$. It moves 0.0157 in. upward and to the left with respect to c_1 or over a distance $c'4$ drawn parallel to cB. The intersection B' of the normals $3B'$ and $4B'$ indicates the movement of B relative to d.

The point a moves a scaled distance $c'5$ horizontally to the left relative to c and a distance $B'6$ upwards and to the right with respect to B. The intersection a' of the normals $5a'$ and $6a'$ gives the travel of a relative to the fixed point d.

Proceeding to the other side of the centre line of the truss e', the intersection of the normal $7e'$ and $8e'$ (too short to be detected in the diagram) gives the relative position of e. The point F moves to the left $D'9$ with respect to D and upward to the right $e'10$ with respect to e, taking up a relative position F'. The end joint g moves horizontally $e'11$ to the right relative to e and $F'12$ upwards and to the left with respect to F, reaching the relative position g'.

Mohr Correction Diagram.—From the Williot diagram it is evident that, if the hypothetical member Dd is held fixed, the end g moves up more than the end a by a distance r equal to the difference of height of the points g' and a'. Since actually, for a level truss, the two ends remain at the same elevation, the truss must be rotated so that this difference is removed.

In order that the horizontal movement of joints may be properly indicated the corrective rotation for Case (a) should be clockwise about a, the fixed end, and for Case (b) clockwise about g, the fixed end for that case. The effect of such rotation is found by the simple device of the Mohr correction diagram.

For Case (a) the Mohr diagram consists of a diagram of the actual truss drawn to scale with its bottom chord vertical and occupying the length $a'g_1$ and the top chord to the *left* of the bottom chord, as shown in Fig. 90(d). The necessary correcting movement of any panel point is then the distance of that designated point in the Mohr diagram to a_1 (or a').

The truth of this is evident from the left half of Fig. 90(e). Rotation causes F to move FF'' downward and to the right. But, from similar triangles,

$$FF'' = \frac{Fa}{ga} \cdot r = \frac{F_1a_1}{g_1a_1} \cdot r = F_1a_1$$

Hence the correcting movement of F is F_1a_1 downward and to the right.

For Case (*b*) the Mohr diagram is drawn in Fig. 90(*d*) with its bottom chord as a_2g' and the top chord again to the *left* of the bottom chord, the correcting rotation being clockwise about g. For the same reasons as those applicable to Case (*a*), the corrective movement of B is $BB'' = B_2g_2$.

Had the point g' come lower than a' in the Williot diagram, a corrective counter-clockwise rotation would have been necessary and the Mohr correction diagrams in Fig. 90(*d*) would have had the top chord to the *right* of the bottom chord in each case.

The true vertical displacement of e, when either the right end or the left end slides, is the vertical distance between e_1 and e' or between e_2 and e' in Fig. 90(*d*). This is $\Delta_e = 0.206$ in. The true horizontal movement of F for Case (*a*) is the horizontal distance between F_1 and $F' = \Delta_F = 0.001$ in. to the right. For Case (*b*) it is the horizontal distance between F_2 and $F' = \Delta_F = 0.09$ in. to the left.

CHAPTER IX

CRANES AND CRANE SUPPORTS

137. Design of a Stationary Jib Crane.—Design a stationary jib crane of the type shown in Fig. 91 for a moving 2-ton load with a mast height of 16 ft. shoulder to shoulder, and with a boom of such length that the centre of the trolley in its extreme outer position is 10 ft. from the centre of the mast. The centre of the boom is to be 12 ft. above the base of the column.

Impact will be taken as 25% of the live load stresses.

Permissible stresses are to be as follows: Tension on net section of tie rods, $p_t = 18{,}000$ lb. per sq. in.; combined compression and flexure on mast or boom, $p_c = p_f = 18{,}000/(1 + l^2/18{,}000\ r^2)$ lb. per sq. in., with a maximum of 15,000, where l = unsupported length and r = corresponding radius of gyration.

Rivets and bolts will be of $\frac{3}{4}$-in. diameter.

Stresses due to the weight of the members and those due to the moment arising from buckling or deflection of members will be neglected.

General Arrangement.—Having regard to the relation of the moments and thrusts, it will be assumed that the overhead tie will connect to the boom at 8 ft. from the centre of the mast. To permit the trolley centre to move 2 ft. farther out, and to accommodate an end stop, the boom will extend 3 ft. past the tie rod connection.

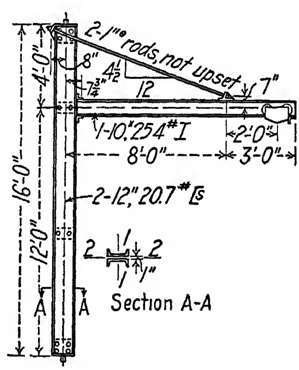


Fig. 91.—Stationary Jib Crane.

Overhead Tie.—The maximum live load reaction at the connection of the tie to the boom will occur when the centre of the trolley is in its extreme outer position, that is 2 ft. from the tie connection. Taking the inner support of the boom as at $7\frac{3}{4}$ in., or 0.65 ft., from the centre of the mast, and adding 25% to the live load of 4000 lb. for impact effect, the maximum vertical effect on the tie rod is, neglecting dead load, $5000 \times 9.35/7.35 = 6360$ lb.

Since the slope of the tie, Fig. 91, is $4\frac{1}{2}$ in 12, the stress in the tie is

$$6360 \times \{(4.5)^2 + (12.0)^2\}^{1/2}/4.5 = 18{,}100 \text{ lb.}$$

Required net area of the tie = 18,100/18,000 = 1.01 sq. in. Use two 1-in. diameter rods, not upset, for which the area = 2 × 0.55 = 1.10 sq. in. at root of thread.

Boom.—Three positions of the live load should be considered for the boom;

that is, when it is (1) at the centre of the 7.35-ft. segment, (2) at the overhead tie connection, and (3) at the extreme outer end of the boom next the stop.

Boom, Case 1.—For a 5000-lb. load (including impact but neglecting the weight of the boom) applied at the centre of the 7.35-ft. segment, the horizontal thrust P_1 applied by the tie approximately 7 in. above the centre of gravity of the boom, assuming the latter to be a 10-in. I-beam, is $2500 \times 12/4.5 = 6670$ lb.

Moment due to eccentricity of application of this thrust $= 6670 \times 7 =$ 46,690 in.-lb.

Moment, including impact effect, due to load at centre of 7.35-ft. segment, assuming no restraint at the ends $= Wl/4 = 5000 \times 7.35 \times 12/4 = 110{,}250$ in.-lb.

Total moment, M_1, at centre of 7.35-ft. segment $= 46{,}690 + 110{,}250$ $= 156{,}940$ in.-lb.

Total maximum stress in the boom on the upper fibre of the 7.35-ft. segment at its centre is the stress, f_c, due to axial thrust considered as centrically applied + stress f_f, due to flexure arising from the trolley load and due to the bending from eccentricity. That is, from Eq. (3), Art. 36

$$f_m = f_c + f_f = P_1/A + M_1 y_e/I,$$

where A = area of section, M_1 = total apparent moment from the two causes; y_e = distance from the centre of gravity of the section to the extreme fibre; and I = moment of inertia of section in the plane of bending. Hence, if a 10-in., 25.4-lb. I be assumed

$$\begin{aligned} f_m &= 6670/7.38 + 156{,}940 \times 5/122.1 \\ &= 900 + 6420 = 7320 \text{ lb. per sq. in.} \end{aligned}$$

Regarding the boom as a strut subjected to both axial and transverse loading, with $l = 7.35 \times 12 = 88$ in. and with r, the radius of gyration, about an axis lying in the plane of the web $= 0.97$, the permissible stress $p_c = 18{,}000/(1 + 88^2/18{,}000 \times 0.97^2) = 12{,}350$ lb. per sq. in. The section assumed is considerably in excess, but a shallower beam would not give sufficient vertical clearance for the trolley to travel on the bottom flange.

Boom, Case 2.—With the 5000-lb. load (including impact) placed at the tie connection, the eccentric horizontal thrust, $P_2 = 5000 \times 12/4.5 = 13{,}330$ lb.

Moment due to eccentricity of $P_2 = 13{,}330 \times 7.0 = 93{,}310$ in.-lb.

Total maximum stress on upper fibre of 7.35-ft. segment is, Eq. (3), Art. 36

$$\begin{aligned} f_m &= 13{,}330/7.38 + 93{,}310 \times 5/122.1 \\ &= 1810 + 3820 = 5630 \text{ lb. per sq. in.} \end{aligned}$$

The assumed section is adequate.

Boom, Case 3.—With the 5000-lb. load 2 ft. from the tie connection, the eccentric horizontal thrust, $P_3 = 6360 \times 12/4.5 = 16{,}960$ lb.

Moment due to eccentricity of $P_3 = 16{,}960 \times 7.0 = 118{,}720$ in.-lb.

Negative moment at centre of 7.35-ft. segment produced by downward reaction of $5000 \times 2/7.35 = 1360$ lb. at the mast $= 1360 \times 44 = 59{,}800$ in-lb.

Net positive moment at centre of 7.35-ft. segment = 118,720 − 59,800 = 58,920 in.-lb.

Total maximum stress on upper fibre of 7.35-ft. segment is

$$f_m = 16{,}960/7.38 + 58{,}920 \times 5/122.1$$
$$= 2300 + 2410 = 4710 \text{ lb. per sq. in.}$$

The section is adequate for this stress.

The condition should be investigated also for the point immediately outside the tie connection. Here, the negative moment due to the load at the end of the boom = 5000 × 24 = 120,000 in.-lb., and the flexural fibre stress is 120,000 × 5/122.1 = 4910 lb. per sq. in. This is within the permissible limit.

Mast.—The mast should be investigated for the three critical positions of the live load considered in designing the boom. By reason of the fact that the maximum buckling effect normal to the plane of the channel webs is at mid-height of the mast, the latter level will be assumed as the critical one. It is true that the moment will be greater at the level of the boom, but the permissible stress would also be greater.

Mast, Case 1.—Assuming the 5000-lb. load (including impact) at the centre of the 7.35-ft. segment of the boom, its line of action will be 4.32 ft. from the centre line of the mast. Taking moments about the top or bottom of the mast, the horizontal reactions will be 5000 × 4.32/16 = 1350 lb., acting to the left at the top and to the right at the bottom.

For the sake of simplicity, consider the forces acting on the lower half of the mast.

Total axial thrust, which may be considered centric, = 5000 lb.

Moment at mid-height of the mast = 1350 × 96 = 129,600 in.-lb.

Assuming the mast to consist of two 12-in., 20-7-lb. channels arranged as shown in Fig. 91, then for the entire section, A = 12.06 sq. in. and I_1 = 256.2 in.^{4}, and the maximum fibre stress at the centre of the height of the mast is

$$f_m = 5000/12.06 + 129{,}600 \times 6/256.2$$
$$= 415 + 3030 = 3445 \text{ lb. per sq. in.}$$

Radius of gyration of two channels 1 in. back to back, as shown in Fig. 91, about axis 2 − 2 is $r_2 = (I_2/A)^{1/2}$. Now, $I_2 = 2(I_0 + A_0y_0^2)$, where I_0 = moment of inertia of one channel about an axis through its centre of gravity parallel to its web; A_0 = area of one channel; y_0 = distance from gravity axis of one channel to gravity axis of mast as a whole. Hence

$$I_2 = 2\{3.9 + 6.03(1.20)^2\} = 25.2 \text{ in.}^4$$

and r_2 = 1.44 in.

Permissible stress on mast considered as strut tending to buckle in plane normal to web = p_c = 18,000/(1 + 192²/18,000 × 1.44²) = 9050 lb. per sq. in.

The assumed section is much in excess for this case of loading.

Mast, Case 2.—With a 5000-lb. load at the tie connection to the boom, the

mast is subjected to a horizontal reaction of 5000 $\times \frac{8}{16}$ = 2500 lb. at the base. The total axial thrust = 5000 lb.

Moment at centre of mast = 2500 $\times$ 96 = 240,000 in.-lb.

For the assumed section, the maximum fibre stress

$$f_m = 5000/12.06 + 240,000 \times 6/256.2$$
$$= 415 + 5620 = 6035 \text{ lb. per sq. in.}$$

The section assumed is, therefore, adequate, as the permissible stress is 9050 lb. per sq. in.

Mast, Case 3.—With the 5000-lb. load at the extreme outer position on the boom, the mast is subjected to a horizontal reaction of 5000 $\times \frac{10}{16}$ = 3125 lb. The total axial thrust = 5000 lb.

Moment at centre of mast = 3125 $\times$ 96 = 300,000 in.-lb.

For the assumed section, that is, two 12-in., 20.7-lb. channels

$$f_m = 5000/12.06 + 300,000 \times 6/256.2 = 7440 \text{ lb. per sq. in.}$$

Since the permissible stress is 9050 lb. per sq. in., the assumed section is adequate. Lighter channels would, however, not be sufficient.

Details.—Filler or separator plates should be inserted between the channels at such distances apart that their maximum spacing centre to centre divided by the radius of gyration r'_2 of a single channel about its gravity axis parallel to its web does not exceed the slenderness ratio of the member, as a whole about the gravity axis parallel to the channel webs, that is 16 $\times$ 12/1.44 = 133.4. Since r'_2 = 0.81, the maximum spacing of separators is, therefore, 0.81 $\times$ 133.4 = 108 in. = 9 ft. This requirement will be met by the use of two separators located as shown in Fig. 91.

Head and foot castings are designed with a 1-in. web to be bolted or riveted between the channels. A casting, attached by four bolts, connects the tie to the boom. All castings should be of steel.

138. Design of an A-Bent for the Support of an Outside Crane Runway.—Design one of the supporting A-shaped bents for an outside crane runway to conform to the following requirements:

General Dimensions.—Length of runway = 200 ft. (10 girder spans of 20 ft. each). Height = 30 ft. to base of runway rail. Span of crane = 60 ft. centre to centre of rails, as shown in Fig. 92.

Loading.—A 20-ton electric travelling crane, imposing on a runway rail, as a maximum, two 36,000-lb. vertical loads, 10 ft. 6 in. apart, plus 25% for impact.

End thrust of crane = 20% of lifting capacity, equally divided amongst the four wheels of the crane and without impact.

Tractive force on the runway rail = 20% of the maximum wheel loads, without impact.

Wind on the side of the crane and the side of the runway girders, 20 lb. per sq. ft. Wind force on the end of crane = 1000 lb. Wind force on the bent = 35 lb. per ft. of height, either in the longitudinal or the transverse direction.

Specification.—

(*a*) The following loading combinations shall be considered:

(1) Dead load + live load + impact;

(2) Dead load + live load + impact + traction + longitudinal wind on bents only;

(3) Dead load + live load + impact + longitudinal wind;

(4) Dead load + live load + impact + end thrust + transverse wind.

(*b*) Permissible stresses in pounds per square inch

Tension on net section...............	18,000
Compression on gross section..........	17,000 − 60 l/r
but not over......................	15,000

These stresses are to be employed for combination (1) but may be increased by 25% for combinations (2), (3) or (4). In no case shall the section be less than that required for combination (1) at the basic permissible stresses.

(*c*) Maximum slenderness ratios, l/r,

For legs, 130;

For other compression members, 175.

(*d*) For two angles riveted back to back on opposite sides of a gusset plate, the full gross section for compression and the full net section for tension are to be considered as effective.

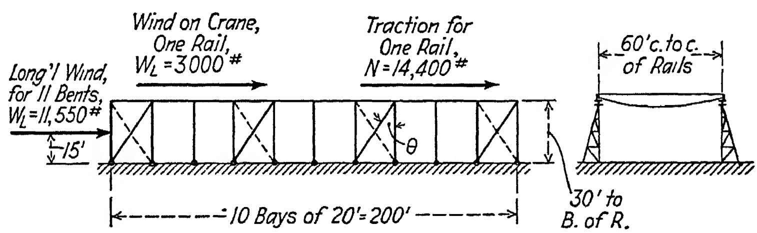


FIG. 92.—Layout of Crane Runway.

Form and Proportions of Bent.—A bent with one vertical leg and one sloping leg, as shown in Fig. 93, will be adopted. This leaves practically the whole space covered by the span of the crane free of obstruction. The height will be divided into four stories averaging 7.5 ft. each, the exact heights to be determined in the detailing. Very steep bracing is likely to make simple connections difficult of attainment, and stories of considerable height are likely to give excessive slenderness ratios for the legs. A base width of one-quarter of the height, or 7.5 ft., will be satisfactory. It will be assumed that the runway girder is 2 ft. deep, which would make the actual height of the bent 28 ft., if the apex point is assumed at the base of rail, as in Fig. 94.

Stresses Due to Vertical Loads.—For simplicity the weight of the bent will be neglected in the computations. The vertical loads will then all be applied at

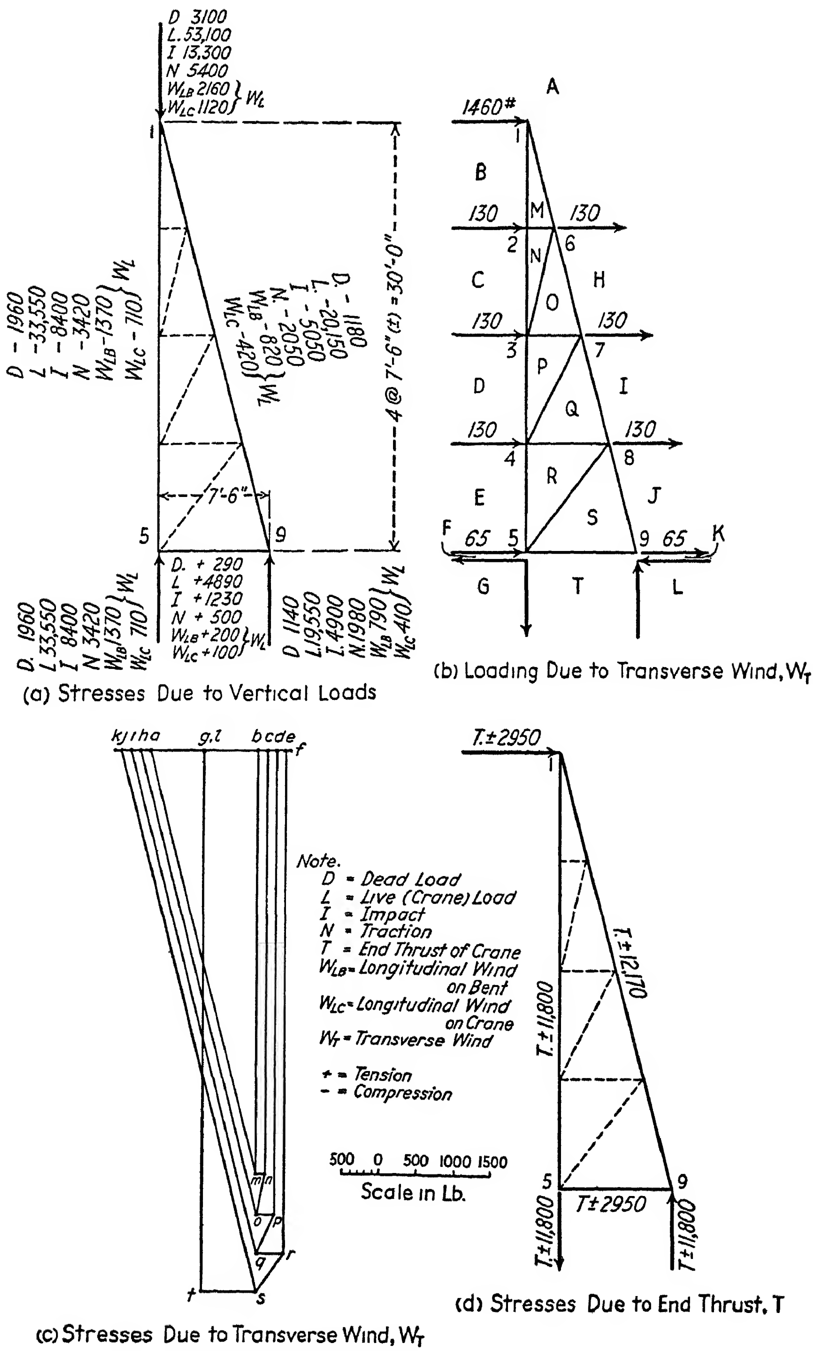


Fig. 93.—Stress Sheet for a Crane Runway Bent.

the apex joint 1. They consist of dead load, live (crane) load, impact (due to the vertical effect of the crane), traction and longitudinal wind.

Assuming the weight of the crane girder with the running rail and attachments to be 155 lb. per lin. ft., the dead load concentration at the top of the bent is $20 \times 155 = 3100$ lb.

The maximum wheel loads of the crane being two loads of 36,000 lb. each, spaced at 10 ft. 6 in., the maximum concentration at the apex of the bent is $36{,}000\left(1 + \frac{9.5}{20}\right) = 53{,}100$ lb. Impact will augment this by $0.25 \times 53{,}100 = 13{,}300$ lb.

The vertical effects of traction and longitudinal wind may be easily seen by an examination of Fig. 92. A total tractive effect for one rail of $0.2 \times 72{,}000 = 14{,}400$ lb. is exerted along the rail. If longitudinal bracing is inserted in four bays, each of them will have to look after 3600 lb. of tractive force, and the vertical effect on a bent to the top of which a bracing diagonal connects will be $3600 \cot \theta = 3600 \times 30/20 = 5400$ lb.

Longitudinal wind force of 35 lb. per ft. of height of each of the eleven bents, or $11 \times 35 \times 30 = 11{,}550$ lb. in all, is exerted at 15 ft. up from the base. Assuming that the bents are not fixed against rotation in the direction of the runway, a reaction of $11{,}550 \times 15/30 = 5775$ lb. will be delivered to the top of the longitudinal bracing system.

If the average depth of the crane bridge be taken as 5 ft., including therein the projected area of the trolley, a longitudinal wind force of $5 \times 30 \times 20 = 3000$ lb. will be delivered to each rail.

In all, therefore, the longitudinal bracing must resist $5775 + 3000 = 8775$ lb. Each braced bay will need to take $8775/4 = 2190$ lb., and the vertical effect at the top of a critical bent is $2190 \cot \theta = 3280$ lb., of which 2160 lb. arises from the wind on the bent and 1120 lb. from the wind on the crane.

Frequently, the vertical loads are considered as being wholly absorbed by the vertical post, the sloping one being then considered merely as a transverse brace for the post. This is in error, on the side of safety, if the seat for the runway girder covers the sloping post as well as the vertical one, as in Fig. 94. In this problem both posts will be considered as resisting vertical load.

The stress in the two legs due to a vertical apex load cannot be found by statical methods alone. A simple solution is effected by considering the relative *vertical* stiffness of the legs and allocating the applied load in proportion thereto. If the area and length of the vertical leg be respectively A_1 and l_1 and the area and sloping length of the inclined post be A_2 and l_2, then under a vertical load of P at the apex, the axial stresses in the two legs are, respectively,

$$P_1 = P \cdot \frac{A_1 l_2}{A_1 l_2 + A_2 l_1} \tag{1}$$

$$P_2 = P \cdot \frac{A_2 l_2}{A_1 l_2 + A_2 l_1} \tag{2}$$

In order to apply these formulae it is necessary to assume the relative areas

of the two legs. Examination of representative designs shows that the area of the sloping leg is commonly about 0.60 of the area of the vertical one, and this ratio will be assumed in the present instance. The ratio of l_2 to l_1 being fixed by the dimensions of the bent, it readily follows from Eqs. (1) and (2) that

$$P_1 = 0.632\,P$$
$$P_2 = 0.379\,P$$

Apportioning the vertical loads to the legs in this manner, the stresses indicated in Fig. 93(*a*) are obtained. For vertical loading the dotted bracing members serve only as auxiliary, or stay, members for the legs. The bottom tie, however, resists the horizontal component in the sloping leg.

Stresses Due to Transverse Wind.—Any intermediate bent may be subjected to three distinct transverse wind forces at the top. The 1000-lb. force on the end of the crane may throw $\frac{1000}{4}\left(1 + \frac{9.5}{20}\right) = 370$ lb. on a bent at the top. Assuming the girders to be 2 ft. deep, and the running rail 0.4 ft. deep, another $2.4 \times 20 \times 20 = 960$ lb. is added at that point. Further, there is a half story of wind properly allocated at the top, that is $\frac{1}{2} \times 7.5 \times 35 = 130$ lb. Consequently, there exists a total horizontal transverse wind force of 1460 lb. at the top.

TABLE 10

STRESSES IN POUNDS IN MEMBERS OF BENT DUE TO VARIOUS LOADS

+ = tension — = compression

Member	Nature of Load							
	Dead Load, D	Live Load, L	Impact, I	Traction, N	Longitudinal Wind		Transverse Wind, W_T	End Thrust, T
					On Bent, W_{LB}	On Crane, W_{LC}		
1 – 2	−1960	−33,550	−8400	−3420	−1370	−710	±5820	±11,800
2 – 3	−1960	−33,550	−8400	−3420	−1370	−710	±5820	±11,800
3 – 4	−1960	−33,550	−8400	−3420	−1370	−710	±6380	±11,800
4 – 5	−1960	−33,550	−8400	−3420	−1370	−710	±6920	±11,800
1 – 6	−1180	−20,150	−5050	−2050	−820	−420	±6000	±12,170
6 – 7	−1180	−20,150	−5050	−2050	−820	−420	±6580	±12,170
7 – 8	−1180	−20,150	−5050	−2050	−820	−420	±7120	±12,170
8 – 9	−1180	−20,150	−5050	−2050	−820	−420	±7660	±12,170
2 – 6	....	....	....	....	....	....	±130	
3 – 6	....	....	....	....	....	....	±570	
3 – 7	....	....	....	....	....	....	±260	
4 – 7	....	....	....	....	....	....	±580	
4 – 8	....	....	....	....	....	....	±390	
5 – 8	....	....	....	....	....	....	±650	
5 – 9	+290	+4,890	+1320	+500	+200	+100	±740	±2,950

For the other panel points of the frame the wind loads, applied equally to the windward and leeward sides, are as indicated in Fig. 93(*b*). If the stories be 7.5 ft. high, an intermediate panel point wind load is $\frac{1}{2} \times 7.5 \times 35 \doteq 130$ lb. The resulting stresses are readily found by the graphical construction of Fig. 93(*c*) and are listed in Table 10.

Stresses Due to End Thrust of Crane.—The total end thrust of the crane being $0.2 \times 40{,}000 = 8000$ lb., a horizontal transverse force of 2000 lb. will be applied to the rail by each wheel. At a bent a maximum concentration of $2000\left(1 + \frac{9.5}{20}\right) = 2950$ lb. is therefore possible. The stresses resulting from this force, which may act either to the right or left, are indicated in Fig. 93(*d*).

TABLE 11

MAXIMUM STRESSES IN POUNDS IN MEMBERS OF BENT DUE TO VARIOUS COMBINATIONS OF LOADS

+ = tension — = compression

Member	Combinations of Loads			
	$(1) = D+L+I$	$(2) = D+L+I+N+W_{LB}$	$(3) = D+L+I+W_L$	$(4) = D+L+I+T+W_T$
1 — 2	−43,910	−48,750	−45,990	−61,530
2 — 3	−43,910	−48,750	−45,990	−61,530
3 — 4	−43,910	−48,750	−45,990	−62,090
4 — 5	−43,910	−48,750	−45,990	−62,630
1 — 6	−26,380	−29,300	−27,620	−44,550
6 — 7	−26,380	−29,300	−27,620	−45,130
7 — 8	−26,380	−29,300	−27,620	−45,670
8 — 9	−26,380	−29,300	−27,620	−46,210
2 — 6				
3 — 6				
3 — 7				
4 — 7				
4 — 8				
5 — 8				
5 — 9	+6,410	+7,110	+6,710	+10,100

Combinations of Stress.—The most exacting combinations of stress in the various members of the bent are indicated in Table 11. As was indicated in the specification at the beginning of this article, basic working stresses are required for combination (1), but increased stresses are permitted for the other three.

Proportioning of Members.—Inspection of Table 11, taking account the fact that the permissible stresses for combinations (2), (3) and (4) may be increased

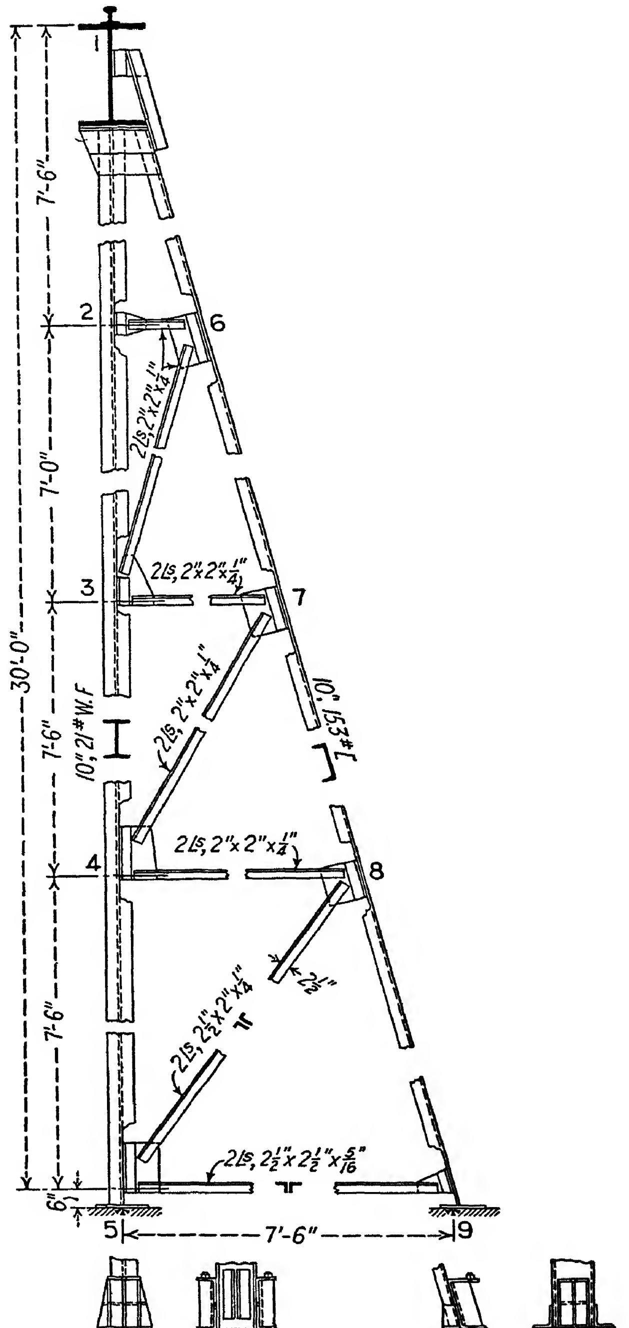


FIG. 94.—A-Bent for the Support of an Outside Crane Runway.

by 25% above the basic working stresses, indicates that, for the vertical leg, the critical case is (4), and the critical story is the bottom one, for which the stress is 62,630 lb.

Although, so far as this leg alone is concerned, it would be permissible to use an 8-in. I-section, it is found that the sloping leg must be at least a 9-in. channel. Since 9-in. I-sections are not readily available, it will be necessary, for convenience in details, to make both legs 10 in. wide, perpendicular to the plane of the bent.

Assuming a 10-in., 21-lb. W.F. for the vertical leg, the least radius of gyration, r, is 1.25 in. and l, the unsupported length, is 90 in. Consequently, $l/r = 90/1.25 = 72$. The permissible stress for combination (4) is then $p_c = 1.25(17{,}000 - 60 \times 72) = 15{,}850$ lb. per sq. in., the required area is $62{,}630/15{,}850 = 3.95$ sq. in., and the provided area is 6.19 sq. in. Were it not for the desirability of conforming to the necessary width of the sloping leg, the assumed section might be substantially reduced.

For the sloping leg the critical stress is also due to combination (4) and is 46,210 lb. in the bottom story. Assuming a 10-in., 15.3-lb. channel, the least r is 0.72 in., l is 90 in. and $l/r = 125$. The permissible stress is then $1.25(17{,}000 - 60 \times 125) = 11{,}875$ lb. per sq. in. and the required area is $46{,}210/11{,}875 = 3.88$ sq. in. For this an area of 4.47 sq. in. is provided.

In order to ensure ruggedness of the bracing, each bracing member will be made of two angles back to back, separated by the gussets at the connections and at intermediate points by washers with rivets not over 3.5 ft. apart. Members connected in this manner are assumed not to suffer in efficiency, either in tension or compression, through being connected by one leg only.

The most important bracing member, 5–9, is, by reason of its liability to accident or misuse, proportioned more by judgment than by calculation. Two $2\frac{1}{2} \times 2\frac{1}{2} \times \frac{5}{16}$ in. angles will be used. These give a slenderness ratio of about $89/0.76 = 117$, or well within the limit of 175 permitted for bracing likely to be subjected to compression. The area is much in excess of that required for the small stress in the member.

In order to keep the slenderness ratio of the diagonal 5–8 within 175 it is necessary to use two angles $2\frac{1}{2} \times 2 \times \frac{1}{4}$ in., with the short legs turned out as shown in Fig. 94. Again, the stress is not the determining factor in design.

All the other bracing members may be made of two $2 \times 2 \times \frac{1}{4}$-in. angles. For the connected legs of these angles it would be necessary to limit the rivets to not over $\frac{5}{8}$ in.

139. Stepped Side Column of a Mill Building.—The stepped side columns of the steel mill building for which a bent is analyzed for wind stresses in Art. 134 not only resist dead, snow and wind load, but also carry the runway girders for a 25-ton travelling crane of 40-ft. span, centre to centre of rails, with 25% impact for vertical effects. Design a column of the type shown in Fig. 95 conforming to the specification of the A.I.S.C. so far as basic permissible stresses are concerned.

Dimensions.—Height of column, 29 ft.; distance of top of rail above column base, 18 ft.; depth of knee brace, 4 ft.; spacing of bents, 16 ft.

Axial Dead and Snow Loads.—Dead load and the reduced snow load deemed appropriate in conjunction with the prescribed wind load are shown in Fig. 95.

Axial Wind Loads.—Wind effects in the column will be assumed as those which arise in it when acting as part of the bent of Art. 134, which is subjected to a combination of wind pressure and suction. As has been pointed out in Art. 134, the apparent axial wind stresses in the columns indicated in the stress sheet, Fig. 89, will not be the true stresses. In Fig. 96(*a*) are shown the stresses in the columns and the auxiliary frames

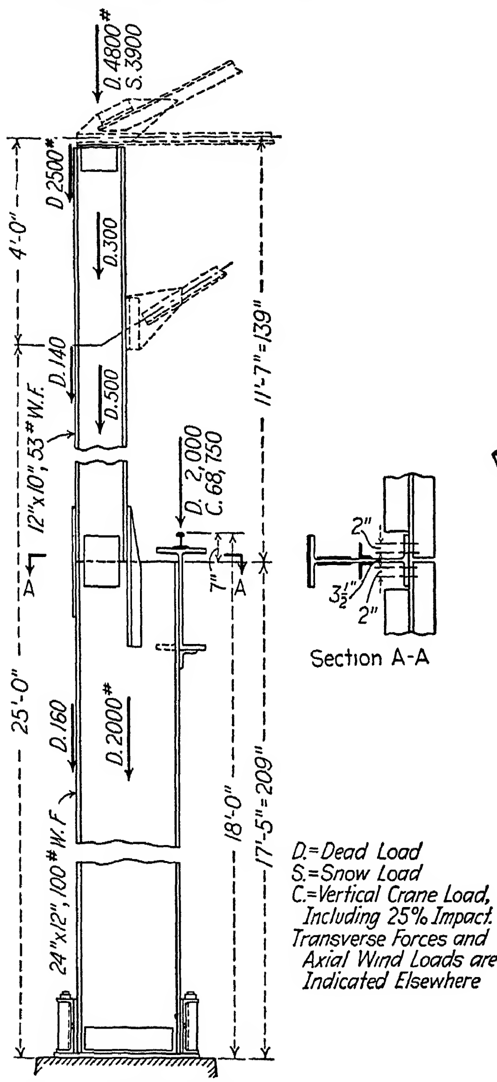


FIG. 95. Side Column of Mill Building Subjected to Dead, Snow, Wind and Crane Loads.

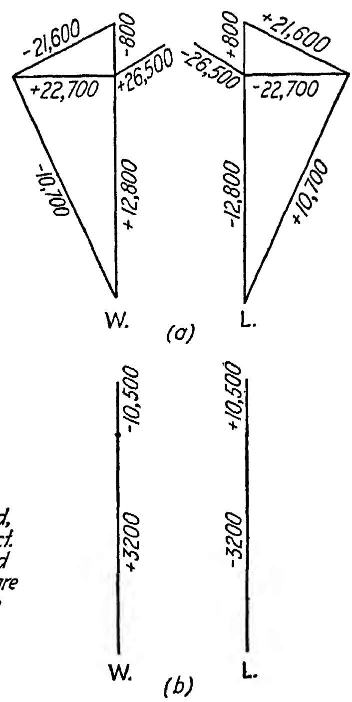


FIG. 96. Apparent and Actual Wind Stresses in Side Columns.

taken from the stress sheet. In Fig. 96(*b*) the true column stresses are shown, these being the algebraic sum of the vertical components of apparent stress in the column and in the inclined auxiliary member at the horizontal section under consideration. Obviously such vertical component as the auxiliary member

appears to carry must in reality be taken by the column as an axial load. The net axial wind loads are assumed to be applied centrically to the upper (narrow) segment of the column.

Axial Crane Load.—For the specified crane, the maximum loads on one rail are two 40,000-lb. loads moving at a fixed distance of 10 ft. apart. (See Ketchum's "Steel Mill Buildings" for a table of typical electric travelling cranes.) Considering 25% impact, these become 50,000 lb. each. The maximum column concentration is

$$C = 50{,}000(1 + \tfrac{6}{16}) = 68{,}750 \text{ lb.}$$

considered as applied in the plane of the inner face of the lower (wide) segment of the column. This load should be considered as having a possible eccentricity, due to the nature of the connection, amounting to 3 in. measured normal to the column web.

Dead Load Moments.—Consider as positive that sense of moments that produces for any level considered the same kind of bending as is indicated for the same level in the wind flexure diagram of Fig. 97.

Assuming the out-to-out width of the upper (narrow) segment of the column in the plane of bending as 12 in., the dead load moment at the level of the knee brace connection, which (see Art. 134) is the critical level in the upper segment of the column, is that due merely to the 2500-lb. load from the eave strut. For the windward column it is, for the level considered,

$$M_w = + 2500 \times 6 = + 15{,}000 \text{ in.-lb.}$$

For the leeward column it is

$$M_l = - 2500 \times 6 = - 15{,}000 \text{ in.-lb.}$$

Assuming the neutral axis of the lower (wide) segment of the column as at its mid-width, the eccentric dead load forces operating at the base of it, which is the critical level for this segment, are the 2500-lb. eave strut load, the 300-lb. girt load, the 4800-lb. dead load reaction from the roof truss, the 800-lb. weight of the upper segment of the column and the 2000-lb. dead load reaction of the crane runway. Adopting the sign convention stated above, and assuming the lower segment of the column Fig. 95 to be 24 in. wide, the dead load moment on the lower segment at the column base is for the windward column

$$M_w = - 2800 \times 12.0 - 5600 \times 6.0 + 2000 \times 12 = - 43{,}200 \text{ in.-lb.}$$

and for the leeward column

$$M_l = + 2800 \times 12.0 + 5600 \times 6.0 - 2000 \times 12 = + 43{,}200 \text{ in.-lb.}$$

Snow-Load Moments.—The snow load acts centrically on the upper segment of the column, and there is no moment therein from this cause.

On the lower segment of the column, the moment due to snow load is -3900

$\times$ 6 = − 23,400 in.-lb. for the windward column and + 23,400 in.-lb. for the leeward column.

Wind-Load Moments.—The wind-load moment at any level in the column arises chiefly from the transverse shear at the plane of contra-flexure acting on an arm equal to the distance between the latter and the level in question.

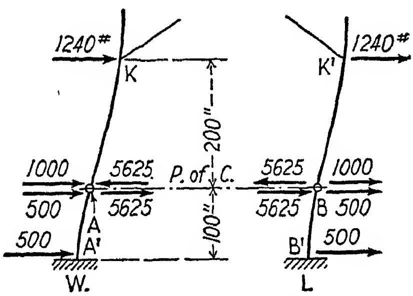


FIG. 97.—Moments in Columns Due to Transverse Action of Wind load.

In Fig. 97 are assembled from Fig. 89 the transverse wind forces applied to both the windward (W) and the leeward (L) column in accordance with the pressure and suction action. At the plane of contra-flexure (P. of C.) there are also shown the shears across each column. Evidently the maximum moments arise at the knee brace connections K or K' and at the actual bases A' or B' of the column. These moments are as given in Table 12.

Added to the above moments at the column bases, there are the moments due to the eccentric application of the axial wind force. This force, for both windward and leeward columns, is indicated in Fig. 96(*b*). The moments at the column bases about the column axis, assuming, as for dead and snow load, that at the column bases clockwise moments are positive, are as given in Table 13.

TABLE 12

MOMENTS IN COLUMNS DUE TO TRANSVERSE ACTION OF FULL WIND LOAD

Column	Point	Moment, In.-Lb.
Windward	K	(5625 − 1000) $\times$ 200 = 925,000
Windward	A'	(5625 + 500) $\times$ 100 = 612,500
Leeward	K'	(5625 − 1000) $\times$ 200 = 925,000
Leeward	B'	(5625 + 500) $\times$ 100 = 612,500

TABLE 13

MOMENTS IN LOWER SECTION OF COLUMNS DUE TO ECCENTRIC APPLICATION OF AXIAL WIND LOAD

Column	Point	Moment, In.-Lb.
Windward	A'	+ 3200 $\times$ 6.0 = + 19,200
Leeward	B'	+ 3200 $\times$ 6.0 = + 19,200

The total wind moments due to both causes are as set forth in Table 14.

TABLE 14

TOTAL WIND MOMENTS AT CRITICAL CROSS SECTIONS OF COLUMNS

Column	Point	Total Wind Moment, In.-Lb.
Windward..........	K	925,000
Windward..........	A'	631,700
Leeward............	K'	925,000
Leeward............	B'	631,700

Crane Load Moments.—The vertical load from the crane, including the impact increment, will produce a moment in the lower (wide) segment of the column of $\pm$ 68,750 $\times$ 12.0 $= \pm$ 825,000 in.-lb. in the plane of the column web.

In addition, there will be a moment in the plane of the column web due to the end thrust of the crane. The total end-thrust effect will be considered as 20% of the lifting capacity of the crane, or 0.20 $\times$ 50,000 = 10,000 lb. If this be applied to the supporting runway equally at the four wheels, the force exerted by each will be 10,000/4 = 2500 lb. The maximum horizontal concentration at a column is then seen from Fig. 98(b) to be 2500(1 + $\frac{6}{16}$) = 3440 lb. This force is applied to the top of the runway rail.

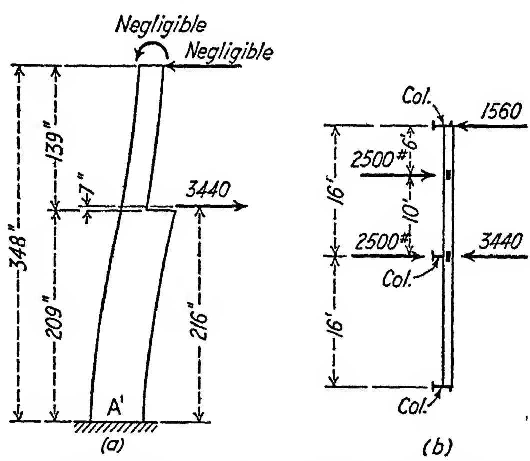


FIG. 98.—Moments in Columns Due to End Thrust of Crane.

Studies of crane-thrust moments in the side columns of mill buildings by the Cross method of moment distribution (*Transactions*, American Society of Civil Engineers, Vol. 96 (1932), p. 48) have shown that under this type of force the two columns of a bent act practically as independent vertical cantilevers with a very small moment at and above the line of application of the thrust.

The moment at the knee brace connections will consequently be neglected, but, as indicated in Fig. 98(a), provision will need to be made at the base A' for a moment of $\pm$ 3440 $\times$ 216 $= \pm$ 743,000 in.-lb. The signs are to be interpreted as under " Dead Load Moments " above.

At the same time, there may be a moment normal to the plane of the column

web arising from the application of live load to the column wholly from one runway girder. Referring to the column detail, Fig. 95, it is seen that the eccentricity may be as much as $2\frac{3}{4}$ in. On this basis the moment, including impact effect, is $68{,}750 \times 2.75 = 189{,}100$ in.-lb.

Proportioning of Column above Runway Girder Seat.—Experience shows that, in general, the most critical section of a member subjected to a series of axial loads and to a comparatively large bending moment is at the point of maximum moment. This, for the upper segment of the column, is at the knee brace connection. Of the two columns the windward one is the more highly stressed.

Having regard to the probability of the coincidence of the various loads mentioned above, and taking advantage of the principle of increasing working stresses for unlikely combinations, provision will be made for the following combinations at the working stresses indicated below, p being the normal permissible stress. As most of the sectional area is required to resist one kind of load, namely, wind, the increase is justifiably kept down to 10%.

Combination	Permissible Stress
(1) Dead load + wind load	p
(2) Dead load + snow load + wind load + crane load	$p + 10\%$

Collecting from above the maximum forces and moments existing at the knee brace connection of the upper segment of the windward column, we have for the two loading combinations, the following:

Combination (1):

Anxial load:

Dead load	7,600 lb.
Wind load	10,500 lb.
Total	18,100 lb.

Moment:

Dead load	15,000 in.-lb.
Wind load	925,000 in.-lb.
Total	940,000 in.-lb.

Combination (2):

Axial load:

Dead load	7,600 lb.
Snow load	3,900 lb.
Wind load	10,500 lb.
Total	22,000 lb.

Moment:

Dead load	15,000 in.-lb.
Wind load	925,000 in.-lb.
Crane thrust (negligible)	
Total	940,000 in.-lb.

To resist these forces and moments, a section consisting of a 12 × 10-in., 53-lb. W.F. will be assumed. The area of this section is 15.59 sq. in. Its radius of gyration in a direction parallel to the web is $r_1 = 5.23$ in., and in a direction normal to the web it is $r_2 = 2.48$ in.

In view of the fact that the girts afford a somewhat uncertain support to the column in a direction normal to the web, the effective length will be assumed as the height between the step and the top, or 139 in. See Fig. 98(*a*).

The maximum slenderness ratio is then $139/2.48 = 56$, and the permissible compressive stress applicable to Combination (1) is then, according to the Specification,

$$p_c = p = 15{,}000 \text{ lb. per sq. in.}$$

and the permissible stress applicable to Combination (2) is

$$p_c' = p + 10\% = 15{,}000 + 1500 = 16{,}500 \text{ lb. per sq. in.}$$

The required areas are, applying Eq. (4) of Art. 36, Combination (1):

$$A_c + A_f = \frac{18{,}100}{15{,}000} + \frac{940{,}000 \times 6.03}{(5.23)^2 \times 15{,}000}$$
$$= 1.21 + 13.82 = 15.03 \text{ sq. in.}$$

Combination (2):

$$A_c + A_f = \frac{22{,}000}{16{,}500} + \frac{940{,}000 \times 6.03}{(5.23)^2 \times 16{,}500}$$
$$= 1.33 + 12.57 = 13.90 \text{ sq. in.}$$

The area of the assumed section, 15.59 sq. in., is ample for either combination.

Proportioning of Column Below Runway Girder Seat.—Consideration of the forces and moments acting on the lower segment of the column leads to the conclusion that for Combination (1) the most critical section is at the base of the leeward column, whereas for Combination (2), the critical section is at the base of the windward column.

In Combination (2) the effect of the tractive force on the crane runway has been neglected, because of the unlikelihood of its concurrence with the other effects that have been considered.

For the lower segment of the column the same permissible stress increase for Combination (2) will be adopted as was employed for the upper segment.

The forces and moments for the two adopted loading combinations are, from what has preceded, as follows:

Combination (1):

Axial load:

Dead load	12,400 lb.
Wind load	3,200 lb.
Total	15,600 lb.

Moment:

Dead load	43,200 in.-lb.
Wind load	631,700 in.-lb.
Total	674,900 in.-lb.

Combination (2):

Axial load:

Dead load	−	12,400 lb.
Snow load	−	3,900 lb.
Wind load	+	3,200 lb.
Crane load	−	68,750 lb.
Total	−	81,850 lb.

Moment, in plane of web:

Dead load	−	43,200 in.-lb.
Snow load	−	23,400 in.-lb.
Wind load	+	631,700 in.-lb.
Crane, weight effect	+	825,000 in.-lb.
Crane, thrust effect	+	743,000 in.-lb.
Total	+	2,133,100 in.-lb.

Moment, normal to plane of web:

Crane weight effect	189,100 in.-lb.

The section assumed for the lower segment of the column will consist of a 24×12-in., 100-lb. W.F., having an area of 29.43 sq. in., a section modulus of 248.9 parallel to the web and one of 33.9 perpendicular thereto.

Since without horizontal stiffeners on the web the effective portion of the web between the fillets should be limited to 40 times the web thickness, the effective area of the column for axial (not flexural) stresses will be $29.43 - (2.16 \times 0.468) = 28.42$ sq. in.

At the critical section, that is at the column base, the column is restrained against lateral buckling, and the permissible stress for loading Combination (1) is consequently $p_c = 18{,}000$ lb. per sq. in.; for Combination (2) it is $p_c' = 18{,}000 + 10\% = 19{,}800$ lb. per sq. in.

The maximum existing stresses for the two combinations are Combination (1):

$$f_c + f_f = \frac{15{,}600}{28.42} + \frac{674{,}900}{248.9}$$
$$= 5480 + 2620 = 8100 \text{ lb. per sq. in.}$$

as compared with 18,000 lb. per sq. in. permitted.

Combination (2):

$$f_c + f_{f1} + f_{f2} = \frac{81{,}850}{28.42} + \frac{2{,}133{,}100}{248.9} + \frac{189{,}100}{33.9}$$
$$= 2880 + 8570 + 5580 = 17{,}030 \text{ lb. per sq. in.}$$

The permissible stress is 19,800 lb. per sq. in., and consequently the section affords considerable margin.

A section 50 in. above the base will be investigated. Here the axial load is practically 81,850 lb., as before, but the wind moment is reduced to 325,500 in.-lb. and the crane-thrust moment to 571,500 in.-lb., giving a total moment of 1,655,400 in.-lb. parallel to the web. Normal to the plane of the web the moment is as before.

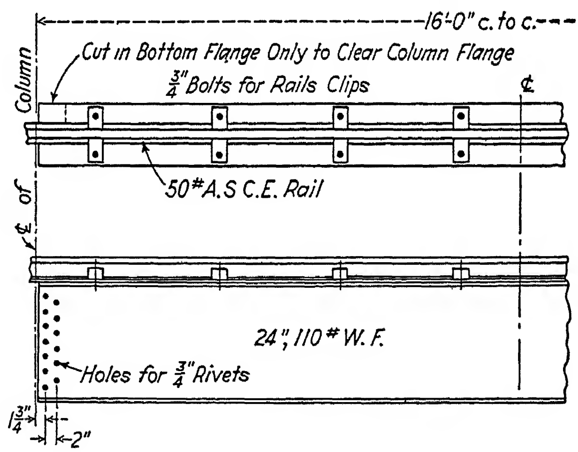


FIG. 99. Crane Runway Girder.

Calculation shows that the maximum extreme fibre stress is 15,120 lb. per sq. in. If the basic permissible stress at 50 in. up from the base be midway between 18,000 lb. per sq. in. and that at mid-height of the wide segment of the column, or 13,320 lb. per sq. in., the increased permissible stress for the load combination considered is $13{,}320 + \frac{1}{2}(18{,}000 - 13{,}320) + 10\% = 17{,}230$ lb. per sq. in. Although there is still a fairly large excess strength here, the next lighter section would not be sufficient. Moreover, the flange width would be too small to afford an attachment of the runway girder to it by the riveting arrangement indicated in Fig. 99.

140. Design of a Crane Runway Girder.—Design the crane runway girder supported by the mill building side column of Art. 139. The span is 16 ft., centre to centre of columns, and the live load is that due to a 25-ton travelling crane of 40-ft. span, centre to centre of rails, with 25% impact for vertical effects. Permissible stresses according to the A.I.S.C. Specification. Rivets, $\frac{3}{4}$ in.

Dead Load.—Assuming a 24-in., 110-lb. W.F. section, the total dead load per lineal foot, including a 50-lb. running rail with clips and bolts, will be about 135 lb.

Live Load.—The vertical loading consists of two 50,000-lb. loads (including 25% impact) moving at a fixed distance of 10 ft. apart, while the horizontal loading consists of two 2500-lb. loads moving similarly.

Maximum Moments and Section for Flexure.—The maximum vertical moment, M_x, which will occur when one of the 50,000-lb. loads is at mid-span, Eq. (6), Art. 141, is as follows:

$$\begin{aligned} \text{Dead load} &= \tfrac{1}{8} \times 135 \times (16)^2 &&= 4{,}320 \text{ ft.-lb.} \\ \text{Live load} &= 50{,}000 \times \tfrac{16}{4} &&= \underline{200{,}000 \text{ ft.-lb.}} \\ \text{Total} & && 204{,}320 \text{ ft.-lb.} \\ & && = 2{,}451{,}840 \text{ in.-lb.} \end{aligned}$$

The maximum horizontal moment, M_y, will occur under the same loading arrangement as the maximum vertical moment, and will therefore be 2,500/50,000 of the vertical moment due to live load, or $200{,}000 \times 2{,}500/50{,}000 = 10{,}000$ ft.-lb. $= 120{,}000$ in.-lb.

In view of the fact that two holes for $\frac{3}{4}$-in. bolts opposite each other may exist in the top flange fairly close to mid-span, a reduction in section modulus will be made to compensate for the loss of compressive value at the holes.

The effective vertical section modulus will be computed on the assumption that the vertical axis does not shift because of perforation, and the approximate result so found will be corrected by application of Eq. (1) of Art. 81, thus:

$$\begin{aligned} &\text{Gross } I \text{ about horizontal gravity axis} && = 3315.0 \text{ in.}^4 \\ &I \text{ of 2 holes about mid-depth of web} && \\ &\quad = 2(0.855 \times 0.875 \times 11.65^2) && = \underline{203.0 \text{ in.}^4} \\ &\text{Net uncorrected } I && = 3112.0 \text{ in.}^4 \\ &\text{Net corrected section modulus} && \\ &\quad = \left\{1.0 - \frac{0.25}{(24.16)^{1/2}}\right\} \times \frac{3112}{12.08} = S_x && = 244.5 \text{ in.}^3 \end{aligned}$$

Since the bottom flange is able to resist only a small part of the horizontal moment, it will be assumed that only the portion of the girder section above mid-depth of the beam is effective for lateral loading. The section modulus of this upper half, taken about its vertical gravity axis, and making allowance for the two bolt holes, is as follows

$$\begin{aligned} &\text{Gross } I \text{ about vertical gravity axis} = \frac{229.1}{2} && = 114.5 \text{ in.}^4 \\ &I \text{ of 2 holes} = 2(0.855 \times 0.875 \times 2.75^2) && = \underline{11.3 \text{ in.}^4} \\ &\text{Net } I && = 103.2 \text{ in.}^4 \\ &\text{Net section modulus} = \frac{103.2}{6.02} = S_y && = 17.1 \text{ in.}^3 \end{aligned}$$

The maximum stress due to the combination of vertical and horizontal moments is, from Art. 95

$$
\begin{aligned}
f_x + f_y &= \frac{M_x}{S_x} + \frac{M_y}{S_y} \\
&= \frac{2,451,840}{244.5} + \frac{120,000}{17.1} \\
&= 10,020 + 7020 = 17,040 \text{ lb. per sq. in.}
\end{aligned}
$$

Since the ratio of laterally unsupported length to flange breadth is $16 \times 12/12.04 = 15.95$, the permissible compressive stress on the top flange allowed by the A.I.S.C. Specification is 17,750 lb. per sq. in.

Maximum Shear and Web Section.—The maximum shear in the girder will be

$$
\begin{aligned}
&\text{Dead load} = \tfrac{1}{2} \times 16 \times 135 && = 1,080 \text{ lb.} \\
&\text{Live load} = 50,000 \times (1 + \tfrac{6}{16}) && = \underline{68,750 \text{ lb.}} \\
&\qquad \text{Total} && = 69,830 \text{ lb.}
\end{aligned}
$$

As the ratio of clear depth between flanges to web thickness is $h/t = 22.45/0.51 = 44$, or less than 60, the permissible shearing stress on the gross area of web is 12,000 lb. per sq. in. The required area is, therefore, $69,830/12,000 = 5.82$ sq. in.

For a 24-in., 110-lb. W.F. the web area provided is $24.16 \times 0.51 = 12.31$ sq. in.

Details.—Assuming that the inside of each flange is cut away to enable the web of the girder to be directly riveted to the column flange, as in Fig. 99, enough rivets will be provided in single shear to carry the maximum reaction, 69,830 lb.

The safe single shear value of power-driven, $\frac{3}{4}$-in. field rivets being $0.442 \times 13,500 = 5960$ lb., the number of rivets required is $69,830/5960 = 12$. These may be conveniently provided as indicated in Fig. 99.

A shelf angle is provided on the column flange for convenience in erection, but is not considered as a part of the permanent connection.

The 50-lb. running rail is secured to the top flange of the girder by cast-iron clips attached to the girder flange by $\frac{3}{4}$-in. bolts.

CHAPTER X

MOVING LOADS ON BEAMS, GIRDERS AND TRUSSES

141. Fundamental Criteria for Maximum Moments, Shears and Floor-Beam Concentrations.—The following formulae serve to determine the critical positions of moving loads on beams, girders and trusses in order to produce maximum values of certain live load functions involved in design. (References: Johnson, Bryan and Turneaure—Modern Framed Structures, Pt. I; Marburg—Framed Structures and Girders, Vol. I; Shedd and Vawter—Theory of Simple Structures.)

For maximum moment at any selected point, C, of a beam or girder not receiving its load through floor beams; or at any point vertically opposite a floor-beam connection of a girder or truss

$$\frac{G_1}{l_1} = \frac{G}{l} \tag{1}$$

For maximum moment at any point vertically opposite a floor-beam connection of a girder or truss having equal panels

$$\frac{G_1}{m} = \frac{G}{n} \tag{2}$$

For maximum moment at a point *not* vertically opposite a floor-beam connection of a girder or truss

$$\frac{G_1 + \frac{k}{p} \cdot G_2}{l_1} = \frac{G}{l} \tag{3}$$

Absolute maximum, or maximum attainable, moment in a beam or girder not receiving its load from floor beams will be at a load P, so placed that

$$e_p = e_g \tag{4}$$

Absolute maximum, or maximum attainable, moment in a beam or girder not receiving its load from floor beams, for two equal loads, P, spaced at a fixed distance a apart, where the span is equal to or greater than 1.7065 a, will be

$$M = \frac{2\,P}{l}\left(\frac{l}{2} - \frac{a}{4}\right)^2 \tag{5}$$

Absolute maximum, or maximum attainable, moment for the above beam or girder where the span is less than 1.7065 a is

$$M = \frac{Pl}{4} \tag{6}$$

Maximum positive shear at any selected point, C, of a beam or girder without floor beams occurs when

$$R_L - \Sigma_A^C P = \text{a maximum} \tag{7}$$

For maximum shear in any panel of a truss or girder with floor beams,

$$\frac{G_2}{p} = \frac{G}{l} \tag{8}$$

For maximum shear in any panel of a truss or girder with floor beams, where all panels are equal,

$$G_2 = \frac{G}{n} \tag{9}$$

For the maximum shear in any panel of a truss or girder with floor beams, due to a uniformly distributed load, the loading extends from the right support to a point lying to the left of the right-hand extremity of the panel in question a distance

$$x = \frac{n - m}{n - 1} \cdot p \tag{10}$$

Exact maximum shear in the panel for the case to which Eq. (10) applies

$$V = \frac{wp}{2(n - 1)} (n - m)^2 \tag{11}$$

Approximate, or conventional, shear in any panel of a truss or girder with floor beams, due to a uniformly distributed load

$$V = \frac{wp}{2n} (n - m)(n - m + 1) \tag{12}$$

For a maximum stress in a diagonal of a curved-chord Pratt truss (Fig. 109), there being no loads to the left of the panel in which the diagonal occurs,

$$\frac{G_2}{p} = \frac{s}{t} \cdot \frac{G}{l} \tag{13}$$

For the case to which Eq. (13) applies, where all panels are equal

$$G_2 = \frac{s}{t} \cdot \frac{G}{n} \tag{14}$$

For the maximum stress in a vertical of a curved-chord Pratt truss (Fig. 109), there being no loads to the left of the panel which lies immediately to the left of the vertical in question, and all panels being equal,

$$G_2 = \frac{s'}{t'} \cdot \frac{G}{n} \tag{15}$$

For a maximum concentration on a floor beam or supporting column

$$\frac{G_1}{p_1} = \frac{G_2}{p_2} \tag{16}$$

For a maximum concentration on a floor beam or supporting column, where adjacent panels are equal

$$G_1 = G_2 \tag{17}$$

In the preceding formulae the symbols have the following significance:

a = fixed distance between two consecutive concentrated loads;

e_g = distance from the centre of gravity of all the loads on a beam to the mid-point of the span;

e_p = distance from the point of absolute maximum moment in a beam to the mid-point of the span;

G = resultant of all the loads on a span, acting at their centre of gravity;

G_1 = in determinations of the condition for maximum moment at any selected point in a beam, or at any floor-beam connection to a girder, or at any panel point of the loaded chord of a truss, the resultant of all loads to the left of the point considered, Eqs. (1) and (2); in determinations of the condition for maximum moment at any point wholly within a panel of a girder or within a panel of the loaded chord of a truss, or at a panel point of the unloaded chord of a Warren truss, resultant of all loads wholly to the left of the panel in which, or opposite which, the moment centre lies, Eq. (3); in determinations of the condition for maximum floor-beam or column concentration, the resultant of all loads in the panel immediately to the left of the floor beam or column considered, Eqs. (16) and (17);

G_2 = total amount of load in any selected panel of a truss or girder with floor beams, Eqs. (3), (8), (9), (13), (14) and (15); total load in the panel immediately to the right of any selected floor beam or column, Eqs. (16) and (17);

k = distance from the particular cross section of a truss or girder with floor beams at which the moment is desired to the nearest panel point to the left, Eq. (3);

l = effective span length of beam, girder or truss;

l_1 = distance from the left support to the cross section of a beam, girder or truss where the moment is desired;

M = bending moment;

m = in determinations of the condition for maximum moment at any floor-

beam connection of a girder, or at any panel point of the loaded chord of a truss, the number of panels from the left support to this point, Eq. (2); in determinations of the condition for maximum shear in a panel when the loading is uniform, the number of panels from the left support to the *right-hand limit* of the panel in which the shear is desired, Eqs. (10), (11) and (12);

n = total number of panels in a span;

P = any concentrated load;

p = length of a panel of a truss or of a girder with floor beams;

p_1 = length of floor panel to the left of any selected floor beam, or girder span to the left of any selected column, Eq. (16);

p_2 = length of floor panel to right of any selected floor beam, or girder span to the right of any selected column, Eq. (16);

R_L = left-hand reaction of a beam, girder or truss;

s = distance from the *left* support of a truss to the point where the prolongation of the sloping chord in any selected panel cuts the prolongation of the line joining the supports, Eqs. (13) and (14) and Fig. 109;

s' = distance from the *left* support of a truss to the point where the prolongation of the sloping chord immediately to the left of any selected vertical cuts the prolongation of the line joining the supports, Eq. (15) and Fig. 109;

$\Sigma_A^C P$ = sum of all applied loads between the left support, A, of a beam and any selected point C;

t = distance from the point where the prolongation of the sloping chord in any selected panel cuts the prolongation of the line joining the supports to the left-hand extremity of the panel, Eqs. (13), (14) and Fig. 109;

t' = distance from the point where the prolongation of the sloping chord immediately to the *left* of any selected vertical cuts the prolongation of the line joining the supports to the vertical under consideration, Eq. (15), Fig. 109;

w = uniformly distributed load per unit of length;

x = distance which a moving uniform load approaching from the right extends past the right-hand extremity of a panel in order that the shear may be a maximum in the panel.

142. Maximum Moment at Selected Point of a Beam.—What wheel of the loading of Fig. 100(a) will produce the maximum moment at a point 6 ft. from one end of a 15-lb. beam, and what will this moment be?

Move the loading from right to left across the span, trying successive loads at the point in question, that is, C. In general, the criterion of Eq. (1), Art. 141, applicable in this case, can be satisfied only by moving a concentrated load across the point.

Move load 1 across the point C, as indicated in Fig. 100(b). When the load is wholly to the right of C, $G_1/l_1 = 0/6 = 0$, and when it is wholly to the left of C, $G_1/l_1 = 10/6 = 1\frac{2}{3}$.

During the movement the value of this quantity must have changed gradually from 0 to $1\frac{2}{3}$, and as it did not at any time attain the value of $G/l = 30/15 = 2$, the condition for a maximum was not produced.

Move load 2 across the point C, as indicated in Fig. 100(c). The left-hand member, G_1/l_1, of Eq. (1), Art. 141, then ranges in value from 0/6 to 20/6, that is from 0 to $3\frac{1}{3}$. As, meanwhile, $G/l = 35/15 = 2\frac{1}{3}$, the condition for a maximum is realized. This will be indicated as Maximum 1.

Obviously moving the loading to the left, so that load 3 is at C, will produce a less serious condition than that depicted in Fig. 100(c), since the load at C would be lighter and the other load on the span would be on the short, rather than on the long, segment. Testing by Eq. (1), G_1/l_1 ranges from $3\frac{1}{3}$ to $5\frac{5}{6}$ while $G/l = 2\frac{1}{3}$, and hence no maximum arises.

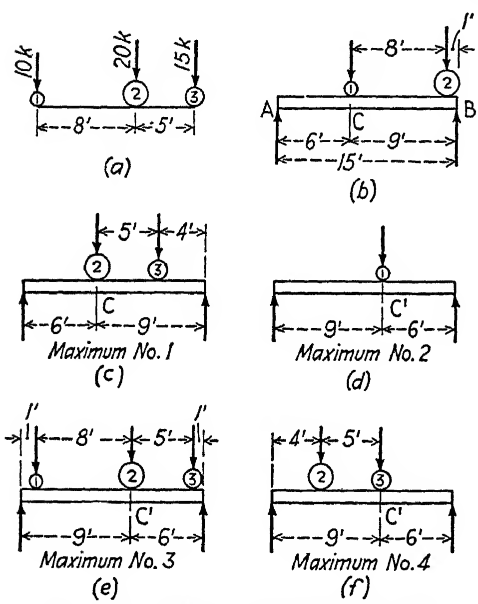


FIG. 100. Maximum Moment at Selected Point in a Beam.

Conceivably a worse situation might arise through moving the load across the span from left to right with load 1 leading, or, what is the same thing, considering the situation at a point C', 6 ft. from the *right*-hand end of the span, with the movement from right to left, as assumed for the point C.

Placing load 1 at C', as shown in Fig. 100(d), will give values of G_1/l_1 ranging from 0 to $10/9 = 1\frac{1}{9}$, while $G/l = 10/15 = \frac{2}{3}$. A condition for a local maximum, therefore, exists. Call this Maximum 2.

With load 2 at C', as shown in Fig. 100(e), G_1/l_1 rises from $10/9 = 1\frac{1}{9}$ to $30/9 = 3\frac{1}{3}$, while $G/l = 45/15 = 3$. This gives Maximum 3.

With load 3 at C', as shown in Fig. 100(f), G_1/l_1 ranges from $20/9 = 2\frac{2}{9}$ to $35/9 = 3\frac{8}{9}$, while $G/l = 35/15 = 2\frac{1}{3}$. This gives Maximum 4.

Inspection of Fig. 100 indicates that Maximum 2 is less serious than any of the others. The moment calculations for the other three maxima are as follows:

Moment for Maximum 1.—Left-hand reaction is

$$R_L = \frac{(20 \times 9) + (15 \times 4)}{15} = 16 \text{ kips.}$$

Moment at C is

$$M = 16 \times 6 = 96 \text{ kip-ft.}$$

Moment for Maximum 3.—Left-hand reaction and moment are respectively

$$R_L = \frac{(10 \times 14) + (20 \times 6) + (15 \times 1)}{15} = 18\tfrac{1}{3} \text{ kips}$$

and

$$M = (18\tfrac{1}{3} \times 9) - (10 \times 8) = 85 \text{ kip-ft.}$$

Moment for Maximum 4.—Left-hand reaction and moment are respectively

$$R_L = \frac{(20 \times 11) + (15 \times 6)}{15} = 20\tfrac{2}{3} \text{ kips}$$

and

$$M = (20\tfrac{2}{3} \times 9) - (20 \times 5) = 86 \text{ kip-ft.}$$

Greatest Maximum Moment.—Comparison of the above results indicate that the greatest possible moment at a point 6 ft. from one end of the 15-ft. beam occurs with only two loads on the span, load 2 being at the point in question and load 3 being on the longer segment, as shown in Fig. 100(*c*). For this the moment is 96 kip-ft.

143. Maximum Attainable Moment in a Beam.—Determine the position of the point of absolute maximum (or maximum attainable) moment in a beam of 20-ft. span due to the passage of the loading indicated in Fig. 101(*a*), and calculate the resulting moment.

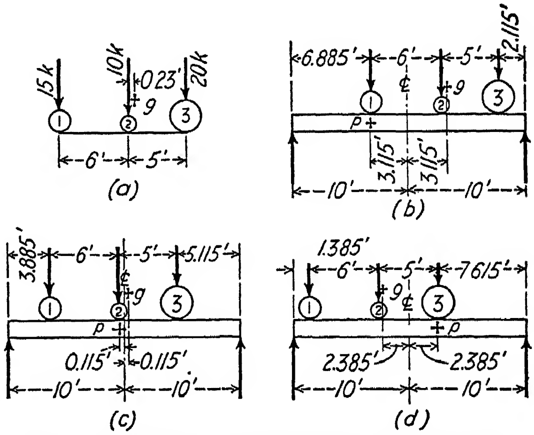


FIG. 101. Maximum Attainable Moment in a Beam.

The criterion of Eq. (4), Art. 141, may generally be satisfied by several different arrangements. In the present case it may be met by three different placings of the loading. In order to ascertain which one gives the greatest moment it will be necessary to calculate the three moments and compare them.

In conformity with the usual convention, consider the loads as moving over the span from right to left.

Load 1 as Critical Load.—The centre of gravity of the loading being 0.23 ft. from load 2 and 4.77 ft. from load 3, Eq. (4) of Art. 141 may be satisfied by placing load 1 at a point p 3.115 ft. to the left of the centre of the beam, the centre of gravity, g, of the loading on the span being 3.115 ft. to the right of mid-span, as shown in Fig. 101(*b*).

For this, the left-hand reaction is 15.51 kips, and the moment at p is

$$M = 15.51 \times 6.885 = 107.0 \text{ kip-ft.}$$

Load 2 as Critical Load.—Eq. (4), Art. 141, is satisfied by placing load 2 a distance of 0.115 ft. to the left of mid-span, the centre of gravity, g, of the loading being 0.115 ft. to the right of mid-span, as shown in Fig. 101(c). For this, the left-hand reaction is 22.26 kips, and the moment at p is

$$M = (22.26 \times 9.885) - (15 \times 6) = 130.0 \text{ kip-ft.}$$

Load 3 as Critical Load.—By moving the loading still farther to the left, the centre of gravity, g, and the load 3 may be so placed as to be equidistant from mid-span, as indicated in Fig. 101(d), this distance being 2.385 ft. The left-hand reaction is then 27.90 kips, and the moment at p is

$$M = (27.90 \times 12.385) - (15 \times 11) - (10 \times 5) = 130.2 \text{ kip-ft.}$$

Absolute Maximum Moment.—Comparison of the above moments indicates that the absolute maximum moment occurs at 2.385 ft. from mid-span, under load 3, and is 130.2 kip-ft.

144. Maximum Attainable Moment in a Beam Carrying Two Equal Loads.—Two loads of 20,000 lb. each, separated by a fixed distance of 8 ft., move over a span (a) of 16 ft., (b) of 10 ft., (c) of 13 ft. What is the loading arrangement for maximum attainable moment in each case, and what is the moment?

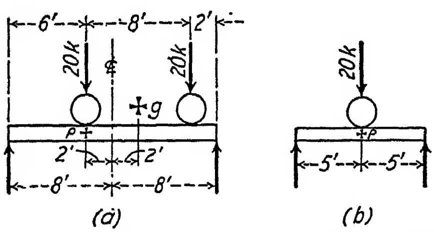


Fig. 102.—Maximum Attainable Moment in a Beam Carrying Two Equal Loads.

Case (a), *16-ft. Span.*—Eq. (4), Art. 141, is satisfied with one 20,000-lb. load placed 2 ft. from mid-span as shown in Fig. 102(a). From Eq. (5), Art. 141, the absolute maximum moment is

$$M = \frac{2 \times 20{,}000}{16}\left(\frac{16}{2} - \frac{8}{4}\right)^2$$
$$= 90{,}000 \text{ ft.-lb.}$$

Case (b), *10-ft. Span.*—Eq. (4), Art. 141, can be satisfied only by placing one of the loads at mid-span, as indicated in Fig. 102(b). The maximum attainable moment is then, from Eq. (6), Art. 141

$$M = \frac{20{,}000 \times 10}{4} = 50{,}000 \text{ ft.-lb.}$$

Case (c), *13-ft. Span.*—For this length of span it is possible to satisfy the criterion of Eq. (4), Art. 141, either by placing one load 2 ft. from mid-span or by placing one load at mid-span. According to Eqs. (5) and (6), Art. 141, a

greater moment is developed by the second arrangement, since the span is less than 1.7065 times the wheel spacing. The maximum moment is

$$M = \frac{20{,}000 \times 13}{4} = 65{,}000 \text{ ft.-lb.}$$

145. Maximum Moment at any Point Vertically Opposite a Floor-Beam Connection of a Truss.—Determine the proper placing of Copper's E60 loading, Fig. 103, on a 175-ft. through Pratt truss span with seven equal panels in order that the maximum moment may arise at the second panel point from the end, that is the point c in Fig. 104. What is the resulting moment for one truss of a single-track bridge?

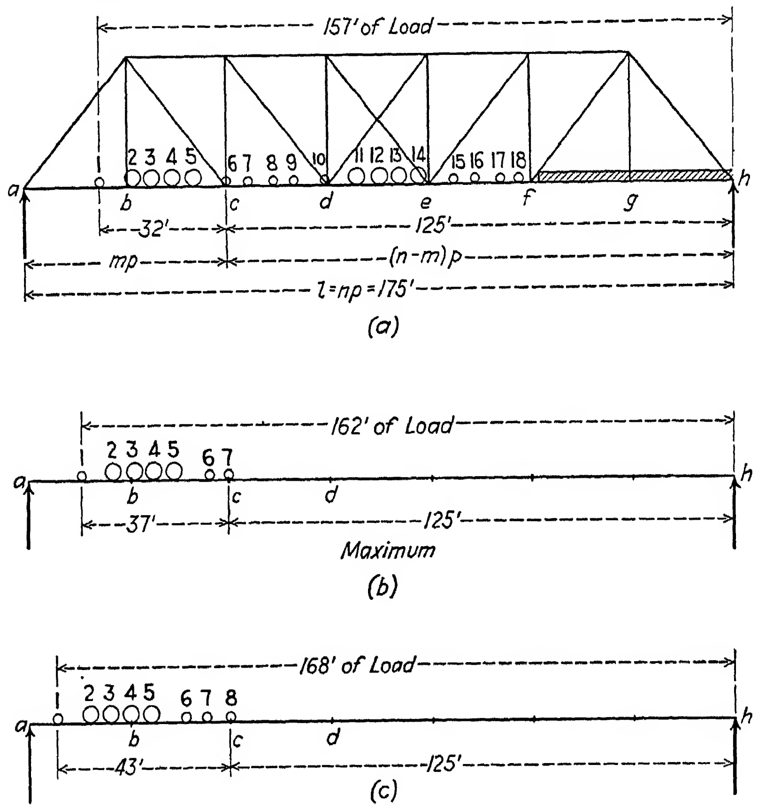


FIG. 104.—Maximum Moment at Any Panel Point of a Pratt Truss.

Experience shows that, for maximum moment at a selected point in a beam or truss: (a) there should be, in general, a relatively large total load on the span; (b) it should be, in so far as is consistent with a large total loading on the span, concentrated near the selected point, and (c) there should be a load at the point.

General conformity to (a) makes it necessary to place the heavy driving

wheels of the first locomotive to the left of the panel point in question, the concentration factor (*b*) being less important in this case than the placing of a heavy total load on the span. It thus happens that wheels 4 or 5 at the panel point *c* will not give appropriate conditions for a maximum moment.

For the problem in hand, try wheel 6 at *c*, as indicated in Fig. 104(*a*). There is then $32 + 125 = 157$ ft. of loading on the span. Consulting the special moment table, Fig. 103, the use of which is obvious, it is seen that there are 18 wheel loads on the span and, in addition, 48 ft. of uniform load amounting to 3 kips per lin. ft. per rail. The total tabular load is then

$$G = 426 + (48 \times 3) = 570 \text{ kips}$$

Proceeding as for the beam problem of Art. 142, and employing the criterion of Eq. (2), Art. 141, which is applicable to the present case, we have, utilizing the moment table for the summation of loads,

$$\frac{G_1}{m} = \frac{135.0}{2} = 67.5 \text{ to } \frac{154.5}{2} = 77.25$$

$$\frac{G}{n} = \frac{570}{7} = 81.43$$

As the movement of wheel 6 across the point *c* did not produce a value of G_1/m equal to the simultaneous value of G/n, the condition for a maximum was not realized.

Try wheel 7 at *c*, as indicated in Fig. 104(*b*). The figures then are

$$\frac{G_1}{m} = \frac{154.5}{2} = 77.25 \text{ to } \frac{174.0}{2} = 87.0$$

$$\frac{G}{n} = \frac{585.0}{7} = 83.57$$

Evidently, at some stage in the movement of wheel 7 over the point *c*, the fraction G_1/m became equal to G/n, and the conditions for a maximum moment were realized.

Try wheel 8 at *c*, as shown in Fig. 104(*c*). The ratios are then

$$\frac{G_1}{m} = \frac{174}{2} = 87 \text{ to } \frac{193.5}{2} = 96.75$$

$$\frac{G}{n} = \frac{603}{7} = 86.14$$

This wheel does not produce a maximum moment.

Wheel 9 at *c* likewise fails to produce a maximum.

The proper arrangement of loading on the span for maximum moment at *c*

is evidently that shown in Fig. 104(*b*), that is with wheel 7 at *c*. Placing the corresponding wheel, 16, of the second locomotive at *c* would be less serious for the truss, as the heavy driving wheels of the first locomotive would be replaced on the span by the comparatively light train load.

The actual moment at the panel point *c* for one truss of a single-track bridge is readily computed for the aid of the moment table, Fig. 103.

Since there is 162 ft. of loading on the span, the reaction at the left support is found by dividing the listed moment about a point 162 ft. from the beginning of the loading by the span length. This reaction is, therefore,

$$R_L = 51{,}337.5/175 = 293.4 \text{ kips}$$

The moment at *c* is the moment of this reaction about *c*, less the moment of all wheel loads to the left of *c*, that is to the left of wheel 7, taken about wheel 7. This latter quantity is read directly from the table. Hence

$$M = 293.4 \times 50 - 3232.5 = 11{,}437.5 \text{ kip-ft.}$$

146. Maximum Moment at a Panel Point of the Unloaded Chord of a Warren Truss.—Determine the maximum moment at the top chord panel point *D* of the 45-ft. pony Warren truss of Fig. 105 due to the passage of two loads of 7.5 and 15.0 kips separated by a fixed distance of 10 ft.

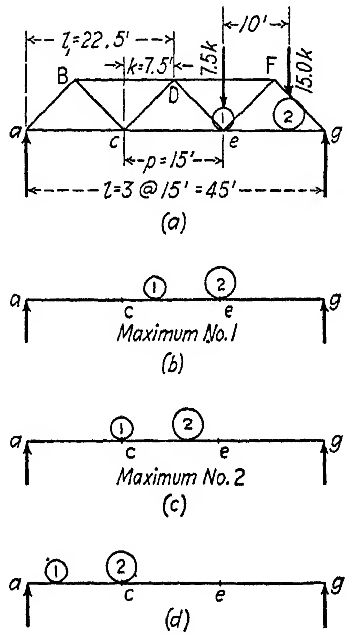


Fig. 105. Maximum Moment at a Panel Point of the Unloaded Chord of a Warren Truss.

For this case, the criterion of Eq. (3), Art. 141, applies. In it $k = 7.5$ ft., $p = 15$ ft., $l_1 = 22.5$ ft. and $l = 45$ ft. Move the load from right to left in the conventional manner with No. 1 leading. For the case in hand, $k/p = \frac{1}{2}$.

Try load 1 at *e*, as indicated in Fig. 105(*a*). Then as 1 passes over the point *e*, the value of the first member of the equation ranges between

$$\frac{G_1 + \dfrac{k}{p} \cdot G_2}{l_1} = \frac{0 + \frac{1}{2} \times 0}{22.5} = 0 \quad \text{and} \quad \frac{0 + \frac{1}{2} \times 7.5}{22.5} = \frac{1}{6}$$

Meanwhile

$$\frac{G}{l} = \frac{22.5}{45} = \frac{1}{2}$$

The condition for a maximum is not realized, as the two members of the equation never become equal for this small movement.

Try load 2 at e, as shown in Fig. 105(b). Then before and after load 2 has passed over the point

$$\frac{G_1 + \frac{k}{p} \cdot G_2}{l_1} = \frac{0 + \frac{1}{2} \times 7.5}{22.5} = \tfrac{1}{6} \quad \text{and} \quad \frac{0 + \frac{1}{2} \times 22.5}{22.5} = \tfrac{1}{2}$$

$$\frac{G}{l} = \frac{22.5}{45} = \tfrac{1}{2}$$

This gives Maximum 1.

Try load 1 at c, as shown in Fig. 105(c). As load 1 passes over c, the ratios have the following range:

$$\frac{G_1 + \frac{k}{p} \cdot G_2}{l_1} = \frac{0 + \frac{1}{2} \times 22.5}{22.5} = \tfrac{1}{2} \quad \text{to} \quad \frac{7.5 + \frac{1}{2} \times 15}{22.5} = \tfrac{2}{3}$$

$$\frac{G}{l} = \frac{22.5}{45} = \tfrac{1}{2}$$

This gives Maximum 2.

Try load 2 at c, as shown in Fig. 105(d). As load 2 passes over c, the ratios range from

$$\frac{G_1 + \frac{k}{p} \cdot G_2}{l_1} = \frac{7.5 + \frac{1}{2} \times 15}{22.5} = \tfrac{2}{3} \quad \text{to} \quad \frac{22.5 + \frac{1}{2} \times 0}{22.5} = 1$$

$$\frac{G}{l} = \frac{22.5}{45} = \tfrac{1}{2}$$

This does not satisfy the criterion for a maximum.

To determine which of the two Maxima, 1 or 2, will be the absolute maximum, it will be necessary to calculate the moments due to the two positions of the loading, Fig. 105(b) and Fig. 105(c), and compare them.

For Maximum 1, the left-hand reaction is

$$R_L = \frac{(7.5 \times 25) + (15 \times 15)}{45} = 9.17 \text{ kips}$$

the concentration at panel point c is

$$\frac{7.5 \times 10}{15} = 5 \text{ kips}$$

and the moment at D is

$$M_1 = 9.17 \times 22.5 - 5 \times 7.5 = 168.75 \text{ kip-ft.}$$

For Maximum 2, the left reaction is

$$R_L = \frac{(7.5 \times 30) + (15 \times 20)}{45} = 11.66 \text{ kips}$$

the concentration at panel point c is

$$7.5 + \frac{15 \times 5}{15} = 12.5 \text{ kips}$$

and the moment at D is

$$M_2 = 11.66 \times 22.5 - 12.5 \times 7.5 = 168.75 \text{ kip-ft.}$$

It thus happens that the two positions of the loading give the same moment at D, that is 168.75 kip-ft. This is the absolute maximum.

147. Maximum Shear at Any Selected Point in a Beam.—Find the maximum attainable shear at a point 10 ft. from one end of one girder of a 48-ft. single-track deck plate girder span due to Cooper's E50 loading.

The loads applicable to one of the girders are five-sixths of the loads for one rail, indicated in Fig. 103.

The proper disposition of the load on the span will be determined by application of the criterion expressed in Eq. (7), Art. 141. According to it any movement of loads that increases the left-hand reaction R_L tends to increase the positive shear, but the passage of any load P over and to the left of the point increases the negative term $\Sigma_A^C P$ and, in itself, tends to reduce the shear.

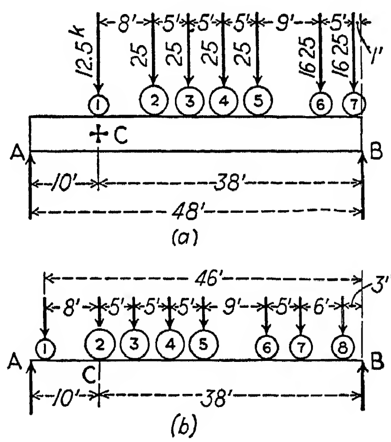


FIG. 106.—Maximum Shear at a Selected Point in a Beam.

As a concentration of loads at, and immediately to the right of, the point C tends to give a large left-hand reaction, the first position of loads requiring consideration will be that with wheel 1 at C, as shown in Fig. 106(a). The left-hand reaction is found by dividing the moment of all loads on the span about a point 38 ft. to the rear of load 1 by the span length. Utilizing the moment table, Fig. 103, this, for E60 loading, is found to be

$$R_L = \frac{3406.5}{48} = 70.97 \text{ kips}$$

or $70.97 \times \frac{5}{6} = 59.1$ kips for E50 loading.

Since there are no wheel loads to the left of C, the shear is, therefore,

$$V_1 = 59.1 - 0 = 59.1 \text{ kips}$$

Try wheel 2 at C, as shown in Fig. 106(b). Proceeding as before, and noting that the moment about support B is the same as the moment about a point 46 ft. from the head of the loading, the left reaction for E60 loading is

$$R_L = \frac{4857.0}{48} = 101.2 \text{ kips}$$

or $101.2 \times \frac{5}{6} = 84.3$ kips for E50 loading.

Load 1 being to the left of C, the shear at C is

$$V_2 = 84.3 - 12.5 = 71.8 \text{ kips}$$

Any further movement of loads to the left would continue to decrease the positive shear. Wheel 2 at C therefore gives the maximum.

148. Maximum Shear in Any Panel of a Truss with Equal Panels.—Determine the proper placing of Cooper's E60 loading, Fig. 103, on a 175-ft. truss span with seven equal panels in order that the maximum shear may arise in the second panel from one end, and find the value of this shear for one truss of a single-track bridge.

Experience shows that for a maximum shear in a panel near one end of a truss carrying a loading such as the one prescribed, one of the driving wheels of the first locomotive should be at the right-hand extremity of the panel.

Try wheel 2 at c, as indicated in Fig. 107(a). There is then 133 ft. of load on the span. Utilizing the moment table of Fig. 103, we have, with the moving of wheel 2 over the point c,

$$G_2 = 15 \text{ to } 45$$

$$\frac{G}{n} = \frac{498}{7} = 71.2$$

At no stage of the movement of wheel 2 over the point c did $G_2 = G/n$, and hence no maximum was realized.

Try wheel 3 at c, as shown in Fig. 107(b). The 138 ft. of load on the span totals 513 kips. Hence

$$G_2 = 45 \text{ to } 75$$

$$\frac{G}{n} = \frac{513}{7} = 73.3$$

This position of loading obviously gives a maximum, which will be called Maximum 1.

Try wheel 4 at c, Fig. 107(c). The 143 ft. of loading amounts to 528 kips. Hence

$$G_2 = 75 \text{ to } 105$$

$$\frac{G}{n} = \frac{528}{7} = 75.4$$

This gives a maximum, which will be called Maximum 2.

Wheel 5 at c does not give a maximum.

Obviously a driving wheel of the second locomotive at c would be less serious than the corresponding wheel of the first locomotive.

The arrangement of loading for absolute maximum shear in the panel bc can be determined only by calculating the shears for Maximum 1 and Maximum 2 and comparing them.

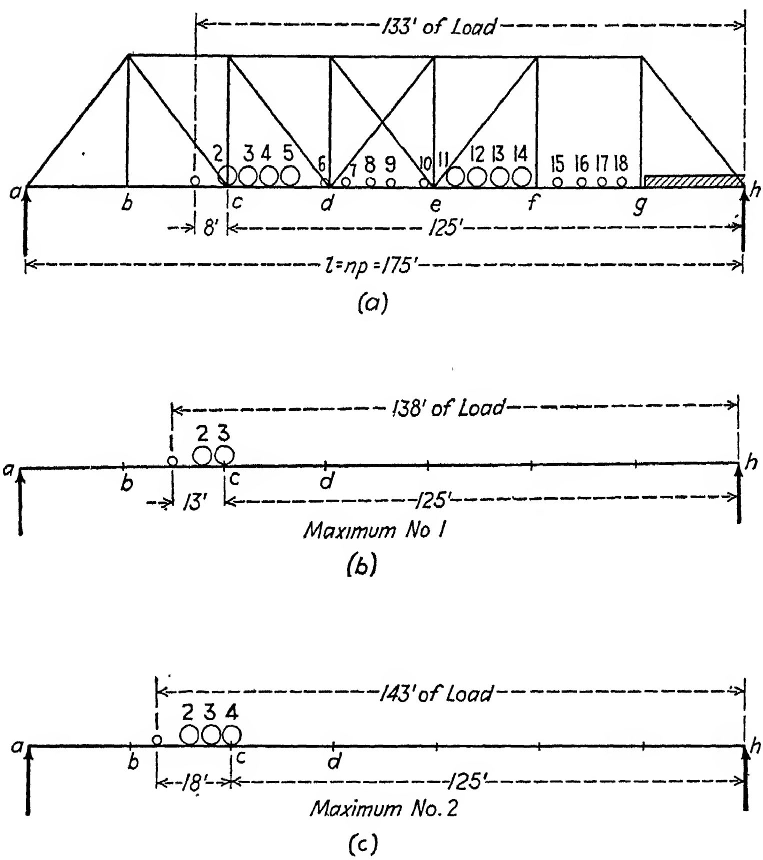


FIG. 107. Maximum Shear in a Selected Panel of a Truss.

For Maximum 1, the left-hand reaction, utilizing the moment table, is

$$R_L = \frac{38{,}161.5}{175} = 218.1 \text{ kips}$$

The concentration at panel point b is, utilizing the table also,

$$C = \frac{345}{25} = 13.8 \text{ kips}$$

The shear in the panel is, therefore, $V_1 = R_L - C = 218.1 - 13.8 = 204.3$ kips.

For Maximum 2, the left-hand reaction is

$$R_L = \frac{40{,}764}{175} = 232.9 \text{ kips}$$

The concentration at b is

$$C = \frac{720}{25} = 28.8 \text{ kips}$$

The shear in the panel is $V_2 = R_L - C = 232.9 - 28.8 = 204.1$ kips.

With wheel 3 at c, the shear is slightly greater than with wheel 4 at that point, and hence the absolute maximum value of the shear for one rail of the E60 loading is 204.3 kips.

149. Exact and Conventional Maximum Shear in a Truss Panel Due to Uniform Load.—A deck Warren highway truss span 90 ft. in length divided into six equal panels carries a moving uniformly distributed load of 1800 lb. per lin. ft. What length of loading must be on the span in order that the maximum positive shear may arise in the third panel from the end, and what is the exact and what is the conventional maximum shear in this panel for one truss?

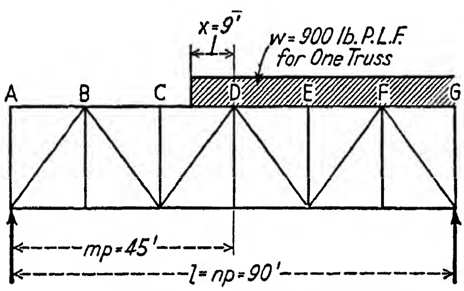


FIG. 108. Shear in a Truss Panel Due to Uniform Load.

According to Eq. (10) of Art. 141, the head of the loading, which is considered as moving on the span from right to left, will, for maximum positive shear in the panel CD, Fig. 108, lie at a distance from D of

$$x = \frac{6 - 3}{6 - 1} \times 15 = 9 \text{ ft.}$$

There will, therefore, be $45 + 9 = 54$ ft. of loading on the span.

For the exact maximum shear, Eq. (11) of Art. 141 applies. The shear in panel CD for one truss is

$$V_c = \frac{900 \times 15}{2(6 - 1)} \times (6 - 3)^2 = 12{,}150 \text{ lb.}$$

For the conventional maximum shear, Eq. (12), Art. 141, applies. The shear is

$$V_c = \frac{900 \times 15}{2 \times 6} (6 - 3)(6 - 3 + 1) = 13{,}500 \text{ lb.}$$

150. Conditions for Maximum Stresses in Diagonals and Verticals of a Pratt Truss with Curved Top Chord and Equal Panels.—For the 8-panel, 200-ft. Pratt truss with curved top chord shown in Fig. 109, determine the appro-

priate placing of Cooper's E60 loading in order that maximum stresses may arise in (1) the main diagonal Cd, (2) the vertical Cc and (3) the possible counter Fg.

(1) *Diagonal Cd.*—For the maximum stress in the main diagonal Cd, Eq. (14) of Art. 141 applies.

By either graphical or analytical means, it is found that $s = 325$ ft. and $t = 375$ ft. Hence, for this panel, the criterion of Eq. (14) becomes

$$G_2 = \frac{13}{15} \times \frac{G}{n}$$

Consider the loading as coming on the span from the right, and place wheel 2 at d. There is then $8 + 125 = 133$ ft. of loading on the span. According to

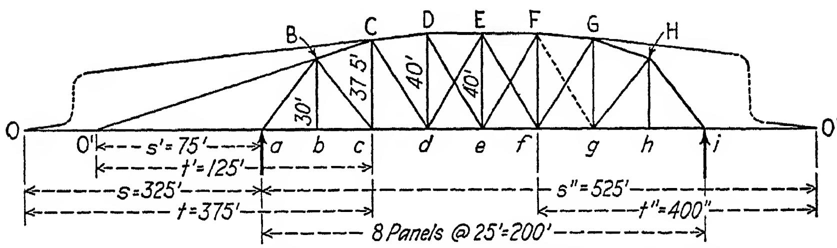


FIG. 109. Maximum Stresses in Diagonals and Verticals of Pratt Truss with Curved Top Chord.

the moment table, this totals 498 kips. As wheel 2 moves over the point to the left, Eq. (14) produces the following ratios:

$$G_2 = 15 \text{ to } 45$$

$$\frac{13}{15} \times \frac{G}{n} = \frac{13}{15} \times \frac{498}{8} = 54.0$$

At no stage is the criterion satisfied, and hence no maximum occurs for this position of the loading.

Try wheel 3 at d. There is then $13 + 125 = 138$ ft. of loading on the span totalling 513 kips. From Eq. (14) we have

$$G_2 = 45 \text{ to } 75$$

$$\frac{13}{15} \times \frac{G}{n} = \frac{13}{15} \times \frac{513}{8} = 55.5$$

A maximum is thus realized with wheel 3 at d.

Wheel 4 at d does not give a maximum.

(2) *Vertical Cc.*—For the maximum stress in the vertical Cc, Eq. (15) of Art. 141 applies. For this vertical $s' = 75$ ft. and $t' = 125$ ft., and Eq. (15) becomes

$$G_2 = \frac{3}{5} \cdot \frac{G}{n}$$

Try wheel 2 at d. The total load on the span being 498 kips, we have

$$G_2 = 15 \text{ to } 45$$

$$\frac{3}{5} \times \frac{G}{n} = \frac{3}{5} \times \frac{498}{8} = 37.4$$

Hence, a maximum arises with wheel 2 at d.

Try wheel 3 at d. Proceeding as before,

$$G_2 = 45 \text{ to } 75$$

$$\frac{3}{5} \times \frac{G}{n} = \frac{3}{5} \times \frac{513}{8} = 38.5$$

Hence, wheel 3 at d does not produce a maximum.

Wheel 4 at d does not give a maximum.

(3) *Assumed Counter Fg.*—Eq. (14) of Art. 141 applies to this as well as to any other diagonal. The distances s and t, which are indicated on Fig. 109 as s'' and t'', are found either graphically or analytically to be 525 ft. and 400 ft., respectively. Eq. (14) then becomes

$$G_2 = \frac{21}{16} \times \frac{G}{n}$$

Try wheel 2 at the panel point g. The total load on the span is 228 kips, and

$$G_2 = 15 \text{ to } 45$$

$$\frac{21}{16} \times \frac{G}{n} = \frac{21}{16} \times \frac{228}{8} = 37.4$$

Wheel 2 at g consequently produces maximum tensile effect in the assumed counter. Wheels 1 and 3 at g do not give appropriate conditions for a maximum.

151. Maximum Floor-Beam Concentration.—The panels of the floor system of a bridge are 25 ft. long. Find the maximum concentration of live load per rail at a floor beam due to Cooper's E60 loading.

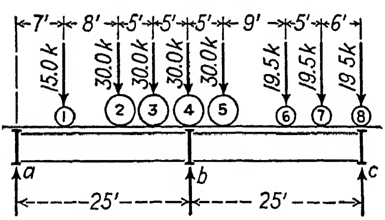


FIG. 110. Maximum Floor-beam Concentration.

For equal floor panels, the condition for maximum floor-beam concentration is attained when Eq. (17) of Art. 141 is satisfied. There may, however, be many arrangements of the loading that will satisfy this criterion. Obviously the absolute maximum concentration will occur when the heavy driving wheels of a locomotive are placed in the vicinity of the floor beam.

By rough trial, it appears that wheel 4 at the selected floor beam b, as shown in Fig. 110, would probably satisfy the rule of Eq. (17). Neglecting wheel 8, which by reason of its position at the extreme limit of one of the adjacent panels, bc, can send no load to the floor beam under consideration, the total load on the

two floor panels is seen by inspection to be 174 kips. Half of this, or 87 kips, must be allocated to the left-hand panel *ab*. This division will occur if, in addition to the 75 kips from wheels 1, 2 and 3, there be assigned to the panel *ab* 12 kips of the 30 kips applied by wheel 4. This position therefore, gives a local maximum.

Exactly the same condition will exist if wheel 13 is placed at the floor beam *b*. The amount of the concentration at *b* is easily found as follows:

Reaction of loads 1, 2 and 3 at *b*	
$= \{15 \times 7 + 30(15 + 20)\}/25$	= 46.2 kips
Load 4 at *b*	= 30.0 kips
Reaction of loads 5, 6 and 7 at *b*	
$= \{19.5(6 + 11) + 30 \times 20\}/25$	= 37.3 kips
Total live load concentration at *b*	= 113.5 kips

152. Exercise Problems on Moving Loads on Beams, Girders and Trusses.—The following exercise problems are based on the principles employed in the solution of the problems of this chapter and Chapter XI. See Appendix I for the answers.

(1) Three loads of 10 kips each, separated by two fixed spaces of 4 ft. each, travel across a 16-ft. beam. What is the greatest attainable moment at a point 5 ft. from one end, and what position of the loading produces it?

(2) Three loads of 20, 10 and 20 kips, separated by two fixed spaces of 5 ft. each, move over a beam of 24-ft. span. Find the maximum moment at a point 10 ft. from one end.

(3) Three loads of 10,000, 15,000 and 10,000 lb., separated by two fixed spaces of 5 ft. each, pass over a beam of 40-ft. span. Find the maximum moment at a point 10 ft. from one end.

(4) Three loads of 10, 20 and 10 kips, separated by fixed distances of 6 ft., travel over a 30-ft. girder. What is the maximum live-load moment at the third point?

(5) Three loads of 10 kips each, separated by two fixed spaces of 9 and 6 ft., pass over a beam of 30-ft. span. Find the maximum moment at the exact centre of the beam. Is this the absolute maximum moment obtainable in the beam, and, if not, where does this absolute maximum occur?

(6) Two loads of 6 and 9 kips, separated by a fixed space of 10 ft., pass over a beam of 20-ft. span. Find the greatest moment that can arise in the beam.

(7) Two loads of 5 and 10 kips, separated by a fixed space of 11 ft., pass over a beam of 16-ft. span. Find the maximum moment in the beam.

(8) Determine the maximum moment produced in a 15-ft. beam by two loads of 11,000 and 20,000 lb. moving over it at a fixed distance apart of 10 ft.

(9) Three loads of 20,000, 6000 and 20,000 lb., separated by two fixed spaces of 6 ft. each, pass over a beam of 20-ft. span. Find the absolute maximum moment in the beam.

(10) Find the maximum moment attainable in a girder of 30-ft. span due to the passage of four loads of 10,000 lb. each separated by three fixed spaces, in the order named, of 5, 10 and 5 ft.

(11) Three loads of 10, 20 and 10 kips, separated by two fixed spaces of 5 ft.,

move over a beam of 20-ft. span. Find the absolute maximum bending moment in the beam.

(12) Four loads of 10 kips each, separated by three fixed spaces of 6 ft. each, move over a beam of 30-ft. span. Find the greatest moment that can arise in the span.

(13) Find the maximum live-load moment on one girder of a deck plate girder span, 36 ft. centre to centre of bearings, due to the passing of two 40-ton double-truck electric railway cars coupled together. The two axles of each truck of a car are 5 ft. centre to centre; the distance centre to centre of trucks is 25 ft., and the distance from the rear axle of the front car to the front axle of the rear car is 10 ft.

(14) What is the absolute maximum moment in a 15-ft. beam due to the passage of two equal loads of 12 kips separated by a fixed space of 9 ft.?

(15) What is the maximum possible moment at the third panel point from the end of the loaded chord of one truss of a deck Warren truss span containing seven 16-ft. panels, if the loading is a train of electric cars of the type and weight specified in Exercise Problem (13)? What wheel should be placed at the panel point in question?

(16) A pony Warren truss consisting of 3 panels of 14 ft. sustains two 10-kip loads moving at a fixed distance apart of 10 ft. Find the greatest possible moment at the centre panel point of the top chord.

(17) Find maximum end shear and maximum bending moment for a girder 18 ft. long from live loads of 20,000 and 40,000 lb. 6 ft. apart.

(18) Three loads of 8, 15 and 12 kips, separated by two fixed spaces of 5 and 10 ft. in the order named, travel over a beam of 20-ft. span. Determine the position of the loading for maximum live-load shear at a point 6 ft. from one end of the beam, and compute this maximum shear.

(19) Determine the maximum shear in the third panel of a deck Warren truss of a single-track bridge containing seven 16-ft. panels, if the loading consists of a train of 40-ton electric cars of the type specified in Exercise Problem (13).

(20) A highway truss span consisting of five panels of 14 ft. each carries a uniformly distributed moving load of 1600 lb. per lin. ft. Find the true maximum live load shear in the second panel from the end for one truss.

(21) What is the theoretical maximum and what is the conventional maximum shear in the second panel of an 8-panel truss of 120-ft. span due to a moving uniform load of 1200 lb. per lin. ft. of truss?

(22) A 60-ft. highway truss span of 4 panels of 15 ft. carries a uniformly distributed moving load of 1600 lb. per lin. ft. of bridge. Compare the correct maximum live-load shear in the second panel of one truss with the maximum found by the conventional method.

(23) A 75-ft. highway truss span of 5 panels of 15 ft. carries a uniformly distributed moving load of 1800 lb. per lin. ft. of bridge. Compare the correct maximum live-load shear in the third panel of one truss with the maximum found by the conventional method.

(24) Three loads of 10, 20 and 30 kips, separated by fixed distances of 6 ft., move across an 8-panel, 160-ft. truss span in either direction, but with the light load always leading. If the two trusses bear equal parts of this loading, find the maximum live-load shear in a truss in the third panel from one end.

(25) A Warren truss highway bridge of 4 panels of 15 ft. each carries a uniformly distributed moving load of 1600 lb. per lin. ft. of floor. Find the conventional

maximum live-load stress in a diagonal of the second panel of a truss if the trusses are $7\frac{1}{2}$ ft. deep.

(26) A road roller of 15 tons' weight, two-thirds of which is on the rear axle, passes over a bridge with floor panels 18 ft. long. If the wheel base is 10 ft., and if any line of stringers may be regarded as taking one-half the load directly applied above it, find the maximum moment on one stringer.

(27) A motor truck of 10 tons' weight, 80% of which is on the rear axle, passes over a bridge with floor panels 17 ft. long. If the wheel base is 10 ft. and if any line of stringers may be regarded as taking one-half of the load directly above it, find the maximum moment on one stringer.

(28) If the front and rear wheel loads of a motor truck are 3000 and 7000 lb. respectively, and the wheel base is 10 ft., find the maximum live-load moment in a bridge stringer of 16-ft. span, assuming that the floor distributes a total of only one-quarter of the loading directly above a stringer to the adjacent stringers.

(29) The front and rear wheels on one side of a motor truck pass over a bridge directly on the line of a stringer. If the stiffness of the floor slab is such as to distribute half the load to the two adjacent stringers, find the maximum moment in the directly loaded stringer if its span is 15 ft., the wheel base of the truck is 10 ft. and the loads on the front and rear wheels are respectively 3000 and 6000 lb.

(30) A motor truck with wheel base of 10 ft. weighing loaded 20,000 lb. with 15,000 lb. on the rear axle passes over a highway truss bridge with panels 15 ft. long. What is the maximum amount of load brought to bear on one floor beam due to the truck?

(31) Two double-truck, 50-ton electric railway cars coupled together pass over a truss span for which the panels are 20 ft. long. The axles bear equal loads and are spaced 5, 20 and 5 ft. apart, and there is a space of 10 ft. from the rear axle of one car to the front axle of the car following. Find the maximum concentration of live load on a floor beam.

(32) The trusses of a highway bridge are 20 ft. 6 in. apart, centre to centre; the clear distance between curbs is 18 ft., and the panel length is 15 ft. Find the maximum possible panel-point concentration due to a 15-ton truck, for which the wheel base is 10 ft., the gauge 6 ft. and the width of the face of the rear wheels 18 in. Assume $\frac{2}{3}$ of the load as carried on the rear axle of the truck.

(33) The panels of a highway bridge are 15 ft. long and the trusses are 20 ft. apart, centre to centre. What is the maximum attainable moment in a floor beam due to a motor truck of 10-ft. wheel base and 6-ft. gauge with 14,000-lb. load on the rear axle and 6000-lb. load on the front axle?

(34) A load of 120,000 lb. on two axles 7 ft. apart passes over a railway bridge having stringer spans of 23 ft. and a spacing of trusses, centre to centre, of 18.ft. What is the maximum live-load moment in a floor beam, neglecting all impact considerations, if two lines of stringers 7 ft. apart are used?

(35) A motor truck with wheel base of 10 ft. and gauge 6 ft. weighing loaded 30,000 lb., with 20,000 lb. on the rear axle, passes over a highway truss bridge with panels 16 ft. long. What is the maximum live-load moment in a floorbeam, if the span of the latter is 18 ft.?

(36) Two adjacent spans of a crane runway are 16 and 22 ft. in length. Find the maximum concentration of loading on the column between them due to a pair of crane wheels 10 ft. apart and carrying a load of 25,000 lb. each.

CHAPTER XI

DESIGN OF PONY WARREN-TRUSS HIGHWAY SPAN

153. Data.—Design the superstructure of a steel pony, or half-through, Warren-truss highway span without sidewalks for a clear opening of 58 ft., with a clear width of 18 ft. between curbs, to accommodate two 15-ton motor trucks of the type shown in Fig. 111 and to conform to the specification of Art. 154.

The flooring will consist of a reinforced concrete slab supported on steel stringers and surfaced with 2 in. of asphaltic concrete. It will be assumed as transferring all lateral and longitudinal forces to the supports without producing important stresses in the steel. Latticed handrails will be employed, and end floor beams will be used. Inverted U-sections for top chords and end posts will be required. Rivets will be $\frac{3}{4}$ in.

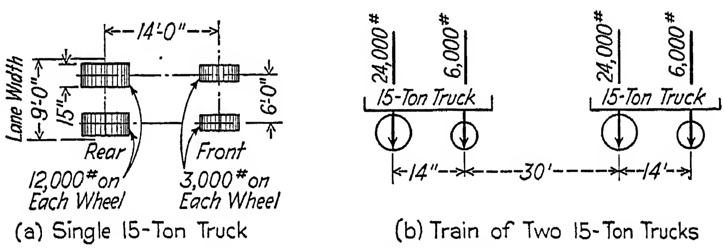


FIG. 111.—Motor Truck Loading.

154. Specification.—The bridge will be designed in accordance with the following abridged special specification:

(1) *Truss Spacing.*—The distance centre to centre of trusses shall be sufficient to provide a clear roadway of 18 ft. between tops of curbs and 19 ft. between handrails.

(2) *Roadways.*—Roadways shall be crowned between curb and the centre not less than 1 in. in 9 ft. Curbs shall be not less than 9 in. high and 7 in. wide. They shall be designed to resist a lateral force of not less than 500 lb. per lin. ft. of curb applied at its top.

(3) *Handrails.*—Handrailings shall extend 3 ft. 9 in. above the adjacent floor surface, and shall be designed to resist a lateral force on the top rail of 150 lb. per lin. ft. and an alternative vertical force of 100 lb. per lin. ft. In the lower half of a latticed steel handrailing the clear perpendicular distance between lattice members shall not exceed 6 in.

(4) *Dimensions for Calculation.*—Span lengths for stress calculation shall be the following distances: For trusses, centre to centre of end bearings; for floor

beams, centre to centre of trusses; for stringers, centre to centre of floor beams. The depth of riveted trusses shall be the distance between the centres of gravity of chord sections.

(5) *Minimum Thickness.*—Material, including webs of I-beams, but excluding fillers and handrails, shall be not less than $\frac{5}{16}$ in. thick.

(6) *Total Loads.*—The structure shall be proportioned for the combination of dead load, live load and impact.

(7) *Live Load.*—The live load shall consist of two 15-ton trucks of the type shown in Fig. 111. These shall be assumed either as moving in the same direction in one traffic lane with a minimum distance of 30 ft. between the rear axle of one truck and the front axle of the other, Fig. 111(*b*), or as moving independently in the two lanes and headed in the same or opposite directions.

(8) *Impact.*—Impact shall be added to the live-load stress in accordance with the formula:

$$I = L \cdot \frac{50}{125 + S} \tag{1}$$

in which I = impact stress;
L = live-load stress;
S = length in feet of the shortest portion of the floor which must be loaded or reloaded with the trucks moving in a forward direction, in order to cause the stress in the member or part considered to rise from zero to a maximum.

(9) *Floor Slab Loading.*—In calculations of moment due to wheel loads on concrete slabs, no distribution of load in the direction of the span of the slab shall be assumed. In the direction perpendicular to the span of the slab, the wheel load shall be considered as distributed uniformly over a width of slab termed the "effective width." For a floor slab supported by longitudinal stringers, in which the critical span and the main reinforcement are perpendicular to the direction of traffic, a wheel load shall be assumed as distributed uniformly over an effective width e of slab given by the formula

$$e = 0.7(2x + t) \tag{2}$$

in which x = distance from the centre of the wheel to the centre of the near support;
t = width of tire.

A slab designed for moment in accordance with the foregoing rule may be considered as adequate for shear without special reinforcement.

(10) *Stringer Loading.*—In calculating moments in longitudinal stringers, no longitudinal distribution of wheel loads shall be assumed, but lateral distribution shall be assumed as follows:

(*a*) Interior stringers supporting a reinforced concrete floor slab, and spaced at not over 10-ft. centres, shall, when two or more traffic lanes exist, be designed

to carry fractions of one, or, if applicable, two wheel loads, in tandem, as given by the formula

$$f = \frac{s}{4.5} \tag{3}$$

in which f = fraction of wheel load under consideration supported by stringer;
s = spacing of stringers in feet, centre to centre.

(*b*) The maximum live load supported by an outside stringer shall be determined by placing a truck in the most unfavorable position and by assuming the flooring to act as a simple beam between stringers.

In calculating end shears and end reactions in stringers, no lateral distribution of any wheel load situated at the end of a stringer shall be permitted. Distribution may, however, be assumed for the wheels of the other axle.

(11) *Floor-Beam Loading.*—In calculating moments in floor beams, no distribution of wheel loads in the direction of the span of the floor beam shall be assumed.

(12) *Moments in Floor Slabs.*—When floor slabs are reinforced for continuity over interior supports, and when the spans are approximately equal, the moment shall be that for a simply supported beam multiplied by the appropriate coefficient from Table 15.

TABLE 15

COEFFICIENTS BY WHICH SIMPLE BEAM MOMENT IN SLAB IS TO BE MULTIPLIED

Type of Load	Intermediate Span		End Span	
	Positive Moment	Negative Moment	Positive Moment	Negative Moment
Uniformly distributed.....	$\frac{2}{3}$	$\frac{4}{5}$	$\frac{2}{3}$	$\frac{4}{5}$
Concentrated at mid-span.	$\frac{4}{5}$	$\frac{2}{3}$	$\frac{4}{5}$	$\frac{2}{5}$

(13) *Reversal of Stress.*—Members in which the static live-load stress may have two maximum values, one of similar sign to the dead-load stress and one of opposite sign shall be proportioned for either (*a*) the full dead-load stress combined with the maximum live-load stress of the same sign with its corresponding impact, or (*b*) the maximum live-load stress of the opposite sign with its corresponding impact, less 70% of the dead load stress, each combination being further increased by an amount equal to one-half the smaller of the two combinations. When the live-load stress is always of opposite sign to the dead-load stress, only combination (*b*) will apply and no further increase shall be made.

(14) *Secondary Stresses.*—With a view to lessening secondary stresses, trusses

should be so designed and detailed that (*a*) the gravity axes of members correspond as closely as possible to the skeleton lines of the truss and (*b*) the centre of gravity of a rivet group connecting a member to a gusset be as close as possible to the gravity axis of the member.

In ordinary trusses without sub-paneling, secondary stresses may be ignored in any member the width of which in the plane of the truss is less than $\frac{1}{10}$ of the length of the member.

(15) *Permissible Stresses.*—Permissible stresses in pounds per square inch are as follows:

STRUCTURAL STEEL

Axial tension on net section	18,000
Bearing compression, except for rivets	24,000
Axial compression on gross section of columns with a maximum of 14,000, where l = unsupported length centre to centre of intersections, and r = least radius of gyration applicable to this length	17,000–60 l/r
Diagonal compression in webs of beams with a maximum of 12,000 where c = clear height between fillets and t = web thickness	17,000–100 c/t
Flexure on extreme fibres of rolled shapes, built-up sections and girders (net section), continuous lateral support being provided	18,000
Flexural compression in the flanges of beams and girders not continuously supported laterally, where l' = spacing of lateral supports in the neighborhood of the maximum moment and b = flange breadth	18,000–170 l'/b
Shear on gross section of webs	12,000

RIVET STEEL

Axial tension on shafts of power-driven rivets where d = diameter of rivet in inches before driving.	7000/d
Shear on power-driven shop rivets	13,500
Shear on power-driven field rivets	12,000
Bearing on power-driven shop rivets	27,000
Bearing on power-driven field rivets	24,000

The above-mentioned values for shear and bearing on rivets shall be reduced 25% where they are countersunk.

INTERMEDIATE GRADE
REINFORCEMENT STEEL

Axial tension	20,000

CONCRETE

Flexural compression, where f_c' = crushing strength at 28 days, in pounds per square inch, $0.4\,f_c'$

(16) *Net Section at Rivet Holes in Tension Members.*—The diameter of rivet holes shall be taken as $\frac{1}{8}$ in. larger than the nominal diameter of the rivet. Allowance shall be made in each component part for as many rivet holes as it contains gauge lines, unless the distance centre to centre of rivet holes, measured on the diagonal, is at least 40% greater than the distance between the gauge lines.

(17) *Effective Area of Angles.*—The effective area of a single angle connected by one leg only and acting in either tension or compression shall be the actual area (allowing for rivet holes in tension angles as in (16)) multiplied by the coefficient,

$$e = 1.0 - 0.18\,u/c \qquad (4)$$

in which u = length of outstanding leg in inches and c = length of connected leg. This reduction shall apply to each angle of a double-angle member connected back to back on the *same* side of a gusset plate.

If a double-angle member is connected with the angles back to back on *opposite* sides of a single gusset plate, the net area (gross, in the case of compression members) shall be considered as fully effective. This shall also be assumed if the angles connect to separate gusset plates and are connected to each other by stay plates located as near as practicable to the gussets.

(18) *Limiting Radius of Gyration.*—The minimum allowable radius of gyration for compression members shall be:

Main compression members. . . . . $r = \dfrac{2\,l + h}{350}$

Wind bracing and other secondary compression members. . . . . $r = \dfrac{2\,l + h}{400}$

where r = least radius of gyration in inches;

l = unsupported length of member in inches in the direction in which r is sought;

h = length of horizontal projection of member in inches, to be included only when considering the axis about which bending is produced by the weight of the member.

For built-up I-sections, the radius of gyration may be computed for the flange material alone, neglecting the web plate (in part or wholly), in which case the latter shall not be counted as effective section for axial compression.

For riveted tension members the least allowable radius of gyration of the gross section shall not be less than the length of the member in inches divided by 250.

(19) *Local Buckling.*—No material in compression shall have an unsupported width of more than 60 times its thickness and not more than 40 times the thick-

ness of the unsupported portion of a plate shall be considered as effective section. The thickness of the outstanding legs of angles in compression, except when reinforced by plates, shall be not less than $\frac{1}{12}$ and $\frac{1}{16}$ of the unsupported length of the outstanding leg for main and secondary members, respectively.

(20) *Pony Truss Bridges.*—In pony truss bridges, the vertical truss members and the floor-beam connections thereto shall be proportioned to resist, in addition to the other stresses herein specified, the stresses due to a lateral thrust at the top chords not less than 2% of the maximum axial stress in the chord members to which the particular vertical connects. The top chord sections shall be so proportioned that the radius of gyration about the vertical axis of the member will be at least twice that about the horizontal axis.

(21) *End Floor Beams.*—End floor beams shall be provided where possible, and they shall be designed for jacking up the spans, under which condition the permissible stresses specified in (15) shall not be exceeded by more than 50%.

(22) *Lateral Bracing.*—Lateral force will be assumed as delivered to the supports by the reinforced concrete floor acting as a horizontal girder. Temporary bracing may be used for convenience in erection.

(23) *Bearings.*—Spans of less than 70 ft. may be arranged to slide upon metal plates with smooth surfaces.

155. General Dimension.—Assuming bearing plates 14 in. wide in the direction of the axis of the bridge, and the distance from the outer edge of such a plate to the face of the abutment as 5 in., the span centre to centre of truss bearings is $58 + 2(\frac{5}{12} + \frac{7}{12}) = 60$ ft.

To give a clear width of 18 ft. between tops of curbs, Art. 154 (1), the centre-to-centre spacing of the trusses, if the distance from the inner top corner of the curb to the centre line of a truss, Fig. 112(*c*), is assumed as 1 ft. 4 in., must be 18 ft. + 2 ft. 8 in. = 20 ft. 8 in. This spacing will make possible the specified minimum clear distance of 19 ft. between handrails and will obviate overlapping of the end connections of the floor beams and outside stringers.

Economic proportions for pony Warren truss spans generally arise when the depth is from $\frac{1}{8}$ to $\frac{1}{10}$ of the span. It will be made 7 ft. between centres of gravity of chords.

Diagonals should for economy be inclined at somewhat less than 45 deg. to the vertical. This requirement will be satisfied and practicable stringer spans brought about by making the panels 12 ft. long, as shown in Fig. 112(*a*).

156. Floor Slab.—For a spacing of trusses of 20 ft. 8 in. centre to centre a stringer spacing of 3 ft. centre to centre will be satisfactory with respect to both economy and practicability of details. As shown in Fig. 112(*c*), the outside stringer is placed at the face of the curb.

A 2-in. asphaltic concrete surfacing will be used on a Portland cement concrete structural slab assumed as 6 in. thick. If both materials be assumed to weigh 150 lb. per cu. ft., the weight of the slab and surfacing will be $150 \times \frac{8}{12} = 100$ lb. per sq. ft.

Placing a rear wheel of a truck in the critical position for slab moment, that is at the centre of slab span, the distance from the centre of the wheel to the

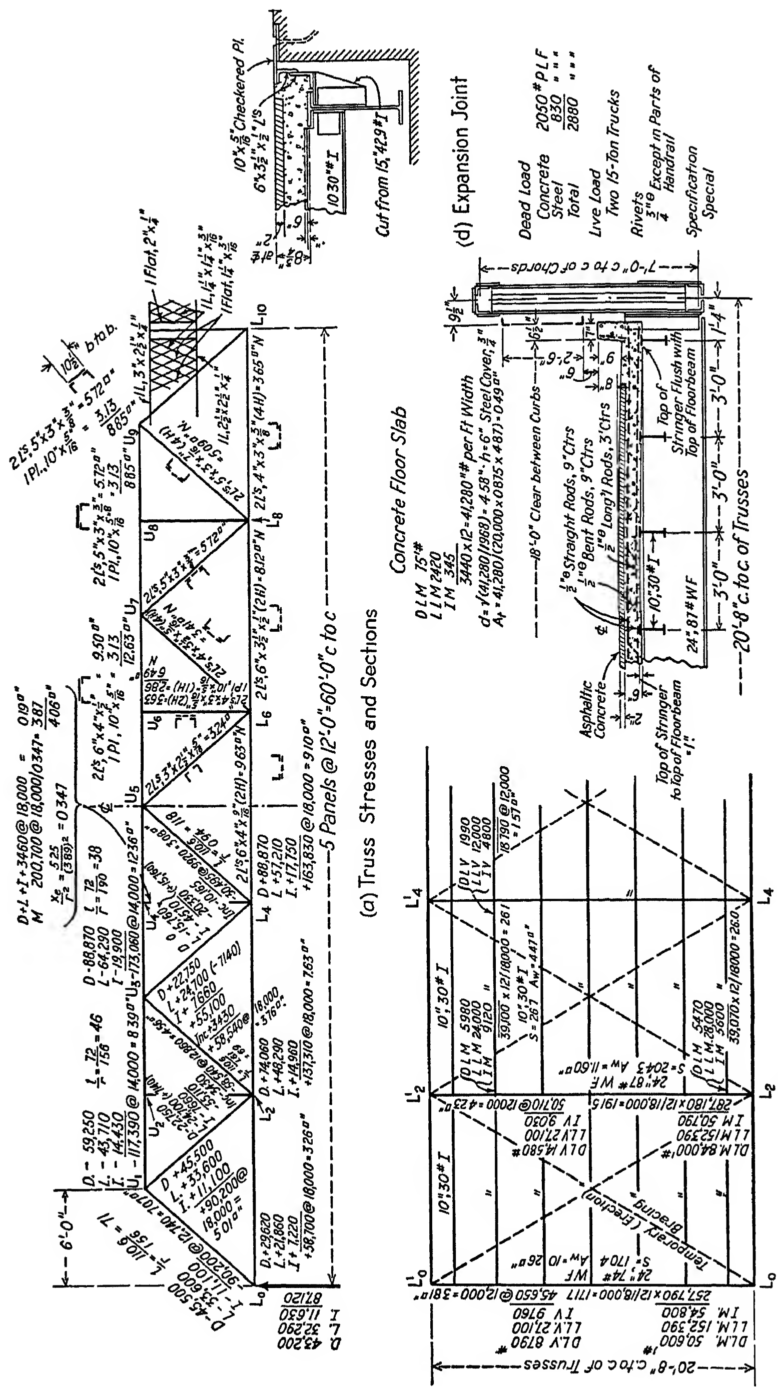


FIG. 112.—Stress Sheet of 60-Ft. Pony Warren-Truss Highway Span

centre of the nearest support is $x = 18$ in. The tire width t being 15 in., the width of slab over which a wheel load may be considered as uniformly distributed in a direction perpendicular to the slab span is, from Art. 154(9)

$$e = 0.7(2 \times 18 + 15) = 35.7 \text{ in.}$$

A 12-in. strip of slab may consequently be considered as bearing a central concentrated load of

$$12{,}000 \times 12/35.7 = 4035 \text{ lb.}$$

The slab will be reinforced for continuity over the interior supports, and hence the moment coefficients for uniformly distributed and concentrated loads given in Table 15 of Art. 154(12) will apply.

The maximum live-load moment in a span of the slab may arise when a rear wheel has moved only 35.7/2, or say, 18 in. on the bridge from the centre of an end floor beam. The loaded distance to be employed in the impact formula of Art. 154(8) for slab moment is only 1.5 ft., and the impact fraction $I/L = 50/(125 + 1.5) = 0.39$.

On a 12-in. strip of an intermediate span of the slab the maximum positive moment is

$$\begin{aligned}
\text{D.L.} &= \tfrac{2}{3} \times \tfrac{1}{8} \times 100 \times (3.0)^2 &&= 75 \text{ ft.-lb.}\\
\text{L.L.} &= \tfrac{4}{5} \times 4035 \times 3.0/4 &&= 2420 \text{ ft.-lb.}\\
\text{I.} &= 0.39 \times 2420 &&= \underline{945} \text{ ft.-lb.}\\
& && \quad 3440 \text{ ft.-lb.}
\end{aligned}$$

or $3449 \times 12 = 41{,}280$ in.-lb. per ft. of width.

The maximum negative moment is

$$\begin{aligned}
\text{D.L.} &= \tfrac{4}{5} \times \tfrac{1}{8} \times 100 \times (3.0)^2 &&= 90 \text{ ft.-lb.}\\
\text{L.L.} &= \tfrac{2}{3} \times 4035 \times 3.0/4 &&= 2018 \text{ ft.-lb.}\\
\text{I.} &= 0.39 \times 2018 &&= \underline{787} \text{ ft.-lb.}\\
& && \quad 2895 \text{ ft.-lb.}
\end{aligned}$$

or $2895 \times 12 = 34{,}740$ in.-lb. per ft. of width.

Proceeding in the same manner, the maximum positive and negative moments for an end span are found to be 41,280 and 21,260 in.-lb.. respectively, per foot of width.

Depth and Reinforcement of Slab.—For any strip of reinforced concrete slab b inches wide and d inches deep to centre of steel, the safe bending moment based on a permissible flexural compressive stress, p_c, in the concrete is

$$M = \tfrac{1}{2}\, p_c k j b d^2 = R b d^2 \tag{5}$$

where k = ratio of depth to neutral axis to depth d;
j = ratio of lever arm of resisting couple to depth d.

Utilizing a concrete with a crushing strength of 2500 lb. per sq. in. at an age

of 28 days, $p_c = 0.4 \times 2500 = 1000$ lb. per sq. in. With a permissible tensile stress in the reinforcing steel of $p_t = 20{,}000$ lb. per sq. in. and a modular ratio of $n = 12$, $R = 164$ and the depth of the structural slab to the centre of the steel required to resist a maximum moment of M on a 12-in. strip must be at least

$$d = (M/12\,R)^{1/2} = (41{,}280/1968)^{1/2} = 4.58 \text{ in.}$$

Adding $1\frac{1}{8}$ in. for the distance between centre of steel and bottom of slab, the total thickness required is 5.71 in. The slab will be made 6 in. thick and will be crowned by varying the height of the stringers from curb to crown, the top of the stringer at the centre of the roadway being placed 1 in. above the top of the floor beam.

The area of reinforcement per foot of width of slab is

$$A_t = \frac{M}{p_t j d} \tag{6}$$

The value of j for the conditions assumed is 0.875 and for the adopted thickness of slab, $d = 6.00 - 1.13 = 4.87$ in.

Hence

$$A_t = 41{,}280/20{,}000 \times 0.875 \times 4.87 = 0.49 \text{ sq. in.}$$

This will be supplied in the form of $\frac{1}{2}$-in. diameter plain rods. At the bottom of the slab straight rods will be run from curb to curb at 9-in. centres, as indicated in Fig. 112(*c*). Alternating with these there are continuous bent rods at 9-in. centres arched up over the stringers. In addition, at the top of the slab, continuous straight rods at 9-in. centres run from curb to curb. This arrangement of steel gives a positive reinforcement at the middle of a 3-ft. span of the slab amounting to $0.1963 \times 12/4.5 = 0.52$ sq. in. per ft. of width, and the same amount for the somewhat smaller negative moment over the stringers.

Longitudinal $\frac{1}{2}$-in. diameter rods at about 3-ft. centres will be inserted to provide for shrinkage and temperature stresses.

Curbs.—Conforming to Art. 154(2), the curbs will be made 9 in. high and 7 in. wide. The reinforcement rods $\frac{1}{2}$-in. diameter at 9-in. centres will be bent up into the curb near its face to serve as reinforcement for the prescribed horizontal force.

157. Interior Stringers.—Assuming a stringer to weigh 32 lb. per lin. ft., the dead load carried by an interior stringer is $3.0\,(150 \times \frac{8}{12}) + 32 = 332$ lb. per lin. ft.

There being two traffic lanes, Eq. (3) of Art. 154(10) applies. Since the stringer spacing is 3 ft., f, the fraction of any applicable wheel load borne by an interior stringer is $3.0/4.5 = 0.667$.

By reason of the relatively short floor panels, the condition for absolute maximum moment in a stringer requires the placing of one of the rear wheels of the truck of Fig. 111(*a*) at the centre of a stringer. The design load is, therefore, a single central concentrated load of $0.667 \times 12{,}000 = 8000$ lb.

The critical loading for moment in a stringer will occur with the movement of the rear axle of the truck from end to mid-span of the stringers. The front axle being off the stringers before the rear axle comes on, evidently the appropriate loaded distance S is 6 ft. The impact fraction from Eq. (1) is therefore $50/(125 + 6) = 0.38$.

Section for Moment.—The total maximum moment in an interior stringer, assuming simple support, is then

$$\begin{aligned}
\text{D.L.} &= \tfrac{1}{8} \times 332 \times (12)^2 &&= 5{,}980 \text{ ft.-lb.}\\
\text{L.L.} &= \tfrac{1}{4} \times 8000 \times 12 &&= 24{,}000 \text{ ft.-lb.}\\
\text{I.} &= 0.38 \times 24{,}000 &&= \underline{9{,}120} \text{ ft.-lb.}\\
& && 39{,}100 \text{ ft.-lb.}
\end{aligned}$$

or $39{,}100 + 12 = 469{,}200$ in.-lb.

Required section modulus is

$$S = M/p_f = 469{,}200/18{,}000 = 26.1 \text{ in.}^3$$

Use a 10-in., 30-lb. I, for which $S = 26.7$ in.^{3} While a lighter 10-in. section could be selected to give the required section modulus, the web would be less than $\frac{5}{16}$ in.—the minimum thickness allowed by the specification.

Section for Shear.—The dead-load end shear is $\frac{1}{2} \times 12 \times 332 = 1990$ lb. The maximum live-load stringer reaction and shear occur when a 12,000-lb. wheel of a truck is as close as possible to the end of the stringer without being on a floor beam. No lateral distribution of this load being permitted, Art. 154(10), the live-load end shear is very nearly 12,000 lb. As the maximum end shear in a stringer is produced in the same way as if the rear axle were leading and no other load had passed over the stringer, the impact fraction is $50/(125+0) = 0.40$, and the impact shear is $0.40 \times 12{,}000 = 4800$ lb. The total shear is consequently 18,790 lb. It is obvious from casual inspection that the web area is ample for both shear and web buckling stresses.

158. Outside Stringers.—The dead load carried by an outside stringer per lineal foot is

$$\begin{aligned}
&\text{Slab and surfacing, } 2.08 \times \tfrac{8}{12} \times 150 &&= 208 \text{ lb.}\\
&\text{Curb, } \frac{9 \times 7}{144} \times 150 &&= 66 \text{ lb.}\\
&\text{Stringer,} &&= \underline{30} \text{ lb.}\\
& && 304 \text{ lb.}
\end{aligned}$$

It is possible that a rear wheel of a moving truck may strike the curb, and, with 15-in. tires, the centre of such wheel would then, allowing for the batter of the curb, be only 8 in. from the centre of the outside stringer. According to Art. 154(10) (*b*), the critical loading for moment on this stringer will consequently be $12{,}000 \times 28/36 = 9333$ lb.

There is small probability that the full impact allowance would arise when the truck is tight against the curb. For this condition the impact fraction will be taken as 0.20.

The maximum moment is then

D.L. $= \frac{1}{8} \times 304 \times (12)^2$	$= 5{,}470$ ft.-lb.
L.L. $= \frac{1}{4} \times 9333 \times 12$	$= 28{,}000$ ft.-lb.
I. $= 0.20 \times 28{,}000$	$= 5{,}600$ ft.-lb.
	39,070 ft.-lb.

A section modulus of 26.0 is required and a 10-in., 30-lb. I furnishes 26.7 in.^{3}, which is adequate.

159. Intermediate Floor Beams.—The span is the distance centre to centre of trusses, Art. 154(4), that is 20.67 ft.

The dead load consists of (1) the weight of the floor beam itself, which may be assumed at 87 lb. per lin. ft. or in all 20.67 × 87 = 1800 lb., uniformly distributed; and (2) the weight of that part of the stringers, floor slab and curb supported by one floor beam.

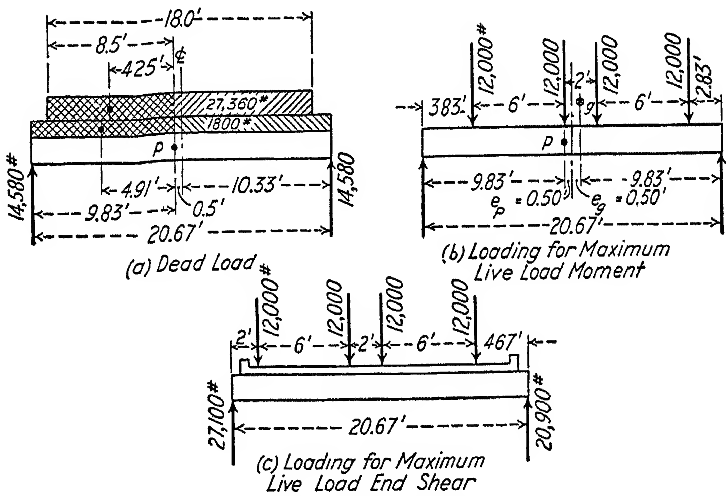


FIG. 113.—Intermediate Floor-beam Loadings.

Load (2), which may be regarded as uniformly distributed over a central 18-ft. length of the floor beam, as shown in Fig. 113(a), consists of one panel of floor and curbs that is 12 ft. In amount it is as follows:

7 stringers at 30 lb. per ft. $= 7 \times 30 \times 12$	$= 2{,}520$ lb.
14 web connections to floor beams at 13 lb.	$= \phantom{00{,}}182$ lb.
Slab and surfacing, $19.17 \times \frac{8}{12} \times 150 \times 12$	$= 23{,}000$ lb.
2 curbs, $2 \times \dfrac{9 \times 7}{144} \times 150 \times 12$	$= 1{,}575$ lb.
Filling over floor beam, $\dfrac{9 \times 0.5}{144} \times 150 \times 18$	83 lb.
Total	$= 27{,}360$ lb.

Section for Moment.—In view of the fact that the maximum live-load and impact moment will occur at a point p, 0.50 ft. from mid-span (see text below), the dead-load moment to be considered is that existing at p. The total dead load on the floor beam is as indicated in Fig. 113(a), and the moment at p is, therefore,

$$(14{,}580 \times 9.83) - \left(1800 \times \frac{9.83}{20.67} \times 4.91\right) - \left(27{,}360 \times \frac{8.50}{18.0} \times 4.25\right) = 84{,}100 \text{ ft.-lb.}$$

The live load will produce the most serious moment in a floor beam when two trucks are on the span with the rear axle of each directly over the floor beam. As the wheel base, Fig. 111(a), is 14 ft., and the stringer span is only 12 ft. for this bridge, there is no part of the front axle load coming to the floor beam over which the rear axles are placed. Each truck, therefore, imposes on the floor beam two maximum concentrations of 12,000 lb. each, 6 ft. apart.

The lateral position of the loading on the bridge to give maximum moment in a floor beam must be such as to satisfy the equation $e_p = e_g$. If, as is physically possible, the two trucks are separated by a distance of only 2 ft. between centres of near wheels, then the critical lateral position of the loading is as shown in Fig. 113(b). The resultant maximum moment occurs at p, under the concentration, which is 0.5 ft. from mid-span. It is

$$\frac{4 \times 12{,}000 \times (9.83)^2}{20.67} - (12{,}000 \times 6) = 152{,}390 \text{ ft.-lb.}$$

In order that the maximum concentration of live load may be brought about, the two trucks must each have advanced $14 + 12 = 26$ ft. from the end of the bridge. The impact fraction is, therefore, $50/(125 + 26) = 0.33$.

Assembling the moments, we have

D.L.	84,000 ft.-lb.
L.L.	152,390 ft.-lb.
I. = 0.33 × 152,390 =	50,790 ft.-lb.
	287,180 ft.-lb.

or $287{,}180 \times 12 = 3{,}446{,}200$ in.-lb.

Required section modulus $= 3{,}446{,}200/18{,}000 = 191.5$ in.^{3}

Use a 24-in., 87-lb. W.F., for which $S = 204.3$ in.^{3}

Section for Shear.—The maximum floor beam reaction occurs when two trucks, moving in either the same or in opposite directions, close together transversely, are as close as possible to one side of the bridge and each has its rear axle over the floor beam under consideration.

Fig. 113(c) shows the disposition. With 15-in. tires on the rear wheels, and with the side of the tires tight against the curb, it is possible to place the central

plane of the wheels on the side of the truck nearest the curb only 2.0 ft. from the central plane of the near truss.

At each of four points a rear wheel applies a load of 12,000 lb. to the floor beam considered. No load is transferred from the front axles, since the wheel base of the trucks is greater than the panel length. The maximum live-load end reaction and shear is, therefore,

$$V = \frac{12{,}000(4.67 + 10.67 + 12.67 + 18.67)}{20.67} = 27{,}100 \text{ lb.}$$

The impact fraction being the same as for moment, the total maximum shear is then

D.L. (Fig. 113(*a*))		14,580 lb.
L.L.		27,100 lb.
I.....$0.33 \times 27{,}100$	=	9,030 lb.
		50,710 lb.

The required area for shear being only 50,710/12,000 = 4.23 sq. in. and the area of the web of the 24-in. 87-lb. W.F. floor beam being $dt = 24.16 \times 0.48 = 11.60$ sq. in., the section is ample.

160. End Floor Beams.—The dead load consists of (1) the weight of the floor beam itself, which will be assumed as a 24-in., 74 lb. W.F., amounting to $20.67 \times 74 = 1530$ lb., uniformly distributed; and (2) the weight of the stringers in half a floor panel, the weight of the stringer end connections, of the brackets extending about 9 in. towards the ballast wall from the web of the floor beam, of a 6.75-ft. length of floor slab, with curbs, parallel to the bridge axis, of the end retaining angle for the slab, of one-half of the expansion plate and the filling over the floor beam and brackets. The nature of these details may be seen from Fig. 112(*d*).

The amount of the dead load applied to this floor beam by the floor is as follows:

7 stringers, 6 ft. at 30 lb. = $7 \times 6 \times 30$	=	1,260 lb.
7 web connections to floor beam, say	=	90 lb.
7 brackets at 50 lb.	=	350 lb.
6.75 ft. of floor slab, 8 in. thick = $6.75 + 19.17 \times \frac{8}{12} \times 150$	=	12,940 lb.
6.75 ft. of curbs = $2 \times 6.75 \times \dfrac{7 \times 9}{144} \times 150$	=	890 lb.
End angle and $\frac{1}{2}$ expansion plate	=	320 lb.
Concrete filling over floor beam and brackets	=	200 lb.
		16,050 lb.

The disposition and magnitude of the dead load are indicated in Fig. 114.

Section for Moment.—The dead-load moment at the critical point P, 0.5 ft.

from mid-span, which is the point at which the maximum live-load moment occurs, is

$$(8790 \times 9.83) - \left(1530 \times \frac{9.83}{20.67} \times 4.91\right) - \left(16{,}050 \times \frac{8.50}{18.0} \times 4.25\right)$$
$$= 50{,}600 \text{ ft-lb.}$$

The live-load moment is, of course, the same as for an intermediate floor beam.

For the maximum concentration of live load to occur, the two trucks need to advance only 14 ft. on the bridge. Hence, the impact fraction is $50/(125 + 14) = 0.36$.

Collecting moments, we have

D.L.	= 50,600 ft.-lb.
L.L. (Art. 159)	= 152,390 ft.-lb.
I. = 0.36 × 152,390	= 54,800 ft.-lb.
	257,790 ft.-lb.

$= 257{,}790 \times 12 = 3{,}091{,}000$ in.-lb.

Required section modulus $= 3{,}091{,}000/18{,}000 = 171.7$ in.^{3}

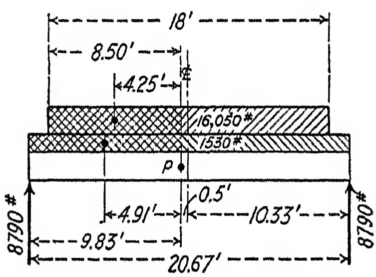


FIG. 114.—End Floor-Beam Dead Load.

A 24-in., 74-lb. W.F., for which $S = 170.4$ in.^{3}, is sufficiently close.

Section for Shear.—The dead-load end shear is apparent from Fig. 114, and the maximum live-load end shear is the same as for an intermediate floor beam. The impact fraction is, however, the same as exists for maximum moment in an end floor beam, that is 0.36. Consequently, the maximum end shear is

D.L. (Fig. 114)	= 8,790 lb.
L.L. (Art. 159)	= 27,100 lb.
I. = 0.36 × 27,100	= 9,760 lb.
	45,650 lb.

It is evident, by inspection, that the web section is adequate.

Sufficiency for Jacking Up Span.—As required by Art. 154(21) an end floor beam shall be sufficiently strong to permit the end of the span to be jacked up by applying the jacks to the beam.

From Art. 161, the total dead load of the span is estimated at 2880 lb. per lin. ft. Hence the weight of the half span is $30 \times 2880 = 86{,}400$ lb., and the load applied by each of two jacks would be $\frac{1}{2} = 86{,}400 = 43{,}200$ lb.

The jacks would be applied near the ends of the beams so as to minimize the moment. Even if they were applied at the quarter-points, and all the dead load were applied at the two ends of the beam, the total moment would be only $43{,}200 \times 20.67/4 = 223{,}000$ ft.-lb. This is less than the total maximum moment for which the beam has already been designed.

The maximum shear due to jacking is less than that due to dead load, live load and impact.

161. Dead-Load Stresses in Trusses.—From the determination of the dead load borne by an intermediate floor beam, Art. 159, the combined weight of the floor slab, surfacing and curbs is seen to be $(23{,}000 + 1575)/12 = 2050$ lb. per lin. ft. For the structure under consideration, the steel may be assumed to weigh $w = 5.5\,l + 500$ lb. per lin. ft., where l = span length in feet. The estimated dead load borne by the trusses is, therefore, as follows:

Slab, surfacing and curbs	2050 lb. per lin. ft.
Steel $= (5.5 \times 60) + 500$	830 lb. per lin. ft.
Total	2880 lb. per lin. ft.

The panel dead load per truss $= \frac{1}{2} \times 2880 \times 12 = 17{,}280$ lb., all of which will be assumed as applied at the bottom chord panel points.

Dead-load reaction of one truss, omitting the half panel load at each support, which does not affect truss stresses, $= 2 \times 17{,}280 = 34{,}560$ lb.

Length of truss diagonal $= (6^2 + 7^2)^{1/2} = 9.2195$ ft.

Secant of angle of slope of diagonal with vertical $= \sec\theta = 9.2195/7 = 1.317$.

The stresses in the web members are found by multiplying the shear in the panel in which the member lies by $\sec\theta$, affixing the appropriate sign. Stresses in the chord members are found by dividing the moment at the opposite panel point by the depth of the truss.

The dead-load stresses are as given in Tables 16 and 17 below and as indicated on the stress sheet, Fig. 112.

162. Live-Load Stresses in Trusses.—More serious live-load effects are produced in the trusses when two trucks travel abreast than when one follows another in same lane at the spacing indicated in Fig. 111(b). Consequently, place a truck with the centre of a rear wheel 8 in. from the curb with another truck abreast of, and as close as possible to, the first, and facing in the same direction, as shown in Fig. 115. The transverse position of the truck wheels on the span will be the same as was found necessary for the maximum end shear in an intermediate floor beam, Art. 159, Fig. 113(c).

It is convenient, in shear and moment calculations, to consider a truss as subjected to two concentrated loads, applied at points along the truss corresponding to the longitudinal position of, and equivalent to, the actual wheel

TABLE 16

DEAD-LOAD SHEARS AND WEB MEMBER STRESSES

+ = tension — = compression

Panel	D. L. Shear, V, Lb.	Member	Stress in Member = 1.317 V, Lb.
L_0L_2	34,560 − 0 = 34,560	L_0U_1	−45,500
		U_1L_2	+45,500
L_2L_4	34,560 − 17,280 = 17,280	L_2U_3	−22,750
		U_3L_4	+22,750
L_4L_6	34,560 − 2 × 17,280 = 0	L_4U_5	0
		U_5L_6	0

loads of the two trucks travelling abreast of each other. The equivalent load P at any point p on the near truss due the rear wheels of the two trucks is, as already found in Art. 159, 27,100 lb., and the corresponding load at q due to the front wheels is one-quarter of this, or Q = 6775 lb. These loads move across the span at a fixed distance of 14 ft. apart.

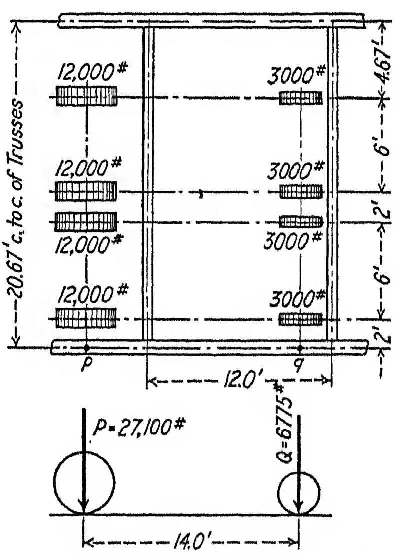


FIG. 115.—Equivalent Truss Loading.

Shears and moments being generally found for the left half of a truss rather than the right, it is convenient to move the loading from right to left with the heavy load P leading. For impact calculations, however, the loads will be considered as having reached their position by moving from left to right with the light load Q leading.

Shears and Web Member Stresses from Concentrated Loading.—The maximum positive shear in a panel will occur when the heavy equivalent load P is over the panel point immediately to the right of the panel under consideration, and the light equivalent load Q is to the right of the heavily loaded one. The theoretical soundness of this assertion may be demonstrated by applying the criterion for maximum shear in any panel of a truss having equal panels, that is Eq. (9), Art. 141,

$$G_2 = \frac{G}{n} \tag{7}$$

TABLE 17

DEAD-LOAD MOMENTS AND CHORD MEMBER STRESSES

+ = tension − = compression

Panel Point	D. L. Moment, M, Ft.-Lb.	Member	Stress in Member $= M/7.0$, Lb.
U_1	$34{,}560 \times 6 = 207{,}360$	L_0L_2	+29,620
L_2	$34{,}560 \times 12 = 414{,}720$	U_1U_3	−59,250
U_3	$34{,}560 \times 18 - 17{,}280 \times 6 = 518{,}400$	L_2L_4	+74,060
L_4	$34{,}560 \times 24 - 17{,}280 \times 12 = 622{,}080$	U_3U_5	−88,870
U_5	$34{,}560 \times 30 - 17{,}280\,(18 + 6) = 622{,}080$	L_4L_6	+88,870

The shears in the successive panels due to the two trucks and the web member stresses resulting therefrom are as given in Table 18.

TABLE 18

MAXIMUM LIVE-LOAD SHEARS AND WEB MEMBER STRESSES DUE TO TWO TRUCKS ABREAST

+ = tension − = compression

Panel	Maximum Concentrated L. L. Shear, V, Lb.	Member	Stress in Member $= 1.317\ V$, Lb.
L_0L_2	$27{,}100 \times \frac{4}{5} + 6775 \times \frac{34}{60} = 25{,}520$	L_0U_1	−33,600
		U_1L_2	+33,600
L_2L_4	$27{,}100 \times \frac{3}{5} + 6775 \times \frac{22}{60} = 18{,}740$	L_2U_3	−24,700
		U_3L_4	+24,700
L_4L_6	$27{,}100 \times \frac{2}{5} + 6775 \times \frac{10}{60} = 11{,}970$	L_4U_5	−15,760
		U_5L_6	+15,760
L_6L_8	$27{,}100 \times \frac{1}{5} = 5{,}420$	L_6U_7	− 7,140
		U_7L_8	+ 7,140

Moments and Chord Stresses from Concentrated Loading.—For maximum moments at the loaded chord panel points of a truss with equal panels, the criterion that must be satisfied is, Eq. (2), Art. 141,

$$\frac{G_1}{m} = \frac{G}{n} \tag{8}$$

For maximum moments at the unloaded chord panel points U_1, U_3 and U_5,

the applicable criterion, as applied to a Warren truss made up of isosceles triangles, is, Eq. (3), Art. 141,

$$\frac{G_1 + \frac{1}{2} G_2}{l_1} = \frac{G}{l} \tag{9}$$

Application of Eq. (8) shows that a maximum moment may be obtained at panel point L_2 by placing either the load P at L_2 with the load Q on the long segment of the span, or load Q at L_2 and load P on the long segment.

In making the test for maximum under the first placement, move the load P over the panel point L_2 from right to left with load Q following. G_1/m, from Eq. (8), then changes gradually from zero to $27{,}100/1 = 27{,}100$. Meanwhile, G/n has continued to be $33{,}875/5 = 6775$. At some stage of the movement these two fractions were evidently equal, and a maximum moment was attained.

The equality required by Eq. (8) may be satisfied also by the second placement, but obviously the maximum effect will be obtained with the first one.

The left-hand reaction for this position of the equivalent loads is, taking moments about the right-hand support,

$$R_L = \frac{27{,}100 \times 48 + 6775 \times 34}{60} = 25{,}520 \text{ lb.}$$

Hence the moment at L_2 is $25{,}520 \times 12 = 306{,}200$ ft.-lb.

By applying Eq. (8) the maximum moment at L_4 is found to occur when the load P is at L_4 and the load Q is 14 ft. to the right of L_4. The left-hand reaction is

$$R_L = \frac{27{,}100 \times 36 + 6775 \times 22}{60} = 18{,}740 \text{ lb.}$$

and the moment at L_4 is $18{,}740 \times 24 = 449{,}800$ ft.-lb.

Next, consider the maximum moments at the upper chord panel points.

For the maximum moment at U_1, try the equivalent load P at L_2 with the load Q 14 ft. to the right of L_2. Apply Eq. (9) above. G_1, being the load to the left of the floor panel opposite which the moment centre U_1 lies, is zero. G_2 changes gradually from zero to 27,100 lb. G, which is the total load on the truss, $= 33{,}875$ lb., and $l_1 = 6$ ft. Hence, when the load P moves to the left over the point L_2, $(G_1 + \frac{1}{2} G_2)/l_1$ changes gradually from zero to $\frac{1}{2} \times 27{,}100/6 = 2258$. Throughout this movement $G/l = 33{,}875/60 = 565$. At one stage of the motion, therefore, the criterion of Eq. (9) must have been satisfied and a maximum moment produced.

The left-hand reaction for this position of the concentrated loads has already been found above to be 25,520 lb. Hence the moment at U_1 is $25{,}520 \times 6 = 153{,}100$ ft.-lb.

If the load P is placed initially wholly in the panel L_0L_2, further movement to the left does not create a proper condition for a maximum moment at U_1. Neither will this load wholly in the panel L_2L_4 satisfy Eq. (9) above. Reversal of the position of the loads would obviously not be so serious for the truss.

Next, consider the conditions for maximum moment at U_3. For this point $l_1 = 18$ ft. Move the 27,100-lb. load over L_4 to the left, with the 6775-lb. load following. $(G_1 + \frac{1}{2} G_2)/l_1$ changes gradually from zero to 753, while G/l retains its value of 565. This gives the maximum.

For this position of the loading, the left-hand reaction, as has already been found, is 18,770 lb. The moment at U_3 is, then $18,740 \times 18 = 337,300$ ft.-lb.

For the maximum moment at U_5, try the 27,100-lb. load P at L_6 with the 6775-lb. load Q to the right of P. Application of Eq. (9) shows no maximum to exist.

If, however, the load is backed still further across the span till the 27,100-lb. load is at L_4 and the 6775-lb. load is to the right, on the long segment, Eq. (9) is also satisfied, and a maximum exists. For this, the left-hand reaction is, as has already been found, 18,740 lb. The moment at U_5 is $18,740 \times 30 - 27,100 \times 6 = 399,600$ ft.-lb.

The stresses in the chord members due to the live load are indicated in Table 19. These are found by dividing the moments at the opposite panel points ascertained above by the depth of the truss, that is, 7 ft.

TABLE 19

MAXIMUM LIVE-LOAD MOMENTS AND CHORD MEMBER STRESSES DUE TO TWO TRUCKS ABREAST

+ = tension — = compression

Panel Point	Maximum L. L. Moment, M, Ft.-Lb.	Member	Stress in Member $= M \div 7.0$, Lb.
U_1	153,100	L_0L_2	+21,860
L_2	306,200	U_1U_3	−43,710
U_3	337,300	L_2L_4	+48,290
L_4	449,800	U_3U_5	−64,290
U_5	399,600	L_4L_6	+57,210

163. Impact Allowances.—Impact allowances representing fractions of the live-load stress as determined by Eq. (1) of Art. 154(8) are added to the dead- and live-load stress to form the total stresses indicated in Fig. 112(a). The loaded distance S is the minimum distance that the loading has to travel in a forward direction to arrive in position to produce the maximum live-load stress. The values of this distance and of the resulting impact fraction are indicated for the various members of the truss in Table 20. The load Q is in every case to the right of the load P.

Members subjected to a live-load stress of either tension or compression are proportioned for a further increment, in accordance with Art. 154(13).

164. Proportioning of Truss Members.—Truss members will be proportioned to withstand the aggregate computed stresses indicated in Fig. 112 at the permissible stresses set forth in the specification, Art. 154(15).

TABLE 20

MINIMUM VALUES OF LOADED DISTANCE S TO BE USED IN IMPACT FORMULA AND CORRESPONDING VALUES OF THE IMPACT FRACTION

Member	Minimum S, Ft.	Load P at Panel Point	Impact Fraction, I/L
L_0U_1, U_1L_2	26	L_2	0.33
L_2U_3, U_3L_4	38	L_4	0.31
L_4U_5, U_5L_6	50	L_6	0.29
L_6U_7, U_7L_8	62	L_8	0.27
L_0L_2, U_1U_3	26	L_2	0.33
L_2L_4, U_3U_5	38	L_4	0.31
L_4L_6	38	L_4	0.31

It is assumed that, both in proportioning and detailing, the requirements of Art. 154(14) respecting secondary stresses will be so observed that the computation of such stresses will be unnecessary. Sections will be selected so that the depth of a diagonal in the plane of the truss will not exceed 110.6/10 = 11.1 in., and the depths of top and bottom chord sections will not exceed 72/10 = 7.2 and 144/10 = 14.4 in., respectively.

End Post, L_0U_1.—The total stress in the end post, made up as shown in Fig. 112(*a*), is 90,200 lb. compression.

Assume as section two angles $5 \times 3 \times \frac{3}{8}$ in., $10\frac{1}{2}$ in. back to back, and a cover plate $10 \times \frac{5}{16}$ in., having a total area of 8.85 sq. in., and arranged as shown in Fig. 116. Main material as thin as $\frac{5}{16}$ in. thick might be used, according to Art. 154(5), but the requirement of Art. 154(19) that the thickness of the outstanding legs of angles in main compression members shall not be less than $\frac{1}{12}$ of the unsupported length (from the beginning of the fillet) necessitates $\frac{3}{8}$-in. material. A plate as wide as 10 in. is chosen to give a radius of gyration about the vertical axis at least twice that about a horizontal axis, thus satisfying Art. 154(20).

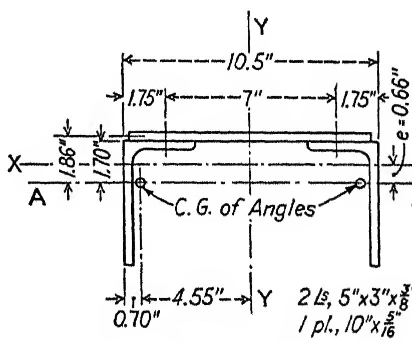


FIG. 116. Section of End Post, L_0U_1.

The distance between rivet lines in the cover plate is 7.0/0.3125 = 22.4 times the thickness of the plate, which is well within the upper limit of 40 set by Art. 154(19) for full effectiveness as a component of the section.

By taking area moments about the gravity axis A–A of the angles, which is

1.70 in. from the back of the shorter legs, the distance e of the neutral axis X–X of the section from the axis A–A is found to be 0.66 in.

The moment of inertia of the whole section about its horizontal neutral axis X–X is

$$I_x = I_A - Ae^2$$

in which I_A is the moment of inertia of the whole section about the horizontal gravity axis of the angles, A is the area of the whole section and e is as already defined. If the moment of inertia of the cover plate about its own gravity axis parallel to A–A be neglected (it is relatively a very small quantity), then

$$I_A = (2 \times 7.37) + (10 \times 0.3125 \times (1.86)^2) = 25.54 \text{ in.}^4$$

and

$$I_x = 25.54 - 8.85 \times (0.66)^2 = 21.68 \text{ in.}^4$$

Radius of gyration about the axis X–X is $r_x = (21.68/8.85)^{1/2} = 1.56$ in.

According to Art. 154(18), the minimum allowable radius of gyration of the end post, for which $h = 72$ in., is

$$r = \frac{(2 \times 110.6) + 72}{350} = 0.84$$

The radius of gyration for the assumed section is therefore adequate.

About the axis Y–Y the moment of inertia I_y is found to be 148.52 in.^{4}, and the radius of gyration r_y is found to be 4.09 in.

The maximum slenderness ratio, l/r, hence occurs in the plane of the truss, and since the length of the end post, centre to centre, is 9.2195 ft. or 110.6 in., it amounts to $110.6/1.56 = 71$.

Applying the column formula of Art. 154(15), the permissible compressive stress is $p_c = 17{,}000 - 60 \times 71 = 12{,}740$ lb. per sq. in.

Area required $= 90{,}200/12{,}740 = 7.07$ sq. in. The provided area, 8.85 sq. in., is ample. A smaller section, such, for example, as using two $4 \times 3 \times \frac{5}{16}$-in. angles and a $10 \times \frac{5}{16}$-in. cover plate will not do.

Top Chord, $U_1U_2U_3$.—The total stress is 117,390 lb.

Assume the same section as for the end post with $r_x = 1.56$ in. and $r_y = 4.09$ in.

According to Art. 154(18), the radius of gyration might be as small as

$$r = \frac{(2 \times 72) + 72}{350} = 0.62 \text{ in.}$$

As the chord is supported in the plane of the truss at points 6 ft. apart, the slenderness ratio in this plane is $72/1.56 = 46$.

In a plane at right angles to the plane of the truss, adequate support is

afforded only at points where the rigid verticals are attached to the chord, that is at U_2, U_4, U_6 and U_8, Fig. 112. At U_3, U_5 and U_7 the support is negligible. Hence, l/r at right angles to the plane of the truss is $144/4.09 = 35$.

Permissible compressive stress $= p_c = 17{,}000 - 60 \times 46 = 14{,}240$, with a maximum of 14,000 lb. per sq. in.

Area required $= 117{,}390/14{,}000 = 8.39$ sq. in. The provided area, 8.85 sq. in., is ample.

Top Chord, $U_3U_4U_5$.—The total stress is 173,060 lb. Assuming for section two angles $6 \times 4 \times \frac{1}{2}$ in. and one plate $10 \times \frac{5}{16}$ in., the area is 12.63 sq. in. and r_x is 1.90 in. The determining l/r is $72/1.90 = 38$, and hence the permissible compressive stress $p_c = 17{,}000 - 60 \times 38 = 14{,}720$, with a maximum of 14,000 lb. per sq. in.

Area required $= 173{,}060/14{,}000 = 12.36$ sq. in., which is within the area provided.

Bottom Chord, L_0L_2.—The total tension is 58,700 lb. and the required net area is $58{,}700/18{,}000 = 3.26$ sq. in.

Assume two $4 \times 3 \times \frac{3}{8}$-in. angles, arranged trough-like as shown in Fig. 112(a), with the 4-in. legs vertical and with battens or tie plates on the 3-in. legs. Allowing, at the end connections, for one line of rivets in each leg, on standard gauges, the distance between these rivet lines, if the angle were developed, would be $3\frac{7}{8}$ in. If the pitch of rivets at the ends were 3 in. in each leg—a reasonable value—then, for the $1\frac{1}{2}$-in. stagger, the diagonal distance between rivets would be 4.15 in. Since this is less than 40% in excess of the distance between gauge lines, two rivet holes will need to be deducted from each angle to satisfy the specification, Art. 154(16). The net area of the section assumed is, therefore, $4.96 - 4 \times 0.875 \times 0.375 = 3.65$ sq. in. According to Art. 154(17) this net area is fully effective and the section is consequently adequate. Art. 154(18) is satisfied, in that the minimum radius of gyration of the gross section is much more than the length of the member divided by 250.

Bottom Chord, L_2L_4.—The maximum tension is 137,310 lb. Area required $= 137{,}310/18{,}000 = 7.63$ sq. in. Use two angles, $6 \times 3\frac{1}{2} \times \frac{1}{2}$ in., having a gross area of 9.00 sq. in., and, deducting two holes, a net area of 8.12 sq. in. The rivet spacing near the edges of the gussets may be arranged so as to make the deduction of only two holes necessary.

Bottom Chord, L_4L_6.—Total tension 163,830 lb. Required area $= 163{,}830/18{,}000 = 9.10$ sq. in. Use two angles, $6 \times 4 \times \frac{9}{16}$ in., having a gross area of 10.62 sq. in., and, deducting two holes, a net area of 9.63 sq. in.

Diagonal U_1L_2.—Total tension $= 90{,}200$ lb. Material provided as shown in Fig. 112(a), two angles, $5 \times 3 \times \frac{7}{16}$ in. On account of lug angles being used, four holes must be deducted.

Diagonal L_2U_3.—Examination of Table 18 shows that this member may have a maximum live-load compression of 24,700 lb., or a maximum live-load tension of 7140 lb. (U_7L_8).

According to the specification, Art. 154(13), this member, having a maximum static live-load stress of the same sign as the dead-load stress, and another one

of opposite sign, must be proportioned for the more serious of the following two combinations:

Combination (a)	
Dead load	$-$ 22,750
Live load	$-$ 24,700
Impact, $-0.31 \times 24{,}700$	$-$ 7,660
	$-$ 55,110
Combination (b)	
0.70 of dead load	$-$ 15,930
Live load	$+$ 7,140
Impact, $+0.27 \times 7140$	$+$ 1,930
	$-$ 6,860

To each of these must be added an increment of one-half of the smaller of the two combinations, giving

Combination (a') $-$ (55,110 + 6860/2) = $-$ 58,540
Combination (b') $-$ (6860 + 6860/2) = $-$ 10,290

Obviously the member must be proportioned for combination (a'), that is 58,540 lb., as indicated in Fig. 112(a). For this, two angles $5 \times 3 \times \frac{3}{8}$ in. will be required because of the limit on the thickness of outstanding legs of compression angles.

Diagonal U_3L_4.—The maximum tension in this member is derived in the same manner as the maximum compression L_2U_3, and amounts to 58,540 lb. Two angles, $4 \times 3\frac{1}{2} \times \frac{5}{16}$ in. with four holes out (lug angles being used) will be sufficient.

Diagonal L_4U_5.—In this diagonal the dead-load stress is zero and the live-load stress, according to Table 18, may be $-15{,}760$ or $+15{,}760$ lb. (U_5L_6). Consequently, in applying the clause of the specification, Art. 154(13), respecting reversal of stress, combinations (a') and (b') give totals of the same magnitude but of opposite sign, thus:

Combination (a)	
Dead load	0
Live load	$-$ 15,760
Impact, $-0.29 \times 15{,}760$	$-$ 4,570
	$-$ 20,330
Combination (b)	
Dead load	0
Live load	$+$ 15,760
Impact, $+0.29 \times 15{,}760$	$+$ 4,570
	$+$ 20,330

Combination (a') $-$ (20,330 + 20,330/2) = $-$ 30,495
Combination (b') + (20,330 + 20,330/2) = + 30,495

In proportioning the member, the compressive stress governs. Two angles $3 \times 2\frac{1}{2} \times \frac{5}{16}$ in. will be sufficient, as indicated in Fig. 112(*a*). The radius of gyration exceeds the permissible minimum of 0.84 for compression diagonals.

Verticals U_2L_2, U_4L_4, *etc.*—These members afford simultaneous support to the top chord both in a vertical and a horizontal plane, and also serve as places of attachment for the handrail.

In resisting any tendency of the chord to move vertically, a vertical needs to exert only a small axial force of tension or compression, which by extension of the application of Art. 154(20), will be assumed as 2% of the maximum axial compression in the chord at the point in question. This force, for U_4L_4 or U_6L_6, would amount to $\pm 0.02 \times 173{,}060 = 3460$ lb.

The duty of preventing horizontal lateral movement of the chord is a more serious one for the vertical, since, according to Art. 154(20), it must be able to resist a lateral force at the chord not less than 2% of the maximum axial stress at that point. For U_4L_4 or U_6L_6 this is 3460 lb.

Consider that the critical section of the vertical is at a point 58 in. below the centre of gravity of the top chord, or about 4 in. above the top of the floor beam. Below this level the vertical is reinforced by the connection angles of the floor beam which are carried up along the inner face of the vertical, the top flange of the floor beam being cut away to permit this. The verticals U_4L_4 and U_6L_6 will thus each have to resist a moment, normal to the plane of the truss, of $3460 \times 58 = 200{,}700$ in.-lb.

Since at the point of maximum moment, which is the section of maximum combined stress, the vertical is restrained from lateral buckling, the permissible compressive stress in flexure is 18,000 lb. per sq. in. of gross section, and the permissible tensile stress in flexure is 18,000 lb. per sq. in. of net section, the latter governs.

In order to facilitate the connection of the floor beams to them, these verticals will be assumed as of two $4 \times 3 \times \frac{5}{16}$-in. angles and a $10 \times \frac{5}{16}$-in. plate, arranged as shown in Fig. 112(*a*), for which the gross area is 7.31 sq. in., and the net area, deducting three $\frac{7}{8}$-in. holes, is 6.49 sq. in. The radius of gyration, normal to the plane of the truss, taking account of three rivet holes, is 3.89 in.

The total net area required in a vertical U_4L_4 or U_6L_6 to withstand 3460-lb. tension and a simultaneous moment of 200,700 in.-lb. is, therefore,

$$A_t + A_f = \frac{3460}{18{,}000} + \frac{200{,}700 \times 5.25}{(3.89)^2 \times 18{,}000} = 4.06 \text{ sq. in.}$$

The provided net area, 6.49 sq. in., cannot be reduced.

For simplicity, all verticals will be made of the same section.

165. Bearings.—The maximum load on an end bearing of a truss will arise when two trucks abreast, headed in the same direction, are as close to the near curb as possible, with their rear axles over the end floor beam. Utilizing the so-called equivalent loads, $P = 27{,}100$ lb., and $Q = 6775$ lb., of Art. 162 and Fig. 115, the live-load concentration is readily found. Two and one-

half panels of dead load are borne by the bearing. The total reaction is, therefore,

D.L. $2\frac{1}{2} \times 17{,}280$	43,200 lb.
L.L. $27{,}100 \times 6775 \times 46/60$	32,290 lb.
I. $0.36 \times 32{,}290$	11,630 lb.
	87,120 lb.

Area required in bearing on the concrete abutments = $87{,}120/600 = 145.2$ sq. in. Bearing and bed plates are each made 14×15 in., as indicated in Fig. 112, the 14-in. dimension being in the direction of the span of the bridge. Reasons of detail make plates of this size necessary.

166. Handrail.—The handrail will be of the latticed type, made up as shown in Fig. 112(*a*). The top of the upper rail must not be less than 4 ft. above the level of the concrete floor adjacent to the curb (Art. 154(3)), and this rail must withstand a horizontal force of 150 lb. per lin. ft. and a vertical, but not simultaneous, force of 100 lb. per lin. ft.

Assuming lateral support at intervals of 6 ft., the lateral bending moment on the upper rail is $\frac{1}{8} \times 150 \times (6)^2 \times 12 = 8100$ in.-lb. Section modulus required = $8100/18{,}000 = 0.45$ in.^{3} Under the vertical loading the required section modulus is 0.30 in.^{3} For a $3 \times 2\frac{1}{2} \times \frac{1}{4}$-in. angle with the 3-in. leg horizontal, the section moduli provided in the vertical and horizontal planes are 0.56 and 0.40 in.^{3} respectively. This is the minimum practicable section to accommodate the necessary details.

The bottom rail is made of a single $2\frac{1}{2} \times 2\frac{1}{2} \times \frac{1}{4}$-in. angle, the vertical straps are $2 \times \frac{1}{4}$ in., and the diagonals are single $1\frac{1}{4} \times 1\frac{1}{4} \times \frac{3}{16}$-in. angles in one direction and $1\frac{1}{4} \times \frac{3}{16}$-in. flats in the other. As required by Art. 154(3), the perpendicular distance between lattice members does not exceed 6 in.

Part II

TIMBER STRUCTURES

CHAPTER XII

TENSION MEMBERS AND TENSION MEMBER DETAILS

167. Design of Timber Tension Member.—Suggest a make-up for a long-leaf yellow pine tension member to carry a total tension of 70,000 lb., using two 4-in. planks dressed on all faces, and assuming that the splices are of the plain wooden fish plate type. Permissible tensile stress, p_t = 1600 lb. per sq. in. of net section.

Required net area = 70,000/1600 = 43.7 sq. in.

Experience shows that in order to provide for the reduction of area brought about by this type of splice, the gross area must be from 50 to 60% greater than the required net area. In this case the gross area should be about $1.55 \times 43.7 = 67.8$ sq. in. Hence a section consisting of two 4×10-in. planks ($3\frac{5}{8} \times 9\frac{1}{2}$ in. dressed size, according to the Southern Pine Manual of Standard Wood Construction), giving a gross dressed section of $2 \times 3.625 \times 9.5 = 68.9$ sq. in., should be sufficient.

Anticipating the splice detail of Art. 168, Fig. 117, allowance should be made for two $1\frac{3}{4}$-in. holes (for $1\frac{5}{8}$-in. bolts) opposite each other in each plank. The net area then becomes $68.9 - 2 \times 1.75 \times 2 \times 3.625 = 43.5$ sq. in., which is sufficient.

168. Design of a Plain Wooden Fish Plate and Filler Splice for Tension Member.—Design a splice of the plain wooden fish plate and filler type for the tension member of Art. 167.

Total tension = 70,000 lb. Member consists of two planks $3\frac{5}{8} \times 9\frac{1}{2}$ in., dressed dimensions, separated by a $2\frac{5}{8}$-in. filler piece.

Permissible stresses on long-leaf yellow pine are to be as follows: Tension, in the direction of the fibres, p_t = 1600 lb. per sq. in. of net area; tension, at right angles to the fibres, p_t' = 100 lb. per sq. in. of net area; bearing compression, parallel to fibres, p_c = 1200 lb. per sq. in.; bearing compression transverse to fibres, p_c' = 350 lb. per sq. in.; shearing, parallel to fibres, p_s = 150 lb. per sq. in.

Permissible flexure on steel bolts, p_f = 24,000 lb. per sq. in.

Assume uniformly distributed pressure on each bolt from each plank through which it passes.

Required net area of fish plates = 70,000/1600 = 43.7 sq. in.

Allowing for $1\frac{5}{8}$-in. bolts ($1\frac{3}{4}$-in. holes) in pairs opposite each other, as indicated in Fig. 117(*a*), the net area may be provided by using two outside fish plates of $2\frac{5}{8} \times 9\frac{1}{2}$ in. dressed dimensions and a filler piece $2\frac{5}{8} \times 9\frac{1}{2}$ in. between the parts being spliced. The net area is then $(3 \times 2.625)\ (9.5 - 2 \times 1.75) = 47.25$ sq. in., which is ample.

Bolt Bearing.—Allowable bearing compressive stress, p_c'', parallel to the fibres, on a cylindrical bolt or pin depends on the ratio, r, of the permissible bearing compression transverse to the fibres to the permissible compression parallel to them. A diagram giving this allowable bearing compression, p_c'', in terms of the permissible bearing compression, p_c, parallel to the fibres on a

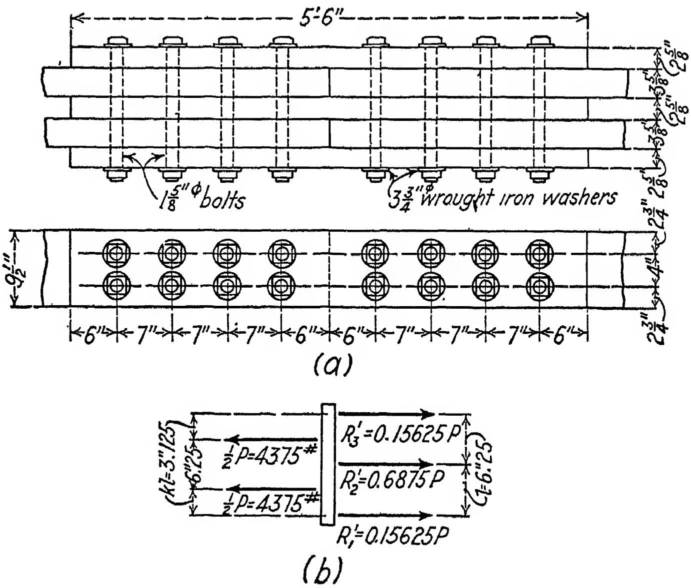


FIG. 117.—Plain Wooden Fish Plate and Filler Splice for Tension.

plane surface is given in Jacoby and Davis's "Timber Design and Construction," Second Edition, p. 96. This may be stated with sufficient accuracy as

$$p_c'' = (0.18 + 1.70\,r)\,p_c$$

In the present problem, $r = p_c'/p_c = 350/1200 = 0.29$, and $p_c'' = (0.18 + 1.70 \times 0.29)1200 = 800$ lb. per sq. in.

Required projected area of bolts $= 70{,}000/800 = 87.5$ sq. in. Using $1\frac{5}{8}$-in. bolts, the number required on each side of the splice, the mimimum bearing width being on the combined thickness of two $3\frac{5}{8}$-in. planks, is $87.5/(1.625 \times 2 \times 3.625) = 7.4$, say 8 bolts.

Bolt Spacing.—Load carried by one bolt is $P = 70{,}000/8 = 8750$ lb.

Since the aggregate thickness of the main planks, or 7.25 in., is less than that of the splicing material, the safe spacing of the bolts will be determined by the

resistance of the former. As there are two shear planes back of each bolt, the *clear*, longitudinal distance between bolt holes should be $8750/2 \times 7.25 \times 150 = 4.02$ in.

An additional length must be provided behind each bolt to provide for the splitting effect, it being assumed that splitting may occur on either longitudinal shear plane.

The magnitude of the splitting force, P', in terms of the longitudinal force, P, applied by a bolt may be taken from the diagram in "Timber Design and Construction," mentioned above, or it may be taken with sufficient accuracy as

$$P' = (0.059 + 0.16\,r)\,P$$

where r is as defined above, and has, for this problem, the value 0.29. Inserting this value, $P' = 0.105\,P = 0.105 \times 8750 = 920$ lb.

Required splitting length in the clear is $920/7.25 \times 100 = 1.27$ in.

Safe spacing of bolts, centre to centre, is, then, $4.02 + 1.27 + 1.63 = 6.92$, say 7 in.

The safe distance from the last bolts to the end of the $3\frac{5}{8}$-in. main planks, may be less than the bolt spacing by one-half the diameter of a bolt, or, $6.92 - 0.81 = 6.11$ in., say 6 in. The same end distance will be used for the fish plates.

Length of Fish Plates.—The length of fish plates and filler piece will be $2(3 \times 7.00 + 2 \times 6) = 66.0$ in.

Flexure in Bolts.—Assuming that the force exerted by each plank on a bolt is applied at the centre of the thickness of the plank, a bolt is in effect a continuous beam of two equal spans with a concentrated load of $P/2 = 8750/2 = 4375$ lb. at the centre of each span. The situation is as shown in Fig. 117(b).

From "Modern Framed Structures," Pt. II, by Johnson, Bryan and Turneaure, the reactions R_1, R_2 and R_3, due to a concentrated load $P/2$ at a distance of kl from the left-hand (in this case, the lower) support are

$$R_1 = \frac{P}{8}(4 - 5\,k + k^3)$$

$$R_2 = \frac{P}{4}(3\,k - k^3)$$

$$R_3 = -\frac{P}{8}(k - k^3)$$

Where two symmetrically placed loads are applied to the bolt, the actual reactions at the three supports are

$$R_1' = R_1 + R_3 = \frac{P}{8}(4 - 6\,k + 2\,k^3)$$

$$R_2' = 2\,R_2 = \frac{P}{2}(3\,k - k^3)$$

$$R_3' = R_1' = \frac{P}{8}(4 - 6\,k + 2\,k^3)$$

Since $k = \frac{1}{2}$, $R_1' = R_3' = 0.15625\,P$, and $R_2' = 0.6875\,P$.

Moment at a concentration $P/2$ is, therefore, $+\,0.15625 \times 8750 \times 3.125 = 4270$ in.-lb., and moment at the centre support is $+\,0.15625 \times 8750 \times 6.25 - 4375 \times 3.125 = -\,5130$ in.-lb.

Since the section modulus, S, of a solid circular section of diameter d is $\pi d^3/32$, it follows that the maximum existing flexural stress on a bolt, which arises at the bolt bearing on the filler piece, is

$$f_f = \frac{M}{S} = \frac{5130}{\pi \times 1.625^3/32} = 12{,}200 \text{ lb. per sq. in.}$$

As the permissible flexural stress, $p_f = 24{,}000$ lb. per sq. in., the bolts are adequate.

As there is only negligible tension in the bolts, light shallow square nuts and wrought-iron washers may be used instead of standard nuts and cast-iron washers.

Edge Distance.—For $1\frac{5}{8}$-in. bolts it is not desirable to have the centre of the bolt hole nearer the side of the timbers than $2\frac{3}{4}$ in., which is the distance shown in Fig. 117(*a*).

169. Design of Tabled Fish Plate Splice for Tension Member.—Design a tabled fish plate splice for a tension member consisting of a solid piece of timber of $5\frac{1}{2} \times 5\frac{1}{2}$ in., dressed dimensions, using two splice plates with one table at each end of each plate, as shown in Fig. 118. Total tension = 17,110 lb.

Permissible stresses on timber (Douglas fir) are as follows: Tension, in the direction of the fibres, $p_t = 1500$ lb. per sq. in. of net area; bearing compression, parallel to fibres, $p_c = 1400$ lb. per sq. in.; shearing, parallel to fibres, $p_s = 140$ lb. per sq. in.

Required net area in the body of each fish plate $= 17{,}110/2 \times 1500 = 5.7$ sq. in. If the fish plates be made of the same nominal depth as the chord, namely, 6 in., or $5\frac{5}{8}$ in. actual (according to the Standards of the Southern Pine Manual), the thickness of each plate in the thinnest part should be $5.7/5.625 = 1.02$ in.

Required bearing area of the notch in each plate against the end of the fibres of the notched member $= 17{,}110/2 \times 1400 = 6.1$ sq. in. The depth of the notch in the plate should be $6.1/5.5 = 1.11$ in.

To satisfy the above two requirements, each fish plate will need to be of 3×6-in. nominal dimension, or $2\frac{5}{8} \times 5\frac{5}{8}$-in. dressed dimensions. By notching it $1\frac{5}{16}$ in., a thickness of $1\frac{5}{16}$ in. is maintained in the body, as compared with the requirements of 1.02 and 1.11 in., respectively.

Required length of table on member being spliced and on each plate, based on the shearing strength parallel to the fibres is $17{,}110/2 \times 5.5 \times 140 = 11.1$ in.

A length of 12 in. for the tables on the fish plates and on the member itself will be adopted, as indicated in Fig. 118. This gives a length of 4 ft. for the fish plates.

Two $\frac{5}{8}$-in. bolts spaced 6 in. apart, with wrought-iron washers, will be placed through each table of the fish plates and through the member. These bolts

carry no calculable stress, but prevent lateral displacement of the plates due to the eccentric application of loading on them.

170. Design of Shear-Pin Splice for Tension Member.—Design a shear pin splice of the form shown in Fig. 119 for the $5\frac{1}{2} \times 5\frac{1}{2}$-in. tension member of Art. 169, using for shear pins $1\frac{1}{2}$-in. extra strong steel pipe with outside diameter of 1.90 in. Total tension, 17,110 lb.

Permissible stresses on Douglas fir: Tension, parallel to fibres, $p_t = 1500$ lb. per sq. in.; bearing compression transverse to fibres, 350 lb. per sq. in.; shearing, parallel to fibres, 140 lb. per sq. in.; compression on shear pins of extra strong steel pipe = 800 lb. per lin. in.; tension on net section of steel bolts, 16,000 lb. per sq. in.

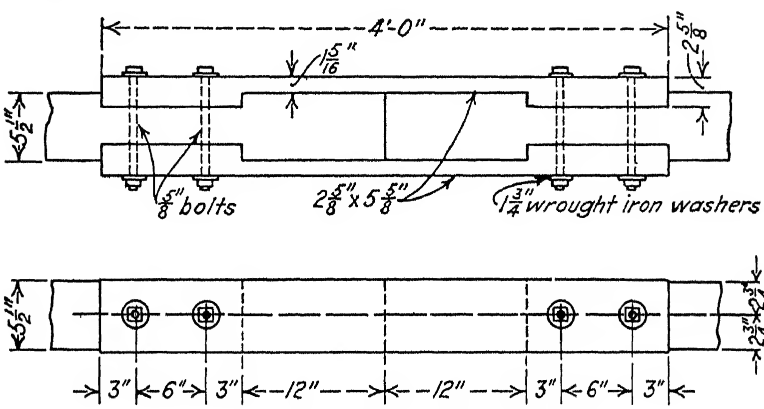


FIG. 118.—Tabled Fish Plate Splice for Tension.

Bolts of a total capacity in tension equal to 50% of the load on the joint must be provided on each side of the splice.

Required net area of each fish plate, as for the problem of Art. 169 = 5.7 sq. in. Using a width of $5\frac{5}{8}$ in., the thickness of each plate at a shear pin should be 1.02 in. Using $2\frac{5}{8} \times 5\frac{5}{8}$-in. plates, and deducting half the diameter of a 2-in. hole for the shear pin, or $\frac{1}{2} \times 2.0 = 1.0$ in., the net thickness provided is $2.625 - 1.0 = 1.625$ in., which is ample.

Number of 1.90-in. pins 5.50 in. long required on each side of splice $= 17{,}110/5.5 \times 800 = 3.9$, say 4.

Tension to be provided for in bolts on one side of splice $= 0.5 \times 17{,}110 = 8550$ lb. One $\frac{5}{8}$-in. bolt with an area of 0.202 sq. in. at root of thread will develop $0.202 \times 16{,}000 = 3230$ lb. Required number of bolts $= 8550/3230 = 2.7$, or 3.

In order to guard against the shearing of the timber back of a shear pin, the spacing of shear pins should be not less than $5.5 \times 800/5.5 \times 140 = 5.7$ in. centre to centre. A spacing of 6 in. will be adopted, and no pin will be nearer the splice than 6 in. The three bolts will be placed as shown in Fig. 119, being symmetrically located with respect to the shear pins.

Area of washer necessary to develop 3230 lb., the value of a bolt, is determined by the permissible bearing on timber perpendicular to the grain and is 3230/350 = 9.2 sq. in. Use plate washers $3 \times \frac{3}{8} \times 3$ in., giving an area of 9.0 sq. in., which is sufficiently close to the requirement.

The 2-in. holes for the shear pins should be bored after the fish plates are bolted in position on the member being spliced.

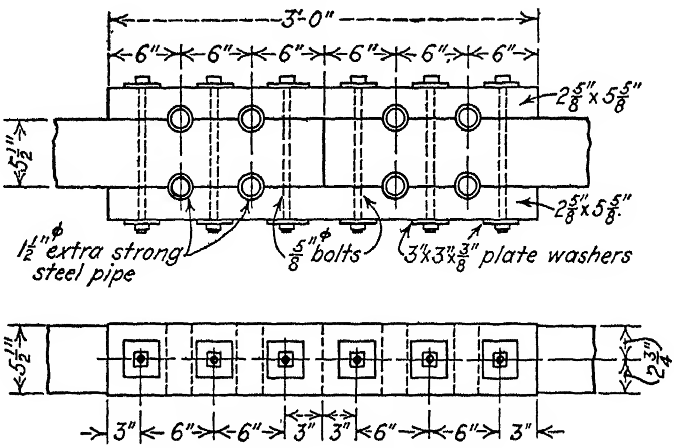


FIG. 119.—Shear Pin Splice for Tension.

171. Exercise Problems on Timber Tension Members and Their Details.—The following exercise problems are based on the problems solved in this chapter. See Appendix I for the answers.

(1) Assuming a loss of 50% of section due to end details, what single section of dressed commercial Douglas fir, not less than $9\frac{1}{2}$ in. deep, will carry a total tension of 35,000 lb.? $p_t = 1500$ lb. per sq. in.

(2) If in the problem of Art. 168, $1\frac{1}{4}$-in. bolts were used instead of $1\frac{5}{8}$-in. bolts, how many would be required, having regard to bearing and flexure?

(3) Assuming the timber sections of the problems of Art. 168 to remain as they were, how far apart should $1\frac{1}{4}$-in. bolts be placed, centre to centre, theoretically?

(4) If in the problem of Art. 169 each fish plate is allowed to have two tables on each side of the splice, what should be the depth of notching and the length of each table?

(5) What is the safe strength of the tension member of Art. 170 based on the main material and not on the splicing?

CHAPTER XIII

COMPRESSION MEMBERS AND COMPRESSION MEMBER DETAILS

172. Design of Timber Column of Intermediate Length.—A square long-leaf yellow pine column 15 ft. high between lateral supports is to be provided to carry a centric axial load of 150,000 lb. Recommend a size, using dressed dimensions. Permissible compressive stress, $p_c = 1300 - l^4/600\, d^4$ lb. per sq. in., where l and d are unsupported length and least lateral dimension, respectively. Neglect the effect of lateral buckling on the intensity of the existing stress.

Assume a column of 12 × 12-in. nominal dimensions, or $11\frac{1}{2} \times 11\frac{1}{2}$ in., dressed dimensions. Ratio $l/d = 180/11.5 = 15.7$.

Permissible stress $p_c = 1300 - (15.7)^4/600 = 1200$ lb. per sq. in.

Required area of post, Eq. (2), Art. 36, $= 150{,}000/1200 = 12.5$ sq. in.

As the 12 × 12-in. post gives a net area of $11.5 \times 11.5 = 132.2$ sq. in., it is sufficient. The next commercial size, a 10 × 10-in. post would not be strong enough.

173. Design of Eccentrically Loaded Column of Intermediate Length.—A square Douglas fir column, dressed on four sides, is provided with cast-iron base and cap carrying unequal girder reactions on two sides and a centric column load from above. See Fig. 120. It supports a 35,000-lb. centric load from an upper column section; a 15,000-lb. girder load 7 in. from the centre of the column; and a 10,000-lb. girder load 7 in. from the centre on the other side.

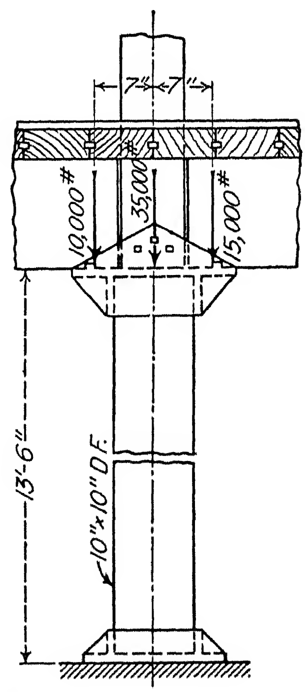


Fig. 120.—Eccentrically Loaded Column.

Permissible compressive stress: For l/d not more than 15, the maximum stress is 1000 lb. per sq. in.; for l/d over 15, $p_c = 1300 - 20\, l/d$. (l = unsupported length of column, and d = least lateral dimension.)

Neglect the effect of the column deflection in computing flexural stress, and neglect the column weight.

Total axial load $= 35{,}000 + 15{,}000 + 10{,}000 = 60{,}000$ lb.

The eccentricity may be found by taking moments of loads about the 35,000-lb. load, and counting measurements to the right positive and those to the left negative, it is $e = (35{,}000 \times 0 + 15{,}000 \times 7 - 10{,}000 \times 7)/60{,}000 = +\ 0.58$ in.

Moment of eccentricity $= 60{,}000 \times 0.58 = 34{,}800$ in.-lb.

Assuming a 10×10-in. post, which when dressed on four sides will be $9\frac{1}{2} \times 9\frac{1}{2}$ in., the area is 90.25 sq. in. and the moment of inertia is 678.8 in.^{4}

Maximum stress on most highly stressed fibre is, from Eq. (3) of Art. 36

$$f_c + f_f = \frac{60{,}000}{90.25} + \frac{(60{,}000 \times 0.58) \times 4.75}{678.8} = 665 + 243 = 908 \text{ lb. per sq. in.}$$

Taking the length of the column as $13\frac{1}{2}$ ft., or 162 in., $l/d = 162/9.5 = 17$, and the permissible stress $p_c = 1300 - 20 \times 17 = 960$ lb. per sq. in. The section is adequate.

174. Design of Long Timber Column Eccentrically Loaded.—A load of 25,000 lb. applied $\frac{1}{4}$ in. off centre on a short diameter is to be carried by a square spruce column, 18 ft. long between lateral supports. Recommend a size, using dressed dimensions, (*a*) neglecting the effect of lateral buckling on existing stress, and (*b*) including this effect. $p_c = 1100(1 - l/60\,d)$ lb. per sq. in. Modulus of elasticity of spruce timber = 1,250,000 lb. per sq. in.

Assume an 8×8-in. column with $7\frac{1}{2} \times 7\frac{1}{2}$ in. dressed dimensions. Ratio $l/d = 216/7.5 = 28.8$. Area $= 7.5 \times 7.5 = 56.3$ sq. in.

Permissible stress, $p_c = 1100(1 - 216/60 \times 7.5) = 573$ lb. per sq. in.

(*a*) *Neglecting Effect of Buckling.*—Existing stress due to 25,000-lb. load, from Eq. (3), Art. 36, is

$$f_c + f_f = \frac{25{,}000}{56.3} + \frac{(25{,}000 \times 0.25) \times 3.75}{\frac{1}{12} \times (7.5)^4} = 444 + 89 = 533 \text{ lb. per sq. in.}$$

This is within the allowable limit.

(*b*) *Including Effect of Buckling.*—Existing stress according to Eq. (6) of Art. 36, letting $C = 15$, a suitable value for a column with flat ends, is

$$f_c + f_f = \frac{25{,}000}{56.3} + \frac{(25{,}000 \times 0.25) \times 3.75}{\frac{1}{12} \times (7.5)^4 - \dfrac{25{,}000 \times (216)^2}{15 \times 1{,}250{,}000}}$$
$$= 444 + 116 = 560 \text{ lb. per sq. in.}$$

This is considerably below the allowable stress, but the next smaller commercial size of square timber, 6×6 in. ($5\frac{1}{2} \times 5\frac{1}{2}$ in. dressed) would be too small.

Observe that inclusion of the buckling effect raises the total existing stress by only 5% for this comparatively stocky column.

175. Capacity of Rectangular Timber Column.—A long-leaf yellow pine column 12 ft. high between lateral supports and of $7\frac{1}{2} \times 11\frac{1}{2}$ in. cross section (dressed dimensions) is to carry an axial load applied 1 in. off centre on a diam-

eter parallel to the long sides. Find the safe load so applied, neglecting the effect of buckling in augmenting the existing stress. $p_c = p_f = 1300 - 25\ l/d$ lb. per sq. in.

The column will be investigated by using Eq. (4) of Art. 36, involving the computation of required cross-sectional area and comparison of it with the existing area. The load P is the unknown.

The permissible compressive and flexural stress, which is based on the tendency of the post to buckle in the direction of the $7\frac{1}{2}$-in. dimension, is $p_c = p_f = 1300 - 25 \times 144/7.5 = 820$ lb. per sq. in.

Required area, from Eq. (4) of Art. 36, is

$$A_c + A_f = \frac{P}{820} + \frac{(P \times 1.00) \times 5.75}{\frac{1}{12} \times (11.5)^2 \times 820}$$

This may be equated to the provided area, $7.5 \times 11.5 = 86.3$ sq. in., and solving, $P = 46{,}600$ lb.

176. Splice for Timber Column.—A square Douglas fir post 6×6 in. in nominal dimensions, and $5\frac{1}{2} \times 5\frac{1}{2}$ in. dressed, is to be spliced at a point where only axial compression exists. Suggest a splice.

There being no flexure to provide for, the most desirable splice is a simple butt joint, as shown in Fig. 121. The abutting ends of the post transfer the compression without aid from the fish plates. The latter, carrying no stress, may be made of 2×6-in. planks ($1\frac{5}{8} \times 5\frac{5}{8}$ in. dressed), and may be secured by two $\frac{5}{8}$-in. bolts on each side of the splice to hold the two segments of the post together. The bolts, which will be provided with $1\frac{3}{4}$-in. diameter wrought-iron washers, may be spaced 6 in. apart on centres and 3 in. from the ends of timbers, as indicated.

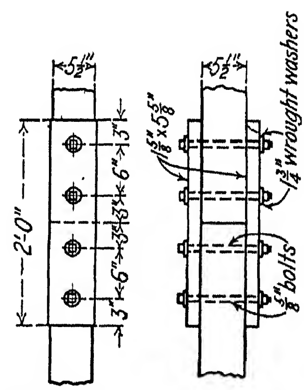


Fig. 121.—Splice for Timber Column.

177. Exercise Problems on Timber Compression Members.—The following exercise problems are based on the problems solved in this chapter. See Appendix I for the answers.

(1) Find the required size of a square dressed Douglas fir column 15 ft. high, carrying a centric axial load of 100,000 lb. $p_c = 1200 - 25l/d$.

(2) A 10×10-in. Douglas fir post 10 ft. high carries an axial load of 64,000 lb. applied on a short diameter and $\frac{3}{4}$ in. off centre. Report on the safety of the column, allowing for dressing and under-run. $p_c = 1300 - 30\ l/d$.

(3) A round timber post, 10-in. diameter and 10 ft. high, carries an axial load of 50,000 lb. applied $\frac{1}{2}$ in. off centre. Report on the safety of the post. $p_c = 1200 - 30\ l/d$.

(4) A 6×6-in. timber post 15 ft. long carries a load of 18,000 lb. applied with an eccentricity of $\frac{1}{2}$ in. on a short diameter. What is the maximum fibre stress arising from all causes, allowing for the effect of deflection. $E = 1{,}500{,}000$.

(5) A 10 × 10-in. timber column 11 ft. 8 in. high supports a 40,000-lb. axial load applied on a short diameter $2\frac{1}{2}$ in. from the centre and a 30,000-lb. axial load applied on the same diameter and $2\frac{1}{2}$ in. on the other side of the centre. Express an opinion as to the safety of the column. $p_c = 1300 - 30l/d$.

(6) Design a square timber post 11 ft. 8 in. high to carry an axial load of 140,000 lb. applied 0.4 in. off centre on a short diameter. $p_c = 1200 - 25l/d$. Use dressed dimensions.

(7) Design a square Douglas fir column 12 ft. 6 in. high for an axial load of 80,000 lb. applied 1 in. off centre on a short diameter. Assume nominal dimensions for the cross section. $p_c = 1200 - 25l/d$.

(8) A 6 × 8-in. timber post 10 ft. long carries an axial load of 30,000 lb. applied on the 8-in. diameter but off centre $\frac{1}{2}$ in. Report on the sufficiency of the post, using nominal dimensions. $p_c = 1200 - 25l/d$.

(9) A timber column 12 ft. long carries an axial load of 100,000 lb. applied $1\frac{1}{2}$ in. off centre. If the section may be rectangular, and not necessarily square, recommend a size. Use nominal dimensions. $p_c = 1300 - 30l/d$.

(10) For two successive stories, each 11 ft. 8 in. high, a column consists of a 10 × 14-in. timber section. At its top and bottom it is amply supported in all directions against buckling, but at the intermediate floor level it is supported only in the direction of the 10-in. dimension. The total load in the upper story of the column is 130,000 lb., and in the lower story it is 154,000 lb., centrically applied in both cases. Express an opinion as to the safety of the column. $p_c = 1400 - 20l/d$.

CHAPTER XIV

BEAMS

178. Sufficiency of Simply Supported Timber Beam.—A simply supported 8×12-in. long-leaf yellow pine beam of 6-ft. span carries a total uniformly distributed load, including its own weight, of 3600 lb. per lin. ft. Report on the safety of the beam, if the permissible stresses in flexure and longitudinal shear are $p_f = 1600$ and $p_s = 150$ lb. per sq. in., respectively. Use nominal dimensions of timber.

Flexure.—Maximum moment, Eq. (1), Art. 72, is $M = \frac{1}{8} \times 3500 \times (6)^2 \times 12 = 194{,}400$ in.-lb. Section modulus of beam $= \frac{1}{6} \times 8 \times (12)^2 = 192$ in.^{3} Maximum existing stress, Eq. (3*a*), Art. 72, is $f_f = 194{,}400/192 = 1010$ lb. per sq. in., which is well within the allowable limit of 1600 lb. per sq. in.

Shear.—Maximum transverse shear, which is at support $= V = \frac{1}{2} \times 6 \times 3600 = 10{,}800$ lb. Maximum intensity of longitudinal (or transverse) shearing stress, is, for a rectangular section

$$f_s = \frac{3}{2} \cdot \frac{V}{A} \tag{1}$$

where V = total shear at the cross section and A = area of cross section. Inserting numerical quantities, $f_s = 3/2 \times 10{,}800/96 = 169$ lb. per sq. in. As the permissible longitudinal shearing stress p_s is only 150 lb. per sq. in., the beam is unsafe in longitudinal shear.

References

Merriman—Mechanics of Materials.
Seely—Resistance of Materials.

179. Design of Restrained Timber Beam.—A timber beam is rigidly fixed to its supports, which are 12 ft. apart in the clear, and carries a total *superimposed* uniformly distributed load of 15,000 lb. Suggest a size for the beam, if the permissible stresses in flexure and in longitudinal shear are $p_f = 1500$ and $p_s = 200$ lb. per sq. in., respectively. Use dressed dimensions. Weight of timber = 4 lb. per ft. B. M.

Flexure.—Neglecting tentatively the weight of the beam itself, the maximum moment, Eq. (1), Art. 72, is $\frac{1}{12} \times 15{,}000 \times 12 \times 12 = 180{,}000$ in.-lb. Required section modulus, Eq. (3*a*), Art. 72, is $S = 180{,}000/1500 = 120$ in.^{3}

A 6×12-in. beam ($5\frac{1}{2} \times 11\frac{1}{2}$ in. in dressed size) has a section modulus of $\frac{1}{6} \times 5.5 \times (11.5)^2 = 121$ in.^{3} Its weight is $(5.5 \times 11.5/12)4 \times 12 = 253$ lb.,

and the additional moment due thereto is $\frac{1}{12} \times 253 \times 12 \times 12 = 3040$ in-lb. Total moment from all causes $= 180{,}000 + 3040 = 183{,}040$ in.-lb. Revised required section modulus is $S = 183{,}040/1500 = 122$. The tentative section is close enough to the requirement for moment.

Shear.—Maximum transverse shear $= \frac{1}{2} \times (15{,}000 + 253) = 7626$ lb. Maximum existing longitudinal (or transverse) shearing stress, Eq. (1), Art. 178, is $f_s = 3/2 \times 7626/(5.5 \times 11.5) = 181$ lb. per sq. in. This is below the permissible stress $p_s = 200$ lb. per sq. in., and hence the assumed section is in all respects satisfactory.

180. Capacity of a Continuous Beam.—A white pine beam 8 in. wide and 12 in. deep is freely supported at two points 20 ft. apart, and is continuous over one support midway between the other two. Assuming the timber to weigh 4 lb. per ft. B. M. and the allowable fibre stress p_f in bending as 800 lb. per sq. in., find the safe load per lineal foot which the beam will carry in flexure in addition to its own weight.

The beam is a continuous one of two equal spans and with zero moment at each of the two end supports. From Eq. (2) of Art. 72 the moment over the centre support, which will be the maximum moment in the span, is

$$M_2 = -\tfrac{1}{8}\,wl^2$$

Applying Eq. (5), Art. 72, to the problem, C being 8, the total safe capacity of the beam in flexure is

$$W = \frac{8\,Sp_f}{l}$$

l being in inches, since S and p_f are based on inch units. The safe load per lineal inch *including* the weight of the beam is

$$w = \frac{8\,Sp_f}{l^2}$$

Now the section modulus S is $\frac{1}{6} \times 8 \times (12)^2 = 192$ in.^{3}, and since each span in *inches* is 120

$$w = \frac{8 \times 192 \times 800}{(120)^2} = 85.3 \text{ lb.}$$

or $85.3 \times 12 = 1024$ lb. per lin. ft.

Subtracting the weight of the beam itself, or $8 \times 4 = 32$ lb. per lin. ft., we have as the safe superimposed capacity, $1024 - 32 = 992$ lb. per lin. ft.

181. Shearing Stress at a Selected Fibre.—Find the intensity of the shearing stress at a depth of 4 in. from the top of an 8×12-in. timber beam, with the 12-in. dimension vertical, if the total shear at the cross section is 10,000 lb.

The general formula for intensity of shearing stress, Eq. (10) of Art. 72, applies; that is

$$f_s = \frac{QV}{It}$$

Q, the statical or area moment of the 4×8-in. horizontal strip above the point in question, and being taken about the neutral axis, is $4 \times 8 \times 4 = 128$ in.^{3} The moment of inertia $I = \frac{1}{12} \times 8 \times (12)^3 = 1152$ in.^{4} The thickness $t = 8$ in. Hence $f_s = 128 \times 10{,}000/(1152 \times 8) = 139$ lb. per sq. in.

182. Sufficiency of a Wooden Floor for Flexure and Deflection.—The flooring in a mill construction building consists of a 4-in. ($3\frac{5}{8}$-in. dressed) spruce under floor and a 1-in. ($\frac{7}{8}$-in. dressed) maple wearing floor. If the span is 10 ft. and the live load is 125 lb. per sq. ft., report on the sufficiency of the under floor for flexure and deflection. Permissible stress in flexure, $p_f = 1000$ lb. per sq. in. $E = 1{,}200{,}000$ lb. per sq. in. for both dead and live load. Maximum allowable deflection = 1/300 of span. Weight of timber = 4 lb. per ft. B. M. Assume under floor as in 10-ft. lengths, with joints at supporting beams.

Loading.—With an under floor of a nominal thickness of 4 in. ($3\frac{5}{8}$ in. actual, according to the Southern Pine Manual), the weight of under and wearing floors combined is $(3.625 + 0.875) \times 4 = 18$ lb. per sq. ft. Adding the live load, the total load is $18 + 125 = 143$ lb. per sq. ft.

Flexure.—As no restraint is possible at the supports of the under floor, owing to the discontinuity of the flooring at these points, the maximum moment in a strip 1 ft. wide is $\frac{1}{8}\,wl^2 = \frac{1}{8} \times 143 \times (10)^2 \times 12 = 21{,}450$ in.-lb. Section modulus of 1-ft. strip is $S = \frac{1}{6} \times 12 \times (3.625)^2 = 26.3$ in.^{3} Existing flexural stress, Eq. (3*a*), Art. 72, $f_f = 21{,}450/26.3 = 817$ lb. per sq. in. This is within the allowable limit of 1000 lb. per sq. in.

Deflection.—Total load on a 1-ft. strip 10 ft. in span $= 10 \times 143 = 1430$ lb. Moment of inertia of 1-ft. strip is $\frac{1}{12} \times 12 \times (3.625)^3 = 47.7$ in.^{4} Maximum deflection, from Eq. (17), Art. 72, is

$$\Delta = \frac{5}{384} \cdot \frac{1430 \times (10 \times 12)^3}{1{,}200{,}000 \times 47.7} = 0.562 \text{ in.}$$

Allowable deflection is only $120/300 = 0.40$ in., and hence the deflection under maximum loading is excessive.

183. Design of Modified Mill Construction Interior Floor Panel.—Design an interior floor panel of beam and girder modified mill construction, as shown in Fig. 122. Beams are not to be spaced closer than 4.5 ft. centre to centre, and to be connected to girders flush on top with beam hangers, and to columns by seating on post caps. No ceiling. Panel, 14×17 ft.

Loads.—Live load, 100 lb. per sq. ft. Weight of Douglas fir and maple, 4 lb. per ft. B. M. Weight of spruce, 3 lb. per ft. B. M.

Materials.—Beams and girders, Douglas fir dressed on four sides; under floor, T. & G. spruce, dressed both sides; wearing floor, 1-in. T. & G. maple, dressed both sides.

Permissible Stresses.—

Spruce:

Flexure, 900 lb. per sq. in.

Douglas fir:

Bearing compression, parallel to fibre, 1300 lb. per sq. in.
Bearing compression, transverse to fibre, 300 lb. per sq. in.
Flexure, 1400 lb. per sq. in.
Shearing, parallel to fibre, 125 lb. per sq. in.

Moduli of Elasticity.—

Spruce, 1,200,000 lb. per sq. in.
Douglas fir, 1,500,000 lb. per sq. in.

Assumptions.—Deflection of flooring, beams or girders not to exceed $\frac{1}{250}$ of span. Dead loads to be increased by $33\frac{1}{3}\%$ for deflection calculations. For reasons of fire resistance, dressed under floor must not be thinner than $2\frac{1}{8}$ in.

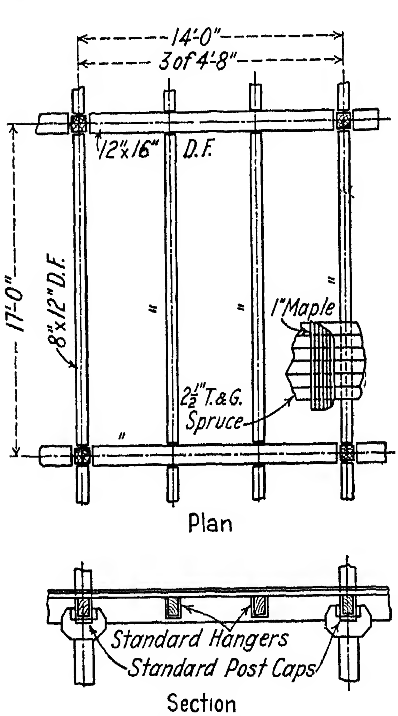


FIG. 122.—Modified Mill Construction Interior Floor Panel.

Layout.—To keep the girder depth down, run the girders in the 14-ft. direction. Place the beams at the third-points and at the columns. A beam spacing of 4.67 ft. is above the lower limit of 4.5 ft. specified.

Flooring Loads.—Live load = 100 lb. per sq. ft. Assume a $2\frac{1}{2}$-in. under floor. When dressed it is $2\frac{1}{8}$ in. thick, and its weight per sq. ft. $= 2.13 \times 3 = 6.4$ lb. Weight of maple floor, of $\frac{3}{4}$-in. net thickness $= 0.75 \times 4 = 3$ lb. per sq. ft. Total dead load of flooring $6.4 + 3 = 9.4$ lb. per sq. ft. Total load on flooring $= 9.4 + 100 = 109.4$ lb. per sq. ft.

Flooring Flexure.—Since the flooring will extend over several 4.67-ft. panels, the moment will be taken at only $\frac{8}{10}$ that for a simple beam.

Maximum moment $= 0.8 \times \frac{1}{8} \times 109.4 \times (4.67)^2 \times 12 = 2860$ in.-lb.

Required section modulus $= 2860/900 = 3.18$ in.^{3}

Section modulus provided $= \frac{1}{6} \times 12 \times (2.13)^2 = 9.08$ in.^{3} This is excessive, but the thickness is the minimum allowable under the specification.

Flooring Shear.—For flooring, longitudinal shearing stresses are unimportant.

Flooring Deflection.—Deflection must not exceed $\frac{1}{250}$ of the span or $56/250 = 0.22$ in.

Total deflection load on strip 12 in. wide and of 4.67-ft., or 56-in., span, increasing the dead load by $33\frac{1}{3}\%$, is $(9.4 \times 1.33 + 100)4.67 = 525$ lb.

Maximum deflection, from Eq. (17), Art. 72, is

$$\Delta = \frac{3.8}{384} \cdot \frac{Wl^3}{EI}$$

where W = total load on span; l = span in inches; E = modulus of elasticity; and I = moment of inertia of strip. Inserting quantities

$$\Delta = \frac{3.8}{384} \cdot \frac{525 \times (56)^3}{1{,}200{,}000 \times \frac{1}{12} \times 12 \times (2.13)^3} = 0.0787 \text{ in.}$$

As the allowable deflection is greater than this, the section is adequate.

Beam Loads.—Each beam, or joist, is of 17-ft. span and carries a strip of floor with its loading 4.67 ft. wide.

Live load per lineal foot of beam = $100 \times 4.67 = 467$ lb.

Load due to flooring = $9.4 \times 4.67 = 44$ lb. per lin. ft. of beam.

Estimated weight of beam itself, assuming a section 8×12 in., or when dressed, $7\frac{1}{2} \times 11\frac{1}{2}$ in. = $(7.5 \times 11.5) \times \frac{4}{12} = 29$ lb. per lin. ft.

Total load carried by a beam = $467 + 44 + 29 = 540$ lb. per lin. ft.

Beam Flexure.—Maximum moment $M = \frac{1}{8}wl^2$, or $\frac{1}{8} \times 540 \times (17)^2 \times 12 = 234{,}000$ in.-lb.

Required section modulus = $234{,}000/1400 = 167.4$ in.^{3}

Section modulus provided = $\frac{1}{6} \times 7.5 \times (11.5)^2 = 165.3$ in.^{3} As this is only slightly below requirement, it is sufficient.

End shear = $540 \times 8.5 = 4600$ lb.

Beam Shear.—Maximum longitudinal shearing stress from Eq. (1) of Art. 178 is

$$f_s = \frac{3}{2} \cdot \frac{V}{A} = \frac{3}{2} \cdot \frac{4600}{7.5 \times 11.5} = 80 \text{ lb. per sq. in.}$$

This is well below the limit of 125 lb. per sq. in.

Beam Deflection.—Deflection must not exceed $17 \times 12/250 = 0.82$ in.

Total deflection load on beam, increasing the dead load by $33\frac{1}{3}\%$ is $\{(44+29) \times 1.33 + 467\} \times 17 = 9600$ lb.

Maximum deflection for 204-in. span is, Eq. (17), Art. 72,

$$\Delta = \frac{5}{384} \cdot \frac{9600 \times (204)^3}{1{,}500{,}000 \times \frac{1}{12} \times 7.5 \times (11.5)^3} = 0.74 \text{ in.}$$

This is within the allowable limit.

Girder Loads.—Each girder is a simple beam of 14-ft. span carrying, in addition to its own weight, equal concentrated loads at the third-points.

Assume a 12×16-in. girder, which when dressed is $11\frac{1}{2} \times 15\frac{1}{2}$ in. and weighs $(11.5 \times 15.5) \times \frac{4}{12} = 59$ lb. per lin. ft.

Each concentrated load on the girder consists of the combined reaction of the two beams framing into the girder from opposite sides. If the adjacent panels are of the same size as the one considered, this is $540 \times 17 = 9200$ lb.

Girder Flexure.—Moment at centre due to weight of girder $= \frac{1}{8} \times 59 \times (14)^2 \times 12 = 17{,}350$ in.-lb.

Maximum moment due to concentrations from beams $= 9200 \times 4.67 \times 12 = 516{,}000$ in.-lb.

Total maximum moment $= 17{,}350 + 516{,}000 = 533{,}350$ in.-lb.

Required section modulus $= 533{,}350/1400 = 381$ in.^{3}

Section modulus of a $11\frac{1}{2} \times 15\frac{1}{2}$-in. girder, is $\frac{1}{6} \times 11.5 \times (15.5)^2 = 460.5$ in.^{3} This is considerably in excess, but the section is the smallest commercially practicable one that will do.

Girder Shear.—End shear $= 59 \times 14/2 + 9200 = 9610$ lb.

Maximum longitudinal shearing stress, Eq. (1) of Art. 178, $= 1.50 \times 9610/(11.5 \times 15.5) = 81$ lb. per sq. in., as compared with a permissible stress of 125 lb. per sq. in.

Girder Deflection.—Maximum allowable deflection $= 14 \times 12/250 = 0.67$ in.

Maximum deflection possible must be calculated by adding the deflection due to the uniformly distributed dead weight of the girder to that due to the two concentrated loads.

Total deflection load due to weight of the girder is the dead load increased by $33\frac{1}{3}\%$, $= 59 \times 1.33 \times 14 = 1100$ lb.

Deflection at centre due to weight of beam is

$$\Delta_1 = \frac{5}{384} \cdot \frac{1100 \times (168)^3}{1{,}500{,}000 \times \frac{1}{12} \times 11.5 \times (15.5)^3} = 0.013 \text{ in.}$$

The dead-load part of each concentrated load is $73 \times 17 = 1240$ lb. Increased by $33\frac{1}{3}\%$, this becomes 1650 lb.

The live-load part is $467 \times 17 = 7950$ lb. Total of each concentrated load for deflection purposes $= 1650 + 7950 = 9600$ lb.

Maximum deflection due to two concentrated loads at the third-points is, Eq. (17) of Art. 72

$$\Delta_2 = \frac{23}{648} \cdot \frac{Pl^3}{EI}$$

where P = each concentrated load in pounds and the other symbols are as defined under "Flooring Deflection." Inserting quantities

$$\Delta_2 = \frac{23}{648} \cdot \frac{9600 \times (168)^3}{1{,}500{,}000 \times \frac{1}{12} \times 11.5 \times (15.5)^3} = 0.302 \text{ in.}$$

Total maximum deflection $= 0.013 + 0.302 = 0.315$ in., as compared with 0.67 in. allowed.

Details.—The general character of the details is shown in Fig. 122. Commercial joist hangers connect beams to girders, except at columns, where steel post caps receive the beam ends.

To ensure an adequate tie across the building in the direction of the beams, deep ridges on the stirrups where the beams rest in them should be provided; or in lieu thereof, $\frac{3}{4}$-in. steel dogs, or $2 \times \frac{1}{4}$-in. straps, properly attached, should

be carried across the top of the girder from each joist or beam to the one opposite.

184. Design of a Timber Rafter.—Design a rectangular timber rafter securely fastened to purlins at each end and inclined at an angle of 30 deg. with the horizontal. See Fig. 123. Span, 10 ft. 6 in., parallel to roof slope. Material, Douglas fir, dressed four sides.

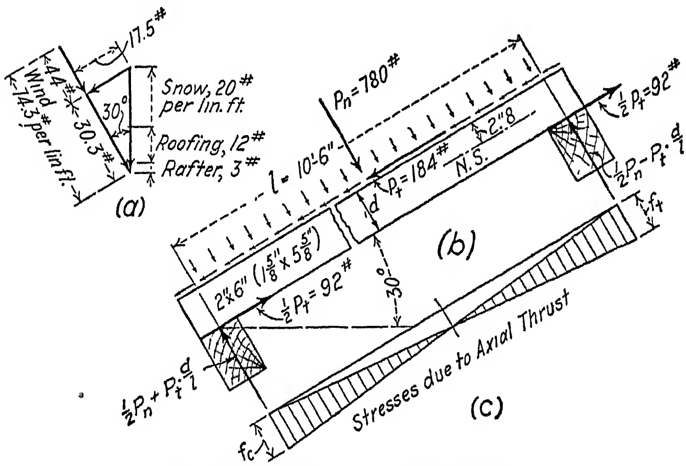


Fig. 123.—Design of a Timber Rafter.

Loads.—Superimposed vertical load of 32 lb. per lin. ft. of rafter, 20 lb. being due to snow and 12 lb. to sheathing and shingles; normal wind load of 44 lb. per in. ft. of rafter; weight of rafter, 4 lb. per ft. B. M.

Permissible Stresses, Etc.—

Flexure, 1500 lb. per sq. in.

Shearing, parallel to fibre, 130 lb. per sq. in.

Modulus of elasticity, 1,600,000 lb. per sq. in.

Assumptions.—Consider the component of vertical forces parallel to the roof as resisted wholly by the rafter, and delivered equally to the two purlins. Allowable deflection, normal to roof = 1/250 of span. Dead load to be increased $33\frac{1}{3}\%$ in calculating deflections.

Flexure.—Vertical superimposed load = 32 lb. per lin. ft. of rafter.

Assume a 2×6-in. rafter, which when dressed is $1\frac{5}{8} \times 5\frac{5}{8}$ in. and weighs $(1.625 \times 5.625)\frac{4}{12} = 3.0$ lb. per lin. ft.

Total vertical load $= 20 + 12 + 3 = 35$ lb. per lin. ft. of rafter.

Component of vertical load normal to roof, as shown in Fig. 123(*a*), $= 35 \cos 30° = 35 \times 0.866 = 30.3$ lb. per lin. ft.

Total load normal to roof $= 30.3 + 44 = 74.3$ lb. per lin. ft.

Since rafters are not continuous over several purlins, they must be calculated as simple beams. The maximum moment due to the normal load is

$$M = \tfrac{1}{8} \times 74.3 \times (10.5)^2 \times 12 = 12{,}300 \text{ in.-lb.}$$

Moment arising from the eccentricity of the tangential force P_t will also be investigated. Component of vertical loading parallel to rafter = 35 sin 30° = 35 × 0.5 = 17.5 lb. per lin. ft., or 17.5 × 10.5 = 184 lb. for the whole rafter. One-half of this, or 92 lb., is resisted at each purlin, and positive and negative normal reactions of $P_t\, d/l$ are developed, where d = depth and l = span. At mid-span the moment is consequently $\frac{1}{2}P_t\, d - P_t\, d/l \times l/2 = 0$; but close to the ends, where the moment due to normal loading is small, it is approximately $\frac{1}{4}P_t\, d$.

Required section modulus = 12,300/1500 = 8.2 in.^{3}

Section modulus provided = $\frac{1}{6} \times 1.625 \times (5.625)^2 = 8.6$ in.^{3}, which is adequate.

Shear.—Maximum end shear = 74.3 × 10.5/2 = 390 lb.

Maximum longitudinal shearing stress, Eq. (1), Art. 178, = 1.5 V/A = 1.5 × 390/(1.625 × 5.625) = 64 lb. per sq. in., which is only about one-half that allowed.

Longitudinal Force.—The elements of loading producing the thrust reaction at the lower end of the rafter produce compressive stresses gradually increasing from zero at the upper end to a maximum at the lower end, while those producing the tensile reaction at the upper end create tensile stresses gradually increasing towards the upper end. As a result, the axial stress is zero at the centre of the rafter, where the flexural stress is a maximum, as will be seen from Fig. 123(*c*).

Deflection.—Total vertical dead load = 12 + 3 = 15 lb. per lin. ft. of rafter.

Component of dead load normal to roof = 15 × cos 30° = 15 × 0.866 = 13.0 lb. per ft. of rafter. For deflection calculations this becomes 1.33 × 13 = 17.3 lb.

Vertical snow load = 20 lb. per ft. of rafter. Component normal to roof = 20 × cos 30° = 20 × 0.866 = 17.3 lb. Wind load = 44 lb. per ft. of rafter.

Total deflection load = 17.3 + 17.3 + 44 = 78.6 lb. per ft. of rafter, or 78.6 × 10.5 = 825 lb. in all.

Maximum deflection is, from Eq. (17), Art. 72

$$= \frac{5}{384} \cdot \frac{Wl^3}{EI} = \frac{5}{384} \times \frac{825 \times (10.5 \times 12)^3}{1{,}600{,}000 \times \frac{1}{12} \times 1.625 \times (5.625)^3} = 0.56 \text{ in.}$$

Allowable deflection = 126/250 = 0.5 in. Although the maximum existing deflection is slightly greater, the design will be adopted, as over half the load is due to wind and an occasional high deflection would not damage the shingle roof.

185. Timber Purlin Subjected to Unsymmetrical Bending.—Design a rectangular timber purlin carrying a temporary corrugated steel roof of $\frac{1}{3}$ pitch. See Fig. 124. Span of purlin, 16 ft. centre to centre of trusses; spacing of purlins, 4.5 ft. centre to centre. Material, spruce, dressed four sides.

Loads.—Vertical superimposed load due to snow, 10 lb., and due to roof covering, 2.5 lb. per sq. ft. of sloping area; normal wind load, 22 lb. per sq. ft. of roof. Weight of spruce, 3 lb. per ft. B. M.

Permissible Stresses, Etc.—

Flexure, 1100 lb. per sq. in.
Shearing parallel to fibre, 100 lb. per sq. in.
Modulus of elasticity, 1,200,000 lb. per sq. in.

Assumptions.—Consider the corrugated steel roofing as incapable of resisting the force parallel to the roof. Let the force tangential to the roof be considered as acting through the centre of gravity of the purlin, thus neglecting the torsional effect. Allowable deflection normal to roof = 1/250 of span. Dead load to be increased $33\frac{1}{3}\%$ in calculating deflections.

Flexure.—Vertical superimposed load due to snow, is $10 \times 4.5 = 45$ lb. per lin. ft. of purlin; due to roofing, $2.5 \times 4.5 = 11.3$ lb.; due to purlin, assuming a 6×10-in. section, which when dressed is $5\frac{1}{2} \times 9\frac{1}{2}$ in., it is $(5.5 \times 9.5)\frac{3}{12} =$ 13.1 lb. per lin. ft. Total, $45 + 11.3 + 13.1 = 69.4$ lb. per lin. ft. of girder.

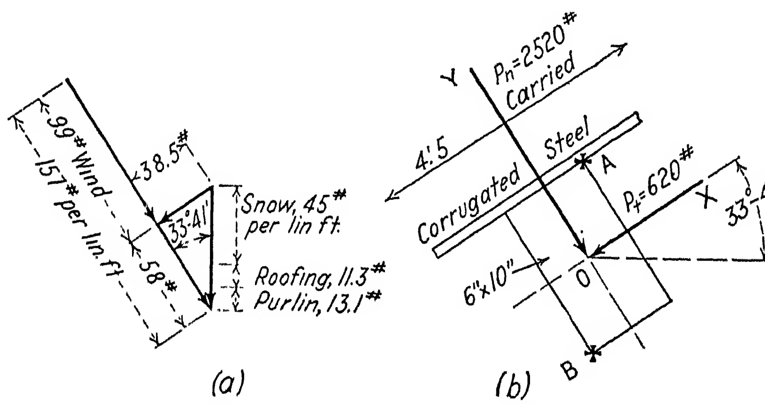


FIG. 124.—Purlin Subjected to Unsymmetrical Bending.

Normal component of vertical loading = 69.4 cos 33° 41′ = 69.4×0.8356 = 58 lb. per lin. ft. of rafter, as shown in Fig. 124(*a*).

Wind force, normal to roof, $= 22 \times 4.5 = 99$ lb. per lin. ft.

Total normal component $= 58 + 99 = 157$ lb. per lin. ft. of rafter.

Maximum moment normal to roof, assuming purlins as discontinuous over trusses, is $M_x = \frac{1}{8} \times 157 \times (16)^2 \times 12 = 60{,}300$ in.-lb.

Section modulus about axis $OX = S_x = \frac{1}{6} \times 5.5 \times (9.5)^2 = 82.7$ in.³

Extreme fibre stress due to normal moment $= M_x/S_x = 60{,}300/82.7 = 729$ lb. per sq. in.

Tangential component of vertical loading = 69.4 sin 33° 41′ = 69.4×0.5546 = 38.5 lb. per lin. ft. of purlin.

Maximum moment tangential to roof, $M_y = \frac{1}{8} \times 38.5 \times (16)^2 \times 12$ = 14,800 in.-lb.

Section modulus about axis $OY = S_y = \frac{1}{6} \times 9.5 \times (5.5)^2 = 47.9$ in.³

Extreme fibre stress due to tangential moment $= M_y/S_y = 14{,}800/47.9$ $= 309$ lb. per sq. in.

Maximum combined compressive stress at A or tensile stress at $B = 729$ $+ 309 = 1038$ lb. per sq. in. This is within the permissible limit of 1100 lb. per sq. in.

Shear.—Total normal load, $P_n = 157 \times 16 = 2520$ lb., and end shear $= 1260$ lb.

Maximum shearing stress due to normal load $= 1.5\ V/A$, Eq. (1), Art. 178, $= 1.5 \times 1260/(5.5 \times 9.5) = 36$ lb. per sq. in. The assumed section is ample to provide both for this and for the shearing stress arising from tangential loading.

There is a torsional moment on the purlin due to the tangential force really acting in the plane of the top of the purlin and not through the centre of gravity of the section. This produces torsional shearing stresses, which, however, are sufficiently provided for in the section assumed.

Deflection.—Permissible deflection normal to roof $= 16 \times 12/250 = 0.77$ in.

Component of loading normal to roof, as found above $= 157$ lb. per lin. ft. of purlin. Of this, the part due to dead load $= (11.3 + 13.1) \cos 33° 41' = 24.4$ $\times 0.8356 = 20.4$ lb. Adding to the total component $33\frac{1}{3}\%$ of this, or 6.8 lb., the total load for deflection calculations $= 163.8$ lb. per lin. ft. of purlin.

Maximum deflection normal to roof is, from Eq. (17), Art. 72.

$$\Delta = \frac{5}{384} \cdot \frac{Wl^3}{EI} = \frac{5}{384} \cdot \frac{(163.8 \times 16) \times (16 \times 12)^3}{1{,}200{,}000 \times \frac{1}{12} \times 5.5 \times (9.5)^3} = 0.51 \text{ in.}$$

This is within the allowable limit of 0.77 in.

186. Design of a Queen-Post Trussed Beam.—Design a queen-post trussed beam having a rough timber top-chord, cast-iron struts and twin tie rods of steel, as indicated in Fig. 125. Span, 30 ft. centre to centre of end bearings, divided into three panels of 10 ft. each. Depth such that the slope of the end section of tie rod is 1 vertical to 4 horizontal.

Load.—1710 lb. per lin. ft. superimposed, plus weight of beam, which will be assumed as 90 lb. per lin. ft.

Materials.—Timber to be undressed long-leaf yellow pine weighing $4\frac{1}{2}$ lb. per ft. B. M.

Tie rods, of soft steel.

Washers, mild steel plates.

Struts, of cast iron.

End supports, of crushed stone concrete.

Permissible Stresses.—

Long-leaf yellow pine:

Compression, parallel to grain, 1500 lb. per sq. in.

Compression, across grain, 350 lb. per sq. in.

Compression, normal to surface which is inclined at an angle θ with direction of grain, by the Howe formula (*Engineering News*, Aug. 1,

1912, p. 190), $n = q + (p - q)(\theta/90)^2$ where q = permissible stress across the grain, and p = permissible stress parallel to grain.
Flexure, 1500 lb. per sq. in.
Combined flexure and compression, 1500 lb. per sq. in.
Shearing parallel to grain, 200 lb. per sq. in.

Soft steel:

Tension, 15,000 lb. per sq. in., net section.

Cast iron:

Compression, on short specimens, 16,000 lb. per sq. in.

Assumptions.—Assume that the tie rods are prestressed sufficiently to hold the top chord at the struts at the same elevation as the end supports under full load. The continuous-beam theory will then apply.

Consider the beam as sufficiently well supported laterally to render unnecessary the reduction of the allowable bending and compressive stresses to compensate for buckling.

References

Merriman—Mechanics of Materials.
Seely—Resistance of Materials.

Section for Shear.—Maximum shear, V, in top chord, occurs at the two struts or posts and on the sides nearest the end supports and is

$$V = \tfrac{6}{10} wp$$
$$= \tfrac{6}{10} \times 1800 \times 10 = 10{,}800 \text{ lb.}$$

Required right sectional area to provide for shearing stress parallel to the fibre, allowing for the fact that the intensity of shearing stress at the neutral axis of a rectangular section is $\frac{3}{2}$ the average stress, is $\frac{3}{2} \times 10{,}800/200 = 81$ sq. in. The section found necessary for combined flexure and compression, in what follows, consisting of three 4×12-in. rough planks, with an area of 144 sq. in., is ample for shear.

Section for Moment and Axial Compression.—Maximum moment in top chord of beam occurs at struts and is $-\frac{1}{10} wp^2$, or numerically, $-\frac{1}{10} \times 1800 \times (10)^2 \times 12 = -216{,}000$ in.-lb.

Horizontal, or centric axial, compression in top chord, $H = \frac{11}{10} wp \cot a$, where a is the angle of slope of the end section of the tie rod. Numerically, $H = \frac{11}{10} \times 1800 \times 10 \times 4 = 79{,}200$ lb.

Assume 3 pieces of 4×12-in. undressed long-leaf yellow pine. Area, A, $= 144$ sq. in. Section modulus, S, $= \frac{1}{6} bd^2 = \frac{1}{6} \times 12 \times (12)^2 = 288$ in.^{3}

Maximum direct compressive stress, $f_c = H/A = 79{,}200/144 = 550$ lb. per sq. in.

Maximum flexural stress, $f_f = M/S = 216{,}000/288 = 750$ lb. per sq. in.

Total maximum stress $= f_c + f_f = 550 + 750 = 1300$ lb. per sq. in.

The stress is lower than is permitted (viz., 1500 lb. per sq. in.), but this is

the most satisfactory section that can be built up of commercial timber. In the excess area, allowance is made both for the approximate character of the theory employed, and for the under-run of the rough timber.

Tie Rod.—Tension in tie rod = $T = \frac{11}{10} wp$ cosec $a = \frac{11}{10} \times 1800 \times 10 \times 4.12 = 81{,}600$ lb. Initial tension does not constitute an extra stress.

Required area = 81,600/15,000 = 5.44 sq. in.

Use two soft steel round rods, $1\frac{7}{8}$-in. diameter having an area in the body of 5.52 sq. in. Ends will be upset to $2\frac{3}{8}$-in. diameter giving an area at root of thread 24% in excess of area in body of rods. (See any structural steel handbook for details of upsets.)

Struts.—Compression = $\frac{11}{10} wp = \frac{11}{10} \times 1800 \times 10 = 19{,}800$ lb.

Required area in compression = 19,800/16,000 = 1.24 sq. in.

Since no ribs should be less than $\frac{5}{8}$ in. thick, the area provided must be greatly in excess of structural requirements. (See detail, Fig. 125.)

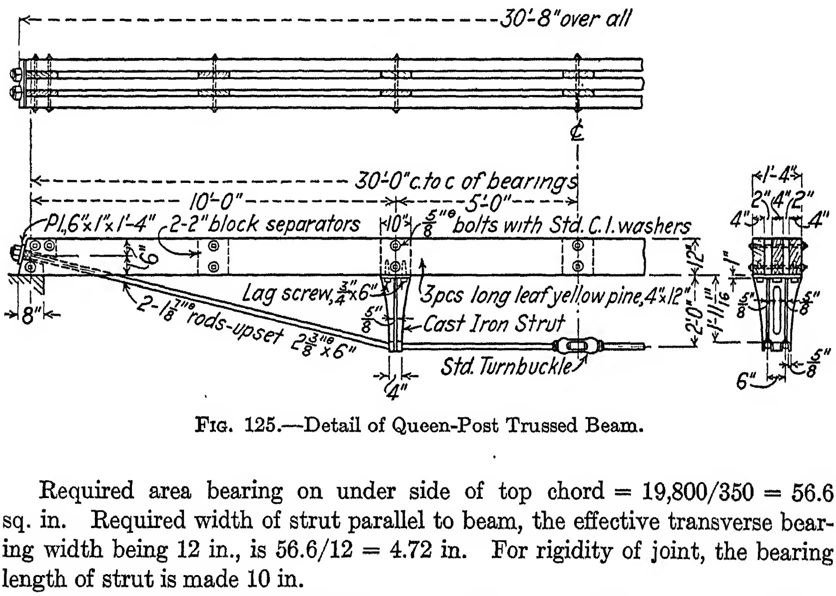


FIG. 125.—Detail of Queen-Post Trussed Beam.

Required area bearing on under side of top chord = 19,800/350 = 56.6 sq. in. Required width of strut parallel to beam, the effective transverse bearing width being 12 in., is 56.6/12 = 4.72 in. For rigidity of joint, the bearing length of strut is made 10 in.

Bearing on Supports.—End reaction, $R = \frac{3}{2} wp = \frac{3}{2} \times 1800 \times 10 = 27{,}000$ lb.

Required bearing area, based on safe compression of long-leaf yellow pine across grain = 27,000/350 = 77 sq. in. Required length of bearing of beam on supports is, therefore, $\frac{77}{12} = 6.4$ in. A length of 8 in. is shown in the detail.

Details.—Details of beam are given in Fig. 125.

Washer plates at ends of tie rods must transmit rod tension to ends of top chord. Tension in rod, $T = 81{,}600$ lb. Required area of plate is governed by bearing on end surface of timber which is inclined at 76° with the grain. This is $n = 350 + (1500 - 350)\left(\frac{76}{90}\right)^2 = 1170$ lb. per sq. in., and area required = 81,600/1170 = 69.8 sq. in. Use a $6 \times 1 \times 16$-in. steel plate, giving area bearing on ends of three 4×12-in. pieces of 72 sq. in.

Wood block separators bolted as shown, are inserted about 5-ft. centres to stiffen timbers against lateral buckling.

Turnbuckles are inserted at centres of tie rods for the purpose of adjustment.

187. Exercise Problems on Timber Beams.—The following exercise problems are based on the problems solved in this chapter. See Appendix I for the answers.

(1) Design a timber beam (use nominal dimensions) for a span of 8 ft. centre to centre to carry a total uniformly distributed load of 1400 lb. per lin. ft. $p_f = 1600$ lb. per sq. in.; p_s (parallel to fibre) $= 150$ lb. per sq. in.

(2) A timber beam 8 by 12 in., 10 ft. long, carries a total uniformly distributed load, including its own weight, of 2000 lb. per lin. ft. Express an opinion as to the safety of the beam against longitudinal shear if the maximum permissible longitudinal shearing stress $p_s = 150$ lb. per sq. in.

(3) An 8×12-in. timber beam of 15-ft. span carries a concentrated load of 9000 lb. at a point 5 ft. from one end. Report on the safety of the beam in longitudinal shear, if the permissible shearing stress is 125 lb. per sq. in. Weight of timber = 4 lb. per ft. B.M.

(4) Two perfectly smooth planks 3×12 in. are placed flat on each other and the combination is made to span an opening of 10 ft. What is the maximum flexural stress in the timber due to a central load of 500 lb.? Neglect the weight of the planks.

(5) Design a cantilever beam of rough (undressed) timber to project 5 ft. out from a wall and carry 1000 lb. per lin. ft., including its own weight. $p_f = 1500$; $p_s = 130$ lb. per sq. in.

(6) A 6×10-in. wooden beam, 15 ft. long, is restrained at both ends and carries a uniformly distributed load. What would be the maximum permissible amount of this load, if the permissible flexural stress on the timber p_f is 1500 lb. per sq. in.?

(7) A 10×12-in. Douglas fir beam of 8-ft. span restrained at both ends carries a total uniformly distributed load of 5000 lb. per lin. ft. Report on its safety. $p_f = 1600$ lb. per sq. in. $p_s = 180$ lb. per sq. in. Use nominal dimensions.

(8) Two children, each weighing 80 lb., use a 2×6-in. rough plank 12 ft. long as a see-saw. Is it safe? $p_f = 1800$ lb. per sq. in.

(9) Design a timber beam of 12-ft. span, using nominal dimensions, if the beam is restrained at both ends and carries a total uniformly distributed load, including own weight, of 18,000 lb. $p_s = 150$; $p_f = 1200$ lb. per sq. in.

(10) A 6×12-in. Douglas fir beam 10 ft. long is fixed at one end and supported on a roller at the other. What is the maximum total safe load per lineal foot, including the weight of the beam, if the permissible stresses in bending and longitudinal shear are 1600 lb. and 150 lb. per sq. in., respectively?

(11) A yellow pine beam carrying a total load of 1000 lb. per ft. uniformly distributed is built rigidly into a wall at one end and at the other end rests on a roller 10 ft. from the wall. If the safe bending and longitudinal shearing stresses on this timber are 1200 and 100 lb. per sq. in., respectively, recommend a size for the beam.

(12) A timber beam is supported on two piers 10 ft. apart centre to centre and overhangs at each end a distance of 3 ft. The load to be carried, including the weight of the beam, is 1200 lb. per lin. ft. Assuming nominal dimensions, suggest a size for the beam. $p_f = 1500$ lb. per sq. in., $p_s = 150$ lb. per sq. in.

(13) An 8×12-in. timber beam of 10-ft. span freely supported at the ends supports a uniformly distributed load, including its own weight, of 1500 lb. per lin. ft. and bears a centric axial compression of 10,000 lb. What is the maximum fibre stress due to the combination of loading?

(14) What is the maximum intensity of longitudinal shearing stress in a timber beam 8 by 12 in. and 15 ft. long, carrying a concentrated load of 6000 lb. at 5 ft. from one end. Neglect the weight of the beam.

(15) Find the maximum intensity of shearing stress in a 6 × 8-in. timber beam 12 ft. long, carrying one concentrated load of 5000 lb. at a point 4 ft. from one end. Weight of timber = 4 lb. per ft. B.M.

(16) What is the intensity of the vertical shearing stress at one-quarter of the depth of an 8 × 12-in. wooden beam if the total vertical shear at the cross section is 12,000 lb.?

(17) The cross section of a beam is an equilateral triangle, 12 in. to the side and the bending is in a median plane. If the total vertical shear is 20,000 lb., find the intensity of shearing stress at a point midway between the upper vertex and the opposite side. $I = \frac{1}{36} bh^3$, where b is the base and h is the height of a triangle.

(18) A 6 × 12-in. timber beam rests on three supports separated by two spaces of 10 ft. A uniformly distributed load of 1600 lb. per lin. ft. is applied to the beam. Is it safe? $p_s = 150$; $p_f = 1500$ lb. per sq. in. Weight of timber, 4 lb. per ft. B.M.

(19) A king-post trussed beam of 20-ft. span is divided by a strut into two equal panels. If the load is 400 lb. per lin. ft. uniformly distributed, find the maximum moment in the beam and the shear on either side of the strut.

(20) A timber beam 6 × 8 in. and 24 ft. long carrying a total uniformly distributed load of 500 lb. per lin. ft. is trussed by placing a centre strut under it with truss rods to the ends. Find the maximum bending stress in the timber.

(21) The top chord of a king-post trussed beam consists of two 3 × 10-in. planks properly separated and fastened together. The truss rod is on a slope of 1 vertical to 4 horizontal. If the total span of the beam is 16 ft. and the total uniformly distributed load is 1000 lb. per lin. ft., find the maximum combined normal stress in the top chord at the strut.

(22) A king-post trussed beam of 18-ft. span carries a total uniformly distributed load, including its own weight, of 900 lb. per lin. ft. If the slope of the tie rod to the horizontal is 1 in 3 and the top chord is composed of a 6 × 10-in. rough timber, find the maximum combined normal stress on a cross section of the chord.

(23) A floor panel in a mill construction building is 15 ft. square. If the live load is 175 lb. per sq. ft., suggest an arrangement of beams and girders, and, neglecting deflection, determine the thickness of the under floor. Average weight of timber = 4 lb. per ft. B.M. Top flooring of 1-in. maple. Permissible bending stress = 1000 lb. per sq. in. Maximum moment on flooring, $\frac{8}{10}$ of that for a simply supported beam.

(24) The under floor of a mill construction building is yellow pine of an actual thickness of 2.6 in. It spans an opening of 8 ft. and carries a total uniformly distributed load, including its own weight, of 100 lb. per sq. ft. Report on its sufficiency if $p_f = 1200$ lb. per sq. in. and E for the combined loading = 1,400,000 lb. per sq. in. Neglect end restraint. Maximum allowable deflection is 1/360 of the span.

(25) The under floor of a mill construction building is Douglas fir of $2\frac{3}{4}$ in. actual thickness. It carries a total uniformly distributed dead load of 16 lb. per sq. ft. and a uniformly distributed live load of 100 lb. per sq. ft. The spacing of joists is 7 ft. centre to centre. Report on the sufficiency of the flooring. $p_f = 1200$ lb. per sq. in. E for dead load = 1,200,000. E for live load = 1,600,000 lb. per sq. in. Maximum allowable deflection $= \dfrac{1}{300}$ of span. Neglect end restraint.

(26) The panels of a mill construction floor are 14 ft. square and beams head into

the girders at the centre point and at columns only. If the total uniformly distributed load above the tops of the beams is 100 lb. per sq. ft. suggest a size for the beams (not the girders) to satisfy flexural requirements. $p_f = 1500$ lb. per sq. in. Weight of timber = 4 lb. per ft. B. M. Neglect deflection.

(27) The panels of a mill construction floor are 15 ft. square and beams head into the girders at the third-points and also into the columns. If the total uniformly distributed load above the tops of the beams is 125 lb. per sq. ft., suggest a size for the beams (not the girders). Use nominal dimensions. $p_f = 1600$ lb. per sq. in. $p_s = 200$ lb. per sq. in. $E = 1{,}500{,}000$ lb. per sq. in. Weight of timber = 4 lb. per ft. B. M. Maximum allowable deflection, 1/300 of span.

(28) A warehouse building 70×100 ft. is to be of mill construction with beams and girders entirely of timber. Suggest a suitable layout for the floor framing and calculate the necessary size of a typical beam and girder, if the total uniformly distributed load, apart from the weight of beams and girders, is 150 lb. per sq. ft. Permissible bending and longitudinal shearing stresses on Douglas fir = 1600 and 150 lb. per sq. in., respectively. Neglect deflection. Weight of timber, 4 lb. per ft. B. M.

(29) The columns in a mill construction building are spaced 12 ft. in one direction and 15 ft. in the other. Suggest an arrangement for the floor framing, and obtain the sizes of the beams and girders and the thickness of the flooring. Live load = 200 lb. per sq. ft. Under floor of spruce weighing 3 lb. per ft. B. M. Top floor of 1-in. maple weighing $4\frac{1}{2}$ lb. per ft. B. M. Beams and girders of long-leaf yellow pine weighing $4\frac{1}{2}$ lb. per ft. B. M. Allowable bending stresses = 900 lb. per sq. in. for spruce and 1500 lb. per sq. in. for long-leaf pine. Neglect end restraint and deflection. Use dressed dimensions.

(30) The roof sheathing for a building is $\frac{7}{8}$ in. thick (1 in. dressed), covers a span of 4 ft. between centres of purlins and is inclined at $\alpha = 30$ deg. with the horizontal. The roof surfacing weighs 1 lb. per sq. ft., and it is assumed that a 3-in. coating of ice covers the entire surface and a wind force of $p = 30$ lb. per sq. ft. on a vertical plane surface is blowing. Investigate the sufficiency of the sheathing for moment and deflection. Weight of timber, 4 lb. per ft. B. M. Weight of ice, 57 lb. per cu. ft. Normal component of wind pressure on roof $p_n = \dfrac{p}{45}\alpha$. $p_f = 1500$ lb. per sq. in. $E = 1{,}500{,}000$. Maximum permissible deflection $= l/250$. Consider moment as $\frac{8}{10}$ of that for a simply supported beam.

(31) In a roof the sheathing is laid directly on rafters spaced $2\frac{1}{2}$ ft. centres. The rafters have a span of 10 ft. up the slope and are inclined at 30 deg. to the horizontal. Find the moment in a rafter due to a coating of 3 in. of ice on the roof. Weight of ice = 57 lb. per cu. ft. Neglect end restraint and tangential effect.

(32) The covering of a roof consists of slates weighing 8 lb. per sq. ft. of sloping area. The snow weighs 10 lb. per sq. ft. of horizontal projection, and the wind pressure normal to the roof surface is 20 lb. per sq. ft. Suggest a thickness for the wooden roof sheathing, if the purlins are spaced 4 ft. centre to centre and $p_f = 1600$ lb. per sq. in. Maximum permissible deflection = 1/300 of span. $E = 1{,}400{,}000$. Slope of roof = 30 deg. Weight of timber = 4 lb. per ft. B. M. Consider moment as $\frac{8}{10}$ of that for a simple beam.

(33) Timber rafters, sloping 1 in 2 to the horizontal, are spaced 28 in. centres and have a span of 11 ft. They support roof sheathing and covering weighing together 9 lb. per sq. ft. of sloping area of roof. The snow load is 15 lb. per sq. ft. of horizontal projection, and the wind load 30 lb. per sq. ft. on a plane normal to the direction of the wind. What is the maximum moment in a rafter due to these weights

and forces, assuming it as freely supported at the ends. Neglect tangential effects and the weight of the rafter.

$$p_n = p \cdot \frac{2 \sin \alpha}{1 + \sin^2 \alpha}$$

(34) Timber purlins spaced 4 ft. centres in a roof having a slope of 20 deg. to the horizontal support wood sheathing so fastened at the ridge and at the eaves as to relieve the purlins of load tangential to the roof. The sheathing and roof covering weigh 12 lb. per sq. ft. of sloping area. Ice 2 in. thick covers the roof and a normal wind force estimated at 18 lb. per sq. ft. of sloping area is acting. What size of purlin (use dressed dimensions) will satisfy the moment requirements? Truss spacing, 15 ft. Neglect end restraint. $p_f = 1600$ lb. per sq. in. Weight of ice, 57 lb. per cu. ft. Weight of timber, 4 lb. per ft. B. M.

CHAPTER XV

TIMBER TRESTLES

188. Design of Timber Bent for a Flume Trestle.—Design a two-post, X-braced timber bent, resting on mud sills, as shown in Fig. 127. Height, from top of mud sills to top of cap, 18 ft. 7 in. Width, centre to centre of batter posts at under side of cap, 6 ft. Batter of posts $1\frac{1}{2}$ in. per ft. Bents spaced 10 ft. centre to centre.

Loads.—Three vertical loads of 7500 lb. applied as shown in Fig. 127. Resultant transverse wind force on flume, 1800 lb. per bent, applied 3 ft. 4 in. above top of cap. Wind force on bent 50 lb. per ft. of height. Weight of Douglas fir, 4 lb. per ft. B. M.

Materials.—Douglas fir, undressed.

Permissible Stresses.—

Douglas fir:

Bearing compression, parallel to fibre, 1500 lb. per sq. in.
Bearing compression, transverse to fibre, 350 lb. per sq. in.
Compression on columns and struts, with effective length not over 15 times the least lateral dimension, 1000 lb. per sq. in.
Compression on columns and struts, with effective length over 15, and not in excess of 50, times the least lateral dimension, $p_c = 1300 - 20\, l/d$, where l = unsupported length, and d = least lateral dimension.
Flexure, 1500 lb. per sq. in.
Shearing, parallel to fibre, 140 lb. per sq. in.
Tension, parallel to fibre, 1500 lb. per sq. in.

Nails:

Transverse resistance in lb., for nails over 40-penny size, $7d$.

where d = penny designation.

Soil:

Bearing pressure, 5 tons per sq. ft.

Assumptions.—As the flume will very rarely be empty, no special provision for anchorages to resist uplift will be made. For stress calculations, the height of bent will be assumed to be 18 ft., with the story heights as shown in Fig. 126. Nominal rough dimensions of timber will be used in the design.

Axial Stresses from Vertical Loading.—Since the bracing carries no calculable stresses except those due to wind, the stress in a post arising from the loads

applied to the cap = vertical load × secant of angle of inclination of post to the vertical. This is 11,250 × 12.1/12 = 11,300 lb. Adding 300 lb. for the part of the weight of the bent borne by the upper segment of the post, and another 400 lb. for the additional weight borne by the lower segment, the dead and live load stresses in these two segments become 11,600 lb. and 12,000 lb., respectively.

Compression in top strut (cap) due to dead and live loading is vertical load at top of post × tangent of angle of batter = 11,250 × 1.5/12 = 1410 lb.

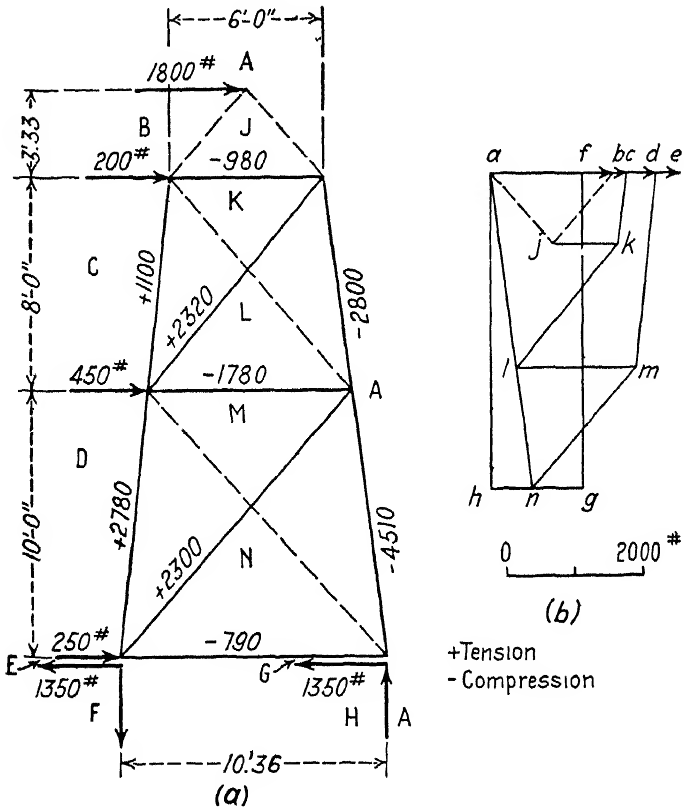


Fig. 126.—Wind Stresses for Flume Bent.

In the bottom strut, which acts as a tie under the vertical load, the stress is the vertical component of the post stress at the bottom × tangent of angle of batter. The vertical component, allowing for half the weight of the bent is approximately 11,250 + 700 = 11,950 lb. Tension in tie = 11,950 × 1.5/12 = 1500 lb.

Wind Stresses.—The wind stresses for the bent are determined graphically in Fig. 126. Horizontal forces as determined from the data are applied at windward panel points and at 3 ft. 4 in. above the top of the bent. Diagonals are

assumed to take tension only. The dotted members above the cap of the bent are merely auxiliary members introduced to make possible a purely graphical solution; they do not modify the stresses in the bent members. Determination of the stresses is made from the top down.

The horizontal components of the reactions at the two post bases are assumed to be each one-half the total horizontal wind force. In drawing the polygon for the windward post base, the length *ef*, equal to 1350 lb., is laid off from right to left on the load line, and the stresses at the post bases naturally follow.

Cap.—Reaction at centre of support, due to the symmetrical concentrated loading $= 3 \times 7500/2 = 11{,}250$ lb.

Moment at centre due to concentrated loads $= (11{,}250 \times 3 - 7500 \times 3.25)12 = 112{,}500$ in.-lb.

Moment due to weight of cap, assuming it to weigh 21 lb. per lin. ft. $= \frac{1}{8} \times 21 \times (6)^2 \times 12 = 1134$ in.-lb.

Total moment at centre $= 112{,}500 + 1134 = 113{,}634$ in.-lb.

Required section modulus $= 113{,}634/1500 = 75.8$ in.^{3}

An 8×8-in. rough section will be used, giving a section modulus of $\frac{1}{6} \times 8 \times (8)^2 = 85.3$ in.^{3}, which is sufficient to allow for under-run.

The axial compression, amounting to 1410 lb. due to vertical loading and 980 lb. due to wind, is amply provided for in the excess area of the section adopted.

Vertical shear immediately inside a post $= 3750$ lb.

Maximum intensity of longitudinal shearing stress, assuming a notch or "gain" of $\frac{1}{2}$ in. over the post, Eq. (1), Art. 178, $= 1.5\ V/A = 1.5 \times 3750/(8 \times 7.5) = 94$ lb. per sq. in., as compared with 140 lb. per sq. in. allowed.

Posts.—Because of the small height of the bent, the posts will be made of the same section from top to bottom, that is, the stress in the bottom story governs.

Stress in lower section of post due to dead and live loading has already been found to be 12,000 lb.

Maximum compressive wind stress in lower section of post, from Fig. 126, is 4510 lb.

Total stress in post $= 12{,}000 + 4510 = 16{,}510$ lb.

The bent is supported by both longitudinal and transverse struts at the dividing point between the stories, and hence the effective length of the lower section $= 10$ ft., or 120 in.

Assume an 8×8-in. section. Value of $l/d = 120/8 = 15$. Hence, from specification, $p_c = 1000$ lb. per sq. in.

Required area $= 16{,}510/1000 = 16.51$ sq. in.

The 8×8-in. section is more than adequate for immediate purposes, but will be employed to provide for effects of deterioration, particularly at the bases.

Bracing.—Each diagonal bracing member will consists of a $1\frac{1}{2} \times 6$-in. plank. This is much in excess of stress requirements, as will be seen from Fig. 126, but it is the smallest practicable commercial section.

The effective length of the intermediate transverse strut is approximately

8 ft., and to keep the ratio of length to least lateral dimension down to 50, the thickness of a single plank will need to be $\frac{96}{50} = 1.92$, say 2 in. For a 2×6-in. plank, the permissible compressive stress $= 1300 - 20 \times \frac{96}{2} = 340$ lb. per sq. in., and the required area for a total compression of 1780 lb. (see (Fig. 126) $= 1780/340 = 5.2$ sq. in. The area provided is $2 \times 6 = 12$ sq. in., or more than needed.

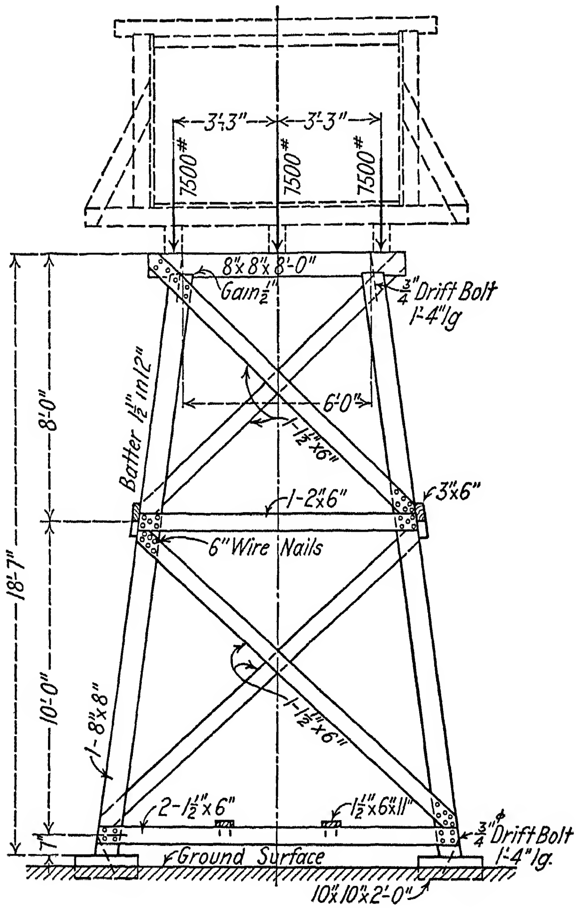


Fig. 127.—Details of Bent for Flume Trestle.

While under vertical loading there is a tension of 1500 lb. in the bottom horizontal member, and under wind a compression of only 790 lb., it should be proportioned as a strut to provide for a possible reversal of stress from an extreme and unexpected wind load.

The effective length of this bottom strut is about 128 in., and the necessary thickness of single plank $= 128/50 = 2.56$ in. A single 3×6-in. plank would satisfy the slenderness requirement. To obviate any tendency of the posts to

twist about their own longitudinal axis, the bottom strut will be made of two $1\frac{1}{2} \times 6$-in. planks, tied across with $1\frac{1}{2} \times 6$-in. battens at the third-points, as shown in Fig. 127.

Longitudinal diagonal bracing between bents (not shown) should be inserted about every fourth or fifth span to give added security against collapse of the trestle longitudinally. A line of longitudinal waling pieces, about 3×6 in., should be run continuously along each side of the trestle at the level of the junction between stories. The latter are indicated in Fig. 127.

Details.—Shifting of the caps is obviated by notching them $\frac{1}{2}$ in. over the post tops and by one $\frac{3}{4}$-in. drift bolt, 1 ft. 4 in. long, driven into each post top, as shown in Fig. 127.

The posts are seated on 10×10-in. mud sills 2 ft. long, with one $\frac{3}{4}$-in. drift bolt, 1 ft. 4 in. long driven into each, as shown. The bearing area of a sill, 240 sq. in., is adequate for the maximum post load of 16,510 lb. at the allowable soil pressure of 5 tons per sq. ft.

Bracing members will be secured to the posts by $60 - d$ (6-in.) wire nails.

Safe working transverse load on one nail $= 7\,d = 7 \times 60 = 420$ lb. The number of nails in each member is readily calculated from the wind stress sheet, Fig. 127.

CHAPTER XVI

WOODEN ROOF PANEL

189. Data.—Design a wooden roof panel consisting of a Howe truss with supported purlins, rafters and roofing. The truss will be made with solid dressed timber chords and compression diagonals and steel rod verticals. The roof covering will be of slate and will be carried on 1-in. dressed spruce sheathing supported by rafters and purlins. Purlins will be located at, or near, panel points, as shown in Fig. 131. Trusses rest on 12-in. brick wall.

Dimensions.—Span, 40 ft. centre to centre of bearings; pitch, $\frac{1}{4}$; spacing of trusses 13 ft. centre to centre.

Loads.—Snow, 20 lb. per sq. ft. of sloping roof surface; wind, 30 lb. per sq. ft. on vertical surface, reduced for intensely normal to roof by formula $p_n = p\alpha/45$, where p = pressure on vertical surface and α = inclination of roof to horizontal in degrees. Weight of spruce and Douglas fir, 3 and 4 lb. per ft. B. M., respectively.

Materials.—Sheathing and rafters, spruce; purlins and truss timber, Douglas fir; tie rods, medium steel; supporting walls, common brick laid in Portland cement mortar.

Permissible Stresses.—

Spruce:

Bearing compression, transverse to grain, 200 lb. per sq. in.
Flexure, 1000 lb. per sq. in.

Douglas fir:

Bearing compression, parallel to grain, 1400 lb. per sq. in.
Bearing compression, transverse to grain, 350 lb. per sq. in.
Bearing compression, normal to surface which is inclined at angle θ with direction of grain, by the Osgood formula (*Engineering News-Record*, Feb. 9, 1928, p. 243).

$$n = \frac{p}{1 + \frac{p - q}{q}\{a\cos^2\theta + (1 - a)\cos^4\theta\}}$$

where p = permissible stress parallel to grain;
q = permissible stress normal to grain, and
a = a constant depending on the kind of timber, being for Douglas fir approximately 1.55.

Bearing compression under washers, not covering full width of member, 25% in excess of above.

Compression on columns not over 10 diameters in length, 1100 lb. per sq. in.

Compression on columns over 10 diameters in unsupported length, $p_c = 1250 - 15\, l/d$, where l = length and d = least lateral dimension.

Flexure, 1500 lb. per sq. in.

Shearing, parallel to grain, 140 lb. per sq. in.

Tension, parallel to grain, 1500 lb. per sq. in.

Brickwork:

Bearing on common brick in Portland cement mortar, 175 lb. per sq. in.

Structural steel:

Tension, net section, 16,000 lb. per sq. in.

Moduli of Elasticity.—

Spruce, 1,200,000 lb. per sq. in.

Douglas fir, 1,500,000 lb. per sq. in.

Assumptions.—The structure is to be designed for the two most severe cases of loading likely to arise, (1) dead load, $\frac{1}{2}$ snow load and full wind load, and (2) dead load, full snow load and $\frac{1}{3}$ wind load. Deflection of sheathing, rafters and purlins must not exceed 1/300 of span. Dead loads are to be increased $33\frac{1}{3}$% in calculating deflections. Purlins are to be calculated for loading normal to roof only.

190. Sheathing.—CASE (1).—Weight of $\frac{3}{16}$-in. slates per square foot of sloping roof surface = 8 lb. Weight of 1-in. spruce sheathing dressed one side $= \frac{7}{8} \times 3 = 2.6$ lb. per sq. ft. One-half snow load $= \frac{1}{2} \times 20 = 10$ lb. per sq. ft. Total vertical load, per square foot of sloping surface, $= 8 + 2.6 + 10 = 20.6$ lb. Component normal to roof having slope of $26° 34' = 20.6 \cos 26° 34' = 20.6 \times 0.8945 = 18.5$ lb. per sq. ft.

Wind force normal to roof, $p_n = p\alpha/45 = 30 \times 26.57/45 = 17.7$ lb. per sq. ft.

Total component normal to roof $= 18.5 + 17.7 = 36.2$ lb. per sq. ft. of sloping surface.

The component parallel to the roof is carried by the sheathing as a small lateral compression, which may be neglected.

The safe span centre to centre of rafters for the sheathing, based on both flexure and deflection, should be found.

Section modulus of 12-in. strip of sheathing $S = \frac{1}{6} \times 12 \times (\frac{7}{8})^2 = 1.53$ in.^{3} Moment of resistance $= Sp_f = 1.53 \times 1000 = 1530$ in.-lb. As the sheathing is continuous over a considerable number of purlins, $M = \frac{1}{10}\, wl^2$, or $\frac{1}{10} \times 36.2 \times l^2 \times 12$ in.-lb. Equating to the moment of resistance, the safe span for flexure is found to be 5.94 ft., or much greater than is permissible either for deflection or rafter spacing.

For sheathing, the shearing stresses are inconsiderable.

To prevent cracking of the slates, the maximum deflection should not be over 1/300 of the span. The limiting span is found by equating the existing deflection, Eq. (17), Art. 72, to the permissible deflection and by solving for l in the resulting expression.

$$\frac{3.8}{384} \cdot \frac{wl^4}{EI} = \frac{l}{300}$$

In this equation, $w = 3.27$ lb. per lin. in. of strip 12 in. wide, the dead load having been increased by $33\frac{1}{3}\%$, as required for deflection calculations; l is the span centre to centre of rafters in inches; $E = 1{,}200{,}000$ lb. per sq. in., the modulus of elasticity of the sheathing; $I = \frac{1}{12} \times 12 \times (\frac{7}{8})^3 = 0.67$ in.^{4}, the moment of inertia of the strip cross section. Solving, $l = 43.6$ in., or greater than would be suitable for the rafter spacing.

Case (2).—Weight of slates and sheathing, 10.6 lb. per sq. ft. of sloping surface, as for Case (1). Adding full snow load, this becomes 30.6 lb. per sq. ft., with a component of $30.6 \times 0.8945 = 27.4$ lb. per sq. ft. normal to the roof.

One-third wind force normal to roof $= \frac{1}{3} \times 17.7 = 5.9$ lb. per sq. ft.

Total component normal to roof $= 27.4 + 5.9 = 33.3$ lb. per sq. ft.

As this is less than for Case (1), the latter governs.

191. Rafters.—The rafters will have a centre-to-centre span of approximately 11.18 ft. (see Fig. 129). They should be so chosen and spaced that the bending strength will be as nearly fully utilized as possible. Assume a 2×8-in. nominal section, which when dressed on four sides measures $1\frac{5}{8} \times 7\frac{1}{2}$ in. It has a section modulus of $S = \frac{1}{6}\,bd^2 = \frac{1}{6} \times 1.625 \times (7.5)^2 = 15.2$ in., and a moment of resistance of $Sp_f = 15.2 \times 1000 = 15{,}200$ in.-lb.

Loads borne by a rafter consist (1), of its own weight $(1.625 \times 7.5)\frac{3}{12} = 3.0$ lb. per lin. ft., vertical load, or normal to the rafter, $3.0 \times 0.8945 = 2.7$ lb. per lin. ft.; (2) of the component of superimposed load normal to roof surface, this for Case (1) being 36.2 lb. per sq. ft. Letting s = spacing of rafters in feet centre to centre, total load in pounds per lineal foot of rafter $= 2.7 + 36.2\,s$.

Spacing.—As the rafters will not be continuous over the purlins, $M = \frac{1}{8}\,wl^2$, or expressing in inch-pounds, and equating to the moment of resistance, $M = \frac{1}{8}(2.7 + 36.2\,s) \times (11.18)^2 \times 12 = 15{,}200$ in.-lb., the span of the rafter on the slope being 11.18 ft. Solving, $s = 2.17$ ft.

This allowable spacing permits the desirable arrangement of rafters shown in Fig. 128, whereby the purlin span of 13 ft., or distance centre to centre of trusses, is divided into 5 spaces of 26 in., and 2 half-spaces of 13 in. This obivates placing a rafter at the purlin centre.

The component of roof load parallel to the roof surface may be neglected, as is evident from Art. 184.

The shearing stresses are negligible.

Deflection.—The deflection of the rafter normal to the roof must not exceed $11.18 \times 12/300 = 0.45$ in.

Vertical dead load of slates and sheathing = 10.6 lb. per sq. ft. of sloping area, or 10.6 × cos 26° 34′ = 9.5 lb. per sq. ft. normal to roof. This becomes 9.5 × 2.16 = 20.6 lb. per lin. ft. of rafter. Adding component of weight of rafter normal to roof, or 2.7 lb., and increasing dead loads $33\frac{1}{3}$%, total dead load for deflection = 1.33(20.6 + 2.7) = 31.1 lb. per lin. ft. of rafter.

Half snow load plus full wind load normal to roof = 10 cos 26° 34′ + 17.7 = 26.6 lb. per sq. ft. of sloping area, or 26.6 × 2.17 = 57.7 lb. per lin. ft. of rafter.

Total load for deflection = 31.1 + 57.7 = 88.8 lb. per lin. ft., or 88.8 × 11.18 = 995 lb. in all.

Maximum deflection (see Art. 183) is for Case (1)—the determining case

$$\Delta = \frac{5}{384} \cdot \frac{Wl^3}{EI} = \frac{5}{384} \cdot \frac{995 \times (11.18 \times 12)^3}{1{,}200{,}000 \times \frac{1}{12} \times 1.625 \times (7.5)^3} = 0.46 \text{ in.}$$

which is practically the allowable deflection, or 0.45 in.

192. Main Purlins.—Since rigid sheathing, capable of taking the component of the roof load parallel to the sloping surface, is used, purlins need be designed only for flexure normal to the plane of the roof. Loading consists of a uniform load due to its own weight, and of 6 concentrated loads normal to the roof of (2.7 + 2.17 × 36.2) × 11.18 = 910 lb. each due to rafter reactions, as shown in Fig. 128.

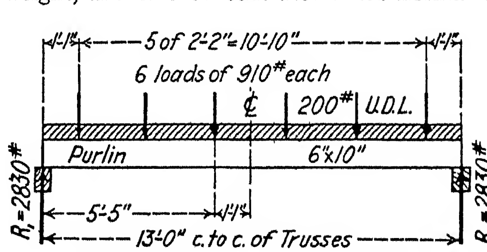


FIG. 128.—Loading of Main Purlin.

Assuming the purlin section as 6 × 10 in., or when dressed on four sides, $5\frac{1}{2} \times 9\frac{1}{2}$ in., the uniform load = (5.5 × 9.5) $\frac{4}{12}$ = 17.4 lb. per lin. ft. Of this, 17.4 cos 26° 34′ = 17.4 × 0.8945 = 15.6 lb. per lin. ft. acts normal to the roof.

Flexure.—Moment at purlin centre due to its own weight = $\frac{1}{8} \times 15.6 \times (13)^2 \times 12$ = 3960 in.-lb.

Maximum moment due to concentrated loading is constant over the central 2 ft. 2 in. and is (910 × 3) × 65 − 910(1 + 2)26 = 106,500 in.-lb.

Total maximum moment = 3960 + 106,500 = 110,460 in.-lb.

Required section modulus = 110,460/1500 = 73.6 in.³ Section modulus of $5\frac{1}{2} \times 9\frac{1}{2}$-in. section = $\frac{1}{6} \times 5.5 \times (9.5)^2$ = 82.7 in.³ The assumed section is adequate.

Shear.—Maximum end reaction normal to roof = (15.6 × $\frac{13}{2}$) + 3 × 910 = 2830 lb.

Maximum shearing stress, Eq. (1), Art. 178, = 1.5 V/A = 1.5 × 2830/(5.5 × 9.5) = 81 lb. per sq. in., as compared with 140 lb. per sq. in. allowed.

Deflection.—The deflection of the purlin normal to the roof must not exceed 13 × 12/300 = 0.52 in.

No appreciable error is involved in considering the purlin load as uniformly

distributed. Normal to the roof, including the normal component of the weight of the purlin, this totals $(910 \times 6) + 15.6 \times 13 = 5660$ lb. Of this $(20.6 \times 2.7) \times 11.18 = 260$ lb. (see deflection calculations for rafters, Art. 191) for each concentration is dead load. Including the force due to the purlin weight or, 200 lb., the addition to the actual load to give the deflection load is $0.33(260 \times 6 + 200) = 590$ lb. Total deflection load $= 5660 + 590 = 6250$ lb.

Maximum deflection normal to roof is

$$\Delta = \frac{5}{384} \cdot \frac{Wl^3}{EI} = \frac{5}{384} \cdot \frac{6250 \times (13 \times 12)^3}{1{,}500{,}000 \times \frac{1}{12} \times 5.5 \times (9.5)^3} = 0.52 \text{ in.}$$

which is exactly the deflection allowed.

193. Ridge Purlins.—Two ridge purlins will be used, one on each side of the open joint, as shown in Fig. 131. Each carries approximately one-half as much load as a main purlin, and while for normal loading a 3×10-in. section —half the size of the main purlins—would be adequate, it has small lateral strength. To provide against some possible lateral flexure, these purlins will be made 4×10 in. nominal, or $3\frac{5}{8} \times 9\frac{1}{2}$ in. dressed.

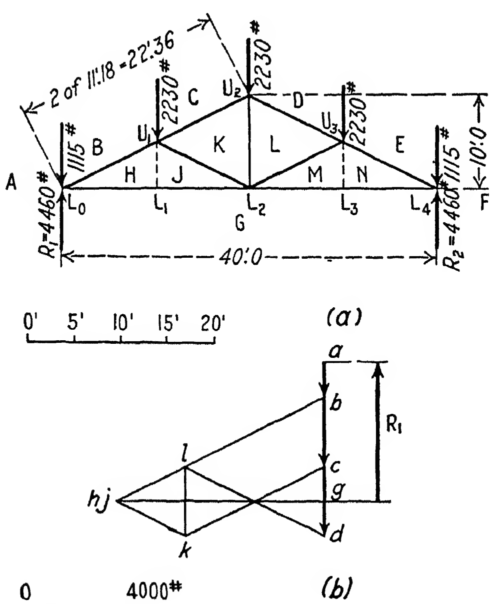


FIG. 129.—Dead Load Stress Sheet.

194. Dead-Load Stresses.—Dead load borne by truss will be assumed as concentrated at top chord panel points.

Weight of slates and sheathing, Art. 190, = 10.6 lb. per sq. ft. of sloping surface. For panel length of 11.18 ft. (see Fig. 129), vertical load carried to panel point U_1, U_2 or U_3 = $11.18 \times 13 \times 10.6 = 1550$ lb.

Weight of 6 rafters 11.18 ft. long, weighing 3 lb. per lin. ft. $= 6 \times 3 \times 11.18 = 205$ lb. applied at a panel point.

Weight of one purlin, weighing 17.4 lb. per lin. ft., Art. 192, or $17.4 \times 13 = 230$ lb., also carried to panel point.

Estimated weight of trusses per square foot of horizontal covered area, found by Ricker's formula to be $w = 0.04\,l + 0.000167\,l^2$, where l = span, centre to centre in feet. This gives for 40-ft. span $w = 1.87$ lb. Horizontal area applicable to one panel point $= 10 \times 13 = 130$ sq. ft. and vertical load at panel point due to truss weight $= 130 \times 1.87 = 245$ lb.

Total panel dead load $= 1550 + 205 + 230 + 245 = 2230$ lb. This is

applied at intermediate panel points and at the ridge, while, as an approximation, one-half a panel load, or 1115 lb., is applied at the two end panel points.

Stresses due to dead load are found graphically in Fig. 129 and are listed in column 2 of Table 21.

195. Snow-Load Stresses.—The full snow load being 20 lb. per sq. ft. of sloping roof surface, the amount of vertical load carried to one panel point = $11.18 \times 13 \times 20 = 2900$ lb. Half panel loads of 1450 lb. are applied at the ends of the truss.

Snow-load stresses may be determined from the dead-load stress diagram, Fig. 129, by proportion, multiplying all dead-load stresses by the factor 2900/2230 = 1.30. These stresses are listed in column 3 of Table 21.

For one-half full snow load the stresses are listed in column 4 of Table 21.

196. Wind Stresses.—Assume that both ends of the truss are rigidly fastened to the brick walls and that the reactions are parallel to the direction of the wind, or normal to the roof surface.

Panel wind load for a normal wind pressure (Art. 190) of 17.7 lb. per sq. ft. = $11.18 \times 13 \times 17.7 = 2580$ lb. One-half panel loads, or 1290 lb., are applied at the windward end of the truss and at the ridge.

Wind stresses are determined graphically in Fig. 130 and listed in columns 5 and 6 of Table 21.

To determine the reactions due to wind, a pole O is chosen and an equilibrium polygon is drawn beginning with the point O' on the line of action of the left-hand reaction, R_1. The dotted lines in Fig. 130(b) represent the rays, parallel to which the sides of the polygon of Fig. 130(a) are drawn. By means of the latter the reactions R_1 and R_2 are found.

Dotted truss members in Fig. 130(a) carry no stress under the wind loading shown.

Stresses due to $\frac{1}{3}$ wind load are listed in column 7 of Table 21.

TABLE 21

STRESSES IN MEMBERS IN POUNDS

(See Fig. 129 for Designation of Members)

Member	Dead Load	Snow Load	One-half Snow Load	Wind from Left	Wind from Right	One-third Maximum Wind Load	Dead Load Plus One-half Snow Load Plus Wind Load	Dead Load Plus Snow Load Plus One-third Wind Load	Maximum Stress
1	2	3	4	5	6	7	8	9	10
L_0U_1	−7450	−9700	−4850	−4580	−3260	−1530	−16,880	−18,680	−18,680
U_1U_2	−5000	−6500	−3250	−2660	−3260	−1090	−11,510	−12,590	−12,590
$L_0L_1L_2$	+6700	+8700	+4350	+5120	+2250	+1710	+16,170	+17,110	+17,110
U_1L_1	0	0	0	0	0	0	0	0	0
U_1L_2	−2500	−3250	−1625	−3220	0	−1070	− 7,345	− 6,820	− 7,345
U_2L_2	+2250	+2930	+1465	+1450	+1450	+ 480	+ 5,165	+ 5,660	+ 5,660

NOTE. + = tension. − = compression.

197. Proportioning of Truss Members.—*Top Chord* $LU_{01}U_2$.—Although the stress in the top chord segment U_1U_2 is substantially less than in L_0U_1, the chord will be made of one piece to avoid splicing.

Maximum stress in L_0U_1, from Table 21, is 18,680 lb.

Assume a solid 6 × 6-in. section, which when dressed is $5\frac{1}{2} \times 5\frac{1}{2}$ in., with an area of 30.25 sq. in. Effective length of member = 11.18 × 12 = 134 in. Permissible working stress = 1250 − 15 × 134/5.5 = 885 lb. per sq. in.

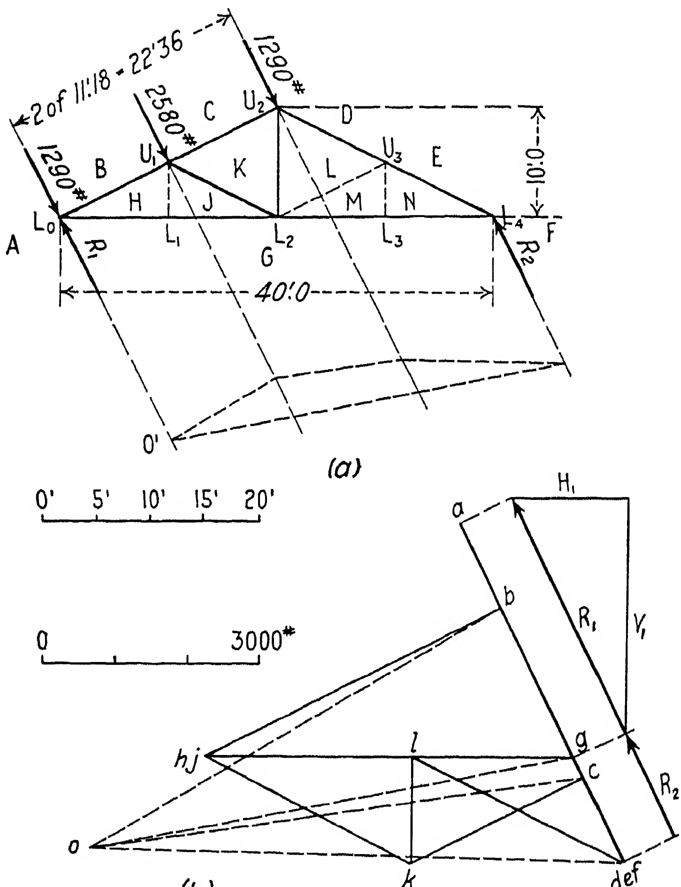


FIG. 130.—Wind Load Stress Sheet.

Required area, neglecting the small bending stress that arises from the purlins not being exactly at the panel points, = 18,680/885 = 21.1 sq. in.

The $5\frac{1}{2} \times 5\frac{1}{2}$-in. section, although excessive, is the smallest single commercial size that is adequate. Hence it will be adopted.

Compression Web Member U_1L_2.—Maximum stress, 7345 lb.

Assume a 4 × 4-in. section, with actual size $3\frac{5}{8} \times 3\frac{5}{8}$ in. and area 13.14 sq. in.

Effective length = 134 in. Permissible stress = 1250 − 15 × 134/3.625 = 695 lb. per sq. in.

Required area = 7345/695 = 10.6 sq. in.

Area provided by $3\frac{5}{8} \times 3\frac{5}{8}$-in. section is sufficient and it will be adopted, subject to possible revision to satisfy joint details.

Bottom Chord $L_0L_1L_2$.—Maximum stress = 17,110 lb., according to Table 21.

Required net area = 17,110/1500 = 11.5 sq. in.

To provide for notching at the joints, the adopted gross section should be 60 to 75% greater than the net area required. In this case adopt, subject to revision in joint detailing, a 6 × 6-in. section, of actual size $5\frac{1}{2} \times 5\frac{1}{2}$ in., giving a section of 30.25 sq. in.

Tension Vertical U_1L_1.—There is no calculable stress in this member, but a $\frac{3}{4}$-in. round rod with net area of 0.302 sq. in. at root of thread will be provided.

Tension Vertical U_2L_2.—Maximum stress, from Table 21, is 5660 lb.

Required net area = 5660/16,000 = 0.35 sq. in.

Use a $\frac{7}{8}$-in. round rod, not upset, having net area at root of thread of 0.419 sq. in.

198. Design of Joints.—Details of the roof panel, including the truss, are shown in Fig. 131. Truss members are made to intersect at a point, so as to reduce secondary stresses. The purlins, by reason of the simple details adopted, are placed slightly to one side of the panel point to clear the joint details.

Joint L_0.—Since at the heel joint the stresses to be provided for are greater than at any other joint in the truss, and the existing stress conditions often call for revision of the tentative section of the connected members, this is the controlling joint of the structure. It should, therefore, be designed first.

The notched type of joint shown in Fig. 131 will be adopted. Two end bearing areas 1–2 and 3–4 will be provided at the end of the top chord and at right angles thereto. The symmetrical location of these with respect to the top chord causes the resultant stress in the latter to pass through the intersection of the bottom chord and the wall reaction, thus eliminating moment of eccentricity.

The permissible bearing pressure on the surfaces 1–2 and 3–4 is governed by the allowable pressure on the bottom chord surfaces, they being inclined at an angle of 63° 26′ with the grain. This value, according to the specification, is

$$n = \frac{1400}{1 + \dfrac{1400 - 350}{350}\{1.55(0.4472)^2 - 0.55(0.4472)^4\}} = 750 \text{ lb. per sq. in.}$$

Required length of each surface, 1–2 and 3–4, = 18,680/750 × 5.5 × 2 = 2.26 in. Each will be made 2.25 in., as shown in Fig. 131. The top chord section does not need to be revised to increase the end bearing area.

Adopting the detail shown, the net right section of the bottom chord which is through the point 4 and the recess for the key is 2.50 × 5.5 = 13.75 sq. in. As only 11.5 sq. in. are required, the bottom chord section already tentatively adopted is now found to be adequate.

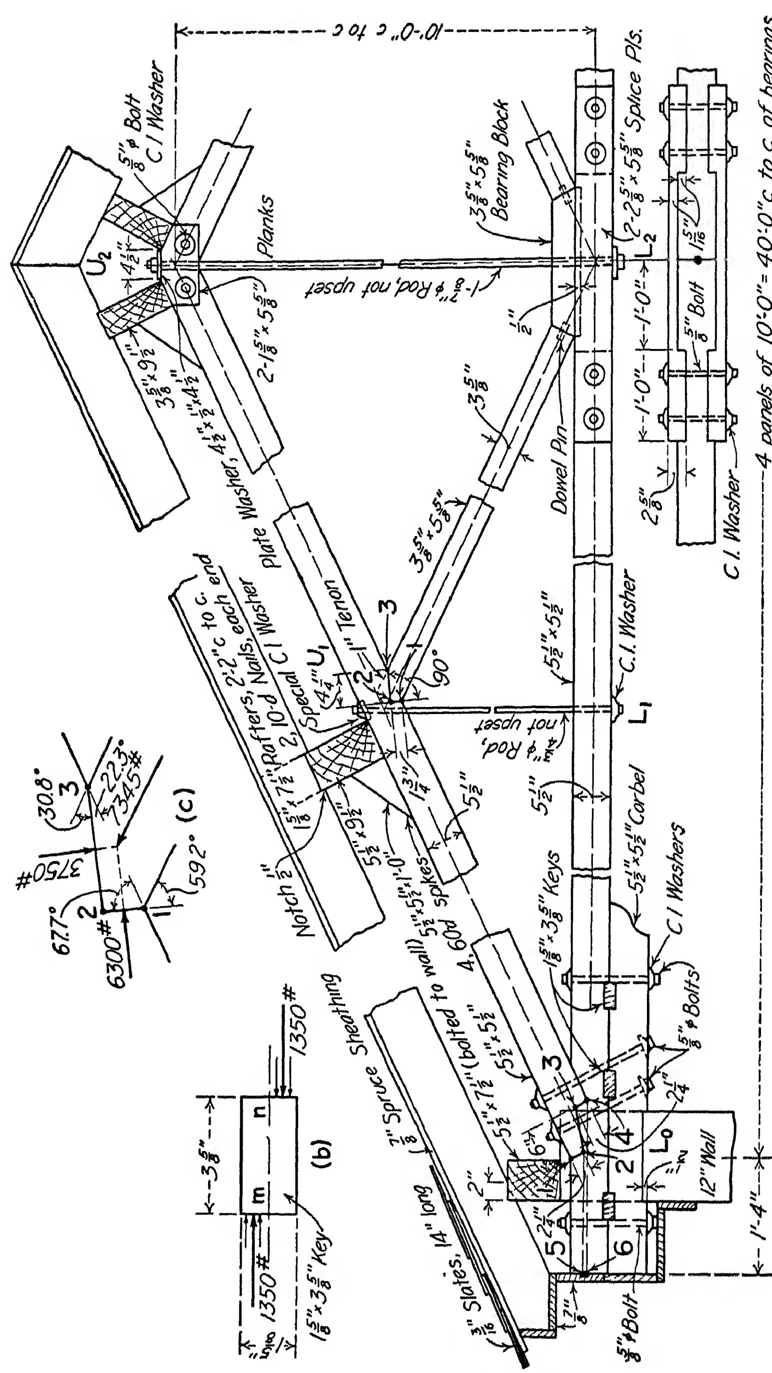


FIG. 131.—Details of Wooden Roof Truss.

To deliver the horizontal component of the top chord stress to the bottom chord, the horizontal shearing area 2–5 must be able to take one-half the horizontal component and the area 4–6 must be able to take the total horizontal component of 17,110 lb.

Required length 2–5 = 8555/5.5 × 140 = 11.1 in. Required length 4–6 = 17,110/5.5 × 140 = 22.2 in. Both these requirements will be met by carrying the bottom chord 1 ft. 4 in. past the centre of the wall bearing.

To hold the top chord in place in the notches in the lower chord, two $\frac{5}{8}$-in. bolts will be passed through the members, as shown in Fig. 131. Although they do not carry any calculable stress, the washers and bearing areas thereunder are usually proportioned on the basis of the full tensile capacity of the bolts being developed.

In order to obviate the further reduction of the bottom chord section by cutting of washer seats, a 6 × 6-in. corbel, $5\frac{1}{2} \times 5\frac{1}{2}$ in. dressed, about 4 ft. long, will be used, as shown in Fig. 131. This corbel will be secured against slipping on the bottom chord by means of keys and will be notched $\frac{1}{2}$ in. over the brick wall to prevent longitudinal shifting of the truss on the wall.

Tensile value of one $\frac{5}{8}$-in. bolt, with net area at root of thread of 0.202 sq. in., is 16,000 × 0.202 = 3230 lb.

For the washers at the upper end of the bolts, the basic safe bearing pressure, according to the specification, is 350 lb. per sq. in., but since the washers do not cover the entire width of the member, the specification permits this to be increased 25%, thus becoming 350 + 88 = 438 lb. per sq. in.

Required bearing area under a washer = 3230/438 = 7.37 in. A standard cast washer for a $\frac{5}{8}$-in. bolt has a net bearing area of 6.69 sq. in. (see Kidder-Parker's "Architects' and Builders' Handbook"). This is sufficient, as there is no calculable stress in the bolt.

The keys between the corbel and the bottom chord must be proportioned to resist, (1) the horizontal component of the assumed tension in the two bolts, or 2 × 3230 sin 26° 34′ = 2890 lb., and (2) the horizontal component of the maximum wind reaction, R_1, which from Fig. 130 is seen to be 1600 lb. Total horizontal force on keys = 2890 + 1160 = 4050 lb.

Assume three 2 × 4-in. keys, of net dimensions $1\frac{5}{8} \times 3\frac{5}{8}$ in. The forces acting on these are a pressure on half of each vertical face, producing a horizontal shear along the plane *m–n*, as shown in Fig. 131(*b*). With three keys the force on each half face is 4050/3 = 1350 lb., and the intensity of pressure on the key, it being $5\frac{1}{2}$ in. long, is 1350/0.5 × 1.625 × 5.5 = 303 lb. per sq. in. As only one-half of the face of the key is under pressure, the allowable bearing stress transverse to the fibres may be taken as 350 + 25% = 438 lb. per sq. in. Shearing stress per key on plane *m–n* = 1350/3.625 × 5.5 = 68 lb. per sq. in. The allowable shearing stress parallel to the fibre is 140 lb. per sq. in. The key is, therefore, sufficiently large. A $\frac{5}{8}$-in. bolt with cast washers at each end, is passed through the chord and the corbel near each key to prevent twisting of the latter.

The bearing stress on the wall must be compared with the allowable pressure

of 175 lb. per sq. in. on the brickwork. The maximum reaction under the two cases of loading will first be found.

Dead-load reaction = 2 × 2230 = 4460 lb.

Full-snow-load reaction = 2 × 2900 = 5800 lb.

Vertical component, V_1, of full-wind-load reaction at windward support, from Fig. 130 = 3200 lb.

Horizontal component, H_1, of full-wind-load reaction is found similarly to be 1600 lb.

Vertical reaction for Case (1), that is, dead load, $\frac{1}{2}$ snow load, and full wind load, = 4460 + 2900 + 3200 = 10,560 lb.

Vertical reaction for Case (2), that is, dead load, full snow load and $\frac{1}{3}$ wind load, = 4460 + 5900 + 1070 = 11,430 lb. This case governs.

Required bearing area on brickwork = 11,430/175 = 65.3 sq. in. The $5\frac{1}{2}$-in. width of corbel bearing on a 12-in. brick wall, gives 5.5 × 12.0 = 66 sq. in., which is adequate.

Required depth of notch to transfer the horizontal component of the wind reaction to the wall depends partly on frictional resistance of the corbel on the wall. Minimum vertical reaction possible with maximum wind force = dead-load reaction + vertical component of wind-load reaction = 4460 + 3200 = 7660 lb. If the coefficient of friction of timber on brickwork be taken as $\frac{1}{3}$, the frictional force developed = $\frac{1}{3}$ × 7660 = 2550 lb., or more than the horizontal reaction of 1600 lb. Nevertheless, the corbel will be notched $\frac{1}{2}$ in. over the wall.

The wall will be carried up between trusses to such a height that a 6 × 8-in. ($5\frac{1}{2} \times 7\frac{1}{2}$) timber plate carried along the top thereof will afford proper bearing support for the rafters. Calculation shows that a 2-in. bearing of the rafters is adequate.

A simple cornice finish is adopted to box in the ends of the trusses and rafters as shown in Fig. 131.

Joint U_1.—A notch joint will be used with the two faces of the notch in the top chord at right angles to each other, as shown in Fig. 131, so that the resultant pressures on them will intersect at a point on the axis of the diagonal U_1L_2, and thus obviate moment of eccentricity.

Assume a notch 1–2, 1.75 in. deep. The face 2–3 then becomes 4.25 in. long.

The two components of the 7345-lb. compressive force in the diagonal, at right angles to 1–2 and 2–3 are found from the diagram shown in Fig. 131(*c*) to be 6300 lb. and 3750 lb., respectively.

The allowable pressure on the face 1–2 will be the pressure on the end of the diagonal, as this face makes an angle of only 59.2° with the grain of this member and an angle of 67.7° with the grain of the chord, as shown in Fig. 131(*c*). Applying the specification,

$$n = \frac{1400}{1 + 3\{1.55(0.5020)^2 - 0.55(0.5020)^4\}} = 680 \text{ lb. per sq. in.}$$

On the face 2–3, the permissible bearing is, for a similar reason, determined with respect to the grain of the chord and is n = 370 lb. per sq. in.

Required area of surface 1–2 = 6300/680 = 9.3 sq. in. With a 4 × 4-in. diagonal, measuring only $3\frac{5}{8}$ in. to the side when dressed, this area cannot be developed without greatly weakening the chord by deep notching. Hence the diagonal section will be revised and made a nominal 4 × 6 in., with the 6-in. face at right angles to the plane of the truss. Bearing area provided then = 1.75 × 5.5 = 9.6 sq. in., which is adequate.

Required area of surface 2–3 = 3750/370 = 10.2 sq. in. Allowing for a $1\frac{1}{8}$-in. slot to accommodate the 1-in. tenon, effective bearing width = 5.50 − 1.13 = 4.37 in. and area provided = 4.37 × 4.25 = 18.6 sq. in., which is more than adequate.

A special cast washer let into the top of the chord about $\frac{1}{2}$ in. is provided for the $\frac{3}{4}$-in. rod vertical which carries no stress except a part of the weight of the two panels of the bottom chord below.

The purlin is offset slightly to clear this washer. Tipping of the purlin is resisted by a triangular block secured to the chord and purlin by four 60–*d* spikes, as shown in Fig. 131.

Joint L_1.—A standard cast-iron washer at the bottom of the $\frac{3}{4}$-in. rod is adequate.

Joint U_2.—The ridge purlins are placed clear of the apex washer on the rod vertical and hence do not affect the bearing under the washer.

Maximum stress in rod vertical = 5660 lb.

Allowable pressure under washer, the bearing surface being at an angle of 26.57° with the grain is n = 380 lb. per sq. in. This may be increased 25%, since the washer does not cover the full width of the member. It thus becomes 475 lb. per sq. in.

Required area under washer = 5660/475 = 11.9 sq. in. A plate washer $4\frac{1}{2} \times 4\frac{1}{2}$ in. will give adequate bearing area, after allowing for the hole for rod, and will be found to serve also for the lower end of the rod. On the basis of flexural strength, a thickness of $\frac{1}{2}$ in. is found to be ample.

Horizontal component of maximum chord stress = 12,590 cos 26° 34′ = 11,300 lb.

Allowable pressure on contact surface of chords, which makes an angle of 63° 26′ with the grain is n = 750 lb. per sq. in.

Required bearing area = 11,300/750 = 15.1 sq. in. With the apex cut to a $4\frac{1}{2}$-in. base for the plate washer, the depth of the vertical contact surface is found to be 5 in., and the area provided, after deducting 1 in. in width for the hole for the rod is 5 × 4.5 = 22.5 sq. in., or much more than required.

Two pieces of nominal 2 × 6-in. plank are bolted by $\frac{5}{8}$-in. bolts to the sides of the chord members to prevent displacement normal to the plane of the truss. Standard cast washers are used with these bolts.

Joint L_2.—A tabled fish plate splice of wooden construction shown in Fig. 131 will be used. This splice is designed in detail in the problem of Art. 169, but for simplicity, cast-iron washers which are used elsewhere in the truss will be adopted here.

Net area of chord through centre of $\frac{3}{4}$-in. hole for $\frac{5}{8}$-in. bolt = (5.5 − 0.75)

$\times$ 2.875 = 13.65 sq. in. This is adequate, since, according to Art. 197, only 11.5 sq. in. are required.

Although the permissible bearing stress under the plate washer for the rod vertical is only 350 lb. per sq. in., the size adopted for the joint U_2, $4\frac{1}{2} \times 4\frac{1}{2}$ in., will be adequate.

Maximum stress is diagonal = 7345 lb.

Permissible compression on wood bearing block, with bearing surface inclined at angle of 63° 26′ with grain of block is n = 750 lb. per sq. in.

Required area = 7345/750 = 9.8 sq. in.

Area provided, deducting 0.6 sq. in. for the dowel pin, = 3.625 $\times$ 5.625 − 0.6 = 19.8 sq. in.

The bearing block is made full width of the chord and is let into the chord $\frac{1}{2}$ in. to preclude shifting longitudinally due to unequal stresses in the two diagonals.

Rafter Connections.—Rafters are notched about $\frac{1}{2}$ in. over purlins, as indicated in Fig. 131, to prevent end shifting. In addition, two 10–*d* nails connect each end of each rafter to the purlin.

APPENDIX I

ANSWERS TO EXERCISE PROBLEMS

Problems of Art. 20.—(1) Section *ABFGCD*; Area = 4.32 sq. in. (2) 138,400 lb. (3) $4 \times 4 \times \frac{3}{8}$-in. angle. (4) $\frac{1}{2}$ in. (5) Section *ABCD* or *EFGH*; 422,000 lb. (6) 17,550 lb. (7) 17,000 lb. (8) $4 \times 3 \times \frac{1}{4}$-in. angle. (9) $\frac{3}{8}$ in. (10) 0.875 P. (11) 6500 lb. per sq. in. (12) Safe; f_t = 17,370 lb. per sq. in. (13) 9180 lb. per sq. in. (14) 16,250 lb. per sq. in. (15) Safe; f_t = 18,320 lb. per sq. in. at section *ABCD*.

Problems of Art. 35.—(1) Five. (2) Three. (3) Four. (4) Six. (5) 22.2 theoretically, 24 practically. (6) 13,500 lb. (7) 14.8%. (8) See Fig. 132 for detail. (9) 6000 lb. and 0 lb. (10) 7640 lb. on rivet 1, 4000 lb. on 2, and 3200 lb. on 3. (11) Adopt two gauge lines $1\frac{3}{4}$ and $3\frac{1}{2}$ in. from the back of the 4-in. legs. Use 6 rivets spaced 2 in. staggered, except for outer space which should be $2\frac{1}{4}$ in. End distance for angles and edge distance for gusset $1\frac{1}{2}$ in. Capacity = 53,000 lb. (12) See Fig. 133. (13) (*a*) *AB* for the full load; area,

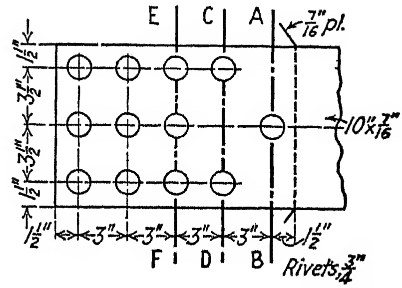


FIG. 132.—Connection of Tension Member for Maximum Efficiency.

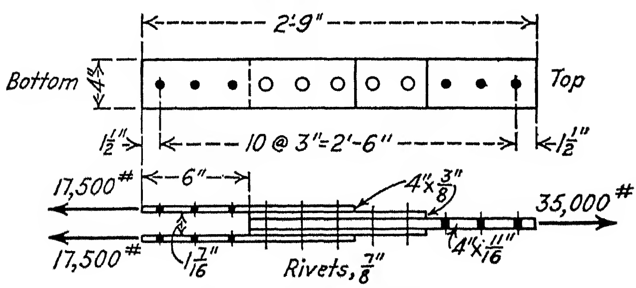


FIG. 133.—Design of Forked Hanger.

30.94 sq. in.; 31.05 sq. in. required. *CD* for partly transferred load (452,000 lb.); area, 28.19 sq. in.; 28.25 sq. in. required. (*b*) 529,000 lb. (*c*) 12.04 sq. in. required; 12.00 sq. in. provided. (*d*) Rivets equal in strength to those through the $\frac{1}{2}$-in. plates and the gussets. Smallest practicable number = 16 shop and 16 field. (*e*) Smallest practicable number = 16.

Problems of Art. 54.—(1) 60,000 lb. (2) 160,000 lb. (3) Safe load, 302,000 lb. (4) Maximum existing stress, 8780 lb. per sq. in.; permissible, 8900 lb. per sq. in. (5) 6-in., 25.0-lb. Carnegie H. (6) 8-in., 34.3-lb. Carnegie H; 8-in., 35-lb. Bethlehem H; or two 6-in., 8-lb. channels with two $8 \times \frac{5}{16}$-in. plates. (7) Unsafe; maximum existing stress, 10,250 lb. per sq. in.; permissible, 9290 lb. per sq. in. (8) Unsafe; maximum existing stress, 14,010 lb. per sq. in.; permissible, 12,710 lb. per sq. in. (9) 118,200 lb. (10) Safe; maximum existing stress, 8330 lb. per sq. in.; permissible, 10,000 lb. per sq. in. (11) 11,270 lb. per sq. in. (12) 6-in., 25.0-lb. Carnegie H. (13) 8-in., 32.6-lb. Carnegie H with web in plane of bending.

Problems of Art. 71.—(1) Six. (2) 2040 lb. (3) 24,200 lb. (4) 2070 lb. Permissible tension, 2650 lb. (5) 10,350 lb. per sq. in. (6) $2\frac{1}{4} \times \frac{5}{16}$-in. bars.

Problems of Art. 112.—(1) 7-in., 15.3-lb. I. (2) 8-in., 19-lb. W.F. (3) 8-in., 17-lb. W. F. (4) (*a*) 76,800 in.-lb. (*b*) 4000 lb. (5) 6000 lb. (6) 14-in., 30-lb. W. F. (7) Safe; f_e = 10,770 lb. per sq. in., p_f = 10,860 lb. per sq. in.; $f_s = f_{dc}$ = 2360 lb. per sq. in. (8) Unsafe; f_e = 15,530 lb. per sq. in.; p_f = 8800 lb. per sq. in.; $f_s = f_{dc}$ = 1145 lb. per sq. in. (9) 12-in., 25-lb. W. F. (10) No; f_e = 9940 lb. per sq. in. p_f = 9250 lb. per sq. in. (11) Safe; f_e = 12,320 lb. per sq. in., p_f = 12,400 lb. per sq. in.; $f_s = f_{dc}$ = 2175 lb. per sq. in. (12) 12-in., 28.0-lb. W. F. (13) 19,200 lb. (14) 19,120 lb. (15) 2686 lb. (16) Unsafe; f_e = 20,550 lb. per sq. in.; $f_s = f_{dc}$ = 4530 lb. per sq. in. (17) 30,930 lb. (18) 0.165 in. (19) Safe; f_{vc} = 10,550 lb. per sq. in. p_{vc} = 13,070 lb. per sq. in. (20) 10-in., 30-lb. I. (21) Unsafe in vertical compression; $f_s = f_{dc}$ = 8000 lb. per sq. in.; f_{vc} = 15,230 lb. per sq. in. $p_{dc} = p_{vc}$ = 10,200 lb. per sq. in. (22) Unsafe in vertical compression; $f_s = f_{dc}$ = 9520 lb. per sq. in.; p_{dc} = 12,040 lb. per sq. in. f_{vc} = 17,600 lb. per sq. in.; p_{vc} = 11,940 lb. per sq. in. (23) Unsafe in vertical compression; $f_s = f_{dc}$ = 9080 lb. per sq. in.; p_{dc} = 10,025 lb. per sq. in.; f_{vc} = 15,550 lb. per sq. in.; p_{vc} = 12,230 lb. per sq. in. (24) Safe at load; unsafe in vertical compression at support, where f_{vc} = 14,040 lb. per sq. in., and p_{vc} = 12,670 lb. per sq. in. (25) Unsafe in vertical compression; f_e = 16,800 lb. per sq. in.; $f_s = f_{dc}$ = 8150 lb. per sq. in.; p_{dc} = 11,520 lb. per sq. in.; f_{vc} = 14,300 lb. per sq. in.; p_{vc} = 13,130 lb. per sq. in. (26) Unsafe in flexure at centre; f_e = 21,650 lb. per sq. in.; otherwise safe. (27) 84,000 in.-lb. (28) Two 5-in., 10-lb. I's. (29) Two 4-in., 7.7-lb. I's (deflection governs). (30) 2530 lb. per lineal ft. (31) 22,240 lb. (32) Safe; f_e = 15,550 lb. per sq. in.; $f_s = f_{dc}$ = 1340 lb. per sq. in. (33) 43,300 lb. (34) 74,900 lb. (35) 2355 lb. per lineal ft. (36) Unsafe; f_e = 16,420 lb. per sq. in. (37) 27,500 lb. (38) Six. (39) 19.45 ft. (40) 3.1, say 4. (41) 5.3 shop, say 6; 6.9 field, say 8. (42) 2 shop; 2.7 field, say 4; $\frac{3}{8}$-in. angles. (43) 8000 lb. (44) 3.9, say 4. (45) 7.5, say 8; one row of 4 in each outstanding leg. (46) 13,020 lb. per sq. in. (47) Yes; f_{vc} = 8030 lb. per sq. in.; p_{vc} = 12,210 lb. per sq. in. (48) 7520 lb. (49) 37,700 lb. (50) 61,800 in.-lb. (51) 113.4 lb. per lin. ft.

Problems of Art. 118.—(1) 487 in.^{3}; 0.508 in. above gravity axis of gross section. (2) 98,040 lb. (3) 36 required in each plate from centre of bearing

to centre of span. Space 6 in. in each row from centre to within about 2 ft. of end of outer cover plate; from there to end space about 4 in. centres. (4) 3500 lb. per lin. ft., determined by flexure. (5) 16.8 and 24.0 ft.

Problems of Art. 131.—(1) No; $f_s = f_{dc} = 6670$ lb. per sq. in. $p_s = p_{dc} = 5420$ lb. per sq. in. (2) 8,010,000 in.-lb. (3) 3,790,000 in.-lb., based on compression flange. (4) Two angles, $6 \times 4 \times \frac{1}{2}$ in. (5) Two angles, $6 \times 6 \times \frac{3}{4}$ in. (6) Safe in shear and diagonal compression; $f_s = f_{dc} = 4170$ lb. per sq. in. Slightly overstressed in tension flange; $f_t = 16{,}010$ and $f_c = 14{,}700$ lb. per sq. in. (7) Safe in shear and diagonal compression; $f_s = f_{dc} = 5040$ lb. per sq. in.; $p_s = p_{dc} = 9470$ lb. per sq. in. Slightly overstressed in tension flange; $f_t = 17{,}050$ lb. per sq. in. $f_c = 15{,}550$ lb. per sq. in. (8) 3490 lb. per lin. ft. (9) 14.44 sq. in. (10) Web, $36 \times \frac{3}{8}$ in. Each flange, two angles $6 \times 6 \times \frac{7}{16}$ in., and two plates $14 \times \frac{3}{8}$ in. (11) 28.9 ft. (12) 8.75 and 10.25 ft. (13) 19.6, 27.7 and 33.9 ft. (14) 23.4 and 25.0 ft. (15) 25.5 and 27.5 ft. (16) 22 rivets. (17) 4.79 in. for top; 4.92 in. for bottom flange. (18) 7.90 in. (19) 7.07 in. (20) 3.49 in. (21) 7.29 in. (22) 4.83 in. (23) 5.02 in. (24) Adequate; 2.28 in. permissible.

Problems of Art. 152.—(1) 65.625 kip-ft.; one load at point and the other two on the 11-ft. segment. (2) 191.67 kip-ft. (3) 218.75 kip-ft. (4) 206.67 kip-ft. (5) 150 kip-ft. Absolute maximum moment occurs at 0.5 ft. from mid-span. (6) 48 kip-ft. (7) 35.6 kip-ft. (8) 68 kip-ft. (9) 112.7 kip-ft. (10) 158.33 kip-ft. (11) 150 kip-ft. (12) 183 kip-ft. (13) 216.8 kip-ft. (14) 45 kip-ft. (15) 1314.3 kip-ft. with wheel 6 at third panel point (32 ft. from end). (16) 160 kip-ft. (17) Shear, 53.3 kips; moment, 213.3 kip-ft. (18) 15-kip load at the point and 8-kip load on the short segment; shear, 14.5 kips. (19) 21.35 kips. (20) 12,600 lb. (21) Theoretical, 46,400 lb.; conventional, 47,300 lb. (22) Correct, 8000 lb.; conventional, 9000 lb. (23) Correct, 6750 lb.; conventional, 8100 lb. (24) 18 kips. (25) 12,730 lb. (26) 22,400 ft.-lb. (27) 16,550 ft.-lb. (28) 21,000 ft.-lb. (29) 11,250 ft.-lb. (30) 16,670 lb. (31) 31,250 lb. (32) 17,650 lb. (33) 57,800 ft.-lb. (34) 279,790 ft.-lb. (35) 74,300 ft.-lb. (36) 38,600 lb.

Problems of Art. 171.—(1) $5\frac{1}{2} \times 9\frac{1}{2}$ in. (2) Ten. (3) 5.48 in. (4) $\frac{5}{8}$ in. deep; 6 in. long. (5) 28,900 lb.

Problems of Art. 177.—(1) $11\frac{1}{2} \times 11\frac{1}{2}$ in. (2) Unsafe; $f_c + f_f = 1042$ lb. per sq. in. $p_c = 920$ lb. per sq. in. (3) Unsafe; $f_c + f_f = 892$ lb. per sq. in.; $p_c = 840$ lb. per sq. in. (4) 890 lb. per sq. in. (5) Safe; $f_c + f_f = 850$ lb. per sq. in.; $p_c = 880$ lb. per sq. in. (6) $13\frac{1}{2} \times 13\frac{1}{2}$ in. (7) 12×12 in. (8) Unsafe; $f_c + f_f = 859$ lb. per sq. in. $p_c = 700$ lb. per sq. in. (9) 12×16 in. (10) Unsafe in the 14-in. direction; $f_c = 1100$ lb. per sq. in.; $p_c = 1000$ lb. per sq. in.

Problems of Art. 187.—(1) 6×10 in. (2) Unsafe; Maximum $f_s = 156$ lb. per sq. in. (3) Safe; Maximum $f_s = 98$ lb. per sq. in. (4) 417 lb. per sq. in. (5) 6×10 in. (6) 667 lb. per lin. ft. (7) Unsafe in shear; $f_s = 250$ lb. per sq. in. (8) Yes; $f_f = 1440$ lb. per sq. in. (9) 8×12 in. (10) 1152 lb. per lin. ft. (11) 8×12 in. (12) 8×12 in. (13) 1275 lb. per sq. in.

(14) 63 lb. per sq. in. (15) 107 lb. per sq. in. (16) 141 lb. per sq. in. (17) 478 lb. per sq. in. (18) Unsafe; $f_s = 211$ lb. per sq. in.; $f_f = 1690$ lb. per sq. in. (19) Moment, 60,000 in.-lb.; shear, 2500 lb. (20) 1690 lb. per sq. in. (21) 1293 lb. per sq. in. (22) 1347 lb. per sq. in. (23) 2 in. nominal, $1\frac{5}{8}$ in. dressed. (24) Safe in flexure; $f_f = 710$ lb. per sq. in.; deflection (0.37 in.) is excessive. (25) Satisfactory; $f_f = 565$ lb. per sq. in.; deflection, 0.198 in. (26) 6×12 in. (27) 6×12 in. (28) Beams framing in at third-points and at columns; beams, 8×12 in.; girders, 12×18 in. (29) Beams framing in at mid-span and at columns; flooring, 3 in. nominal, $2\frac{5}{8}$ in. dressed; beams, $7\frac{1}{2} \times 15\frac{1}{2}$ in. dressed; girders, $11\frac{1}{2} \times 17\frac{1}{2}$ in. (30) Satisfactory; $f_f = 455$ lb. per sq. in.; maximum deflection, 0.19 in. (31) 4650 in.-lb. (32) Dressed, 1 in. (33) 18,000 in.-lb. (34) Nominal, 4×8 in., dressed, $3\frac{5}{8} \times 7\frac{1}{2}$ in.

APPENDIX II

DERIVATION OF CERTAIN FORMULAE

A1. Eq. (6), Art. 1, for Area Required for Combined Stress.—If the effect of deflection is to be considered, evidently the final term of Eq. (4) of Art. 1 would be altered in the same ratio as the final term of Eq. (3) is altered to give the final term of Eq. (5). Hence, the final term of Eq. (6) becomes

$$\frac{I}{r^2 p_t} \times \frac{My_e}{I + \frac{Pl^2}{CE}} = \frac{My_e}{\left(r^2 + \frac{Pl^2}{CAE}\right)p_t}$$

A2. Equations of Art. 58 Respecting Anchorage of a Column Base.—Determination of anchor bolt tension and compressive stress on the pedestal at the toe of a column base due to overturning moment can be made with accuracy only by taking into account the elastic properties of the construction. A convenient method of doing this is one developed by Wendt (see *Engineering Record,* Oct. 17, 1914, p. 441). The method is applicable in all cases where the base plate tends to lift off the masonry at the windward edge, but is not applicable where there is no tension in the windward anchor bolts (J.E. Lothers, *Engineering News-Record,* Jan. 3, 1935, p. 5).

If the column is subjected to an axial load R', with an eccentricity e', and an additional overturning moment M', it is convenient to replace the two, shown dotted in Fig. 134(a), by a single force $R = R'$ acting with an eccentricity of

$$e = \frac{M' + R'e'}{R}$$

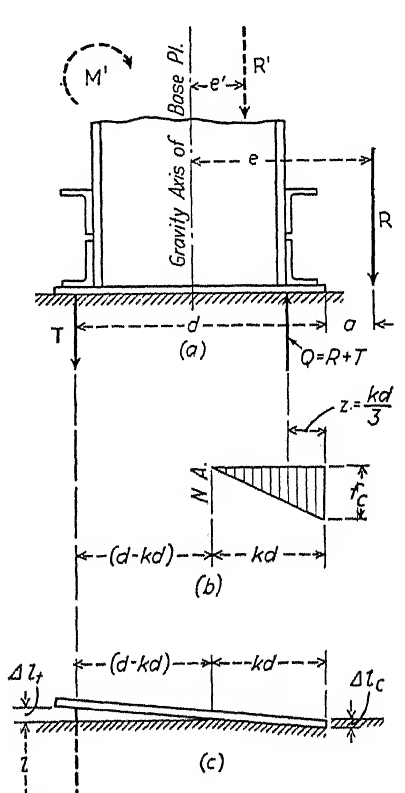


FIG. 134. Anchorage of Column Base.

Three equations may be set up to make possible the determination of the three independent unknowns, which are the anchor bolt tension, T, the existing maximum compressive stress, f_c, and the width of the pressure triangle, kd, Fig. 134(b).

First, since the sum of the vertical forces is zero, we have, if Q be the total compression,

$$R + T - Q = 0 \tag{1}$$

Second, the sum of the moments at the contact of the base with the pedestal must be zero. If the substitute single force R act at a distance a from the toe of the base (positive if outside, and negative if inside, the toe) and if the distance from the toe to the active anchor bolts be d,

$$R\left(\frac{kd}{3} \pm a\right) - T\left(d - \frac{kd}{3}\right) = 0 \tag{2}$$

Third, if the base plate be considered as remaining a plane under stress, the displacements of the bottom of the plate from the original plane of the top of the pedestal will be proportional to the distance from the neutral axis, Fig. 134(c). If the effective vertical depth of the pedestal be considered as the embedment length, l, of the anchor bolts, and the vertical displacements of the plate at the anchor bolt and the compressed edge of the plate be Δl_t and Δl_c, respectively, then

$$\frac{\Delta l_t}{\Delta l_c} = \frac{d - kd}{kd} \tag{3}$$

Applying Hooke's law,

$$\Delta l_t = \frac{Tl}{A_g E_s}$$

$$\Delta l_c = \frac{f_c l}{E_c}$$

where A_g = gross area of the active anchor bolt;
E_s = modulus of elasticity of the embedded anchor bolt, making due allowance for the effect of the encasement; and
E_c = modulus of elasticity of the concrete in compression under the base plate, making allowance for the relation of the loaded area to the area of the top of the pedestal.

Making use of these expressions, and letting $E_s/E_c = n$, Eq. (3) may be written

$$f_c = \frac{T}{nA_g} \cdot \frac{kd}{d - kd} \tag{4}$$

Introducing in Eq. (1) the value of R from Eq. (2) and the value of Q, which is $\frac{1}{2} f_c bkd$, where b = breadth of the base, we have

$$T \cdot \frac{d - \dfrac{kd}{3}}{\dfrac{kd}{3} \pm a} + T - \frac{T}{2nA_g} \cdot \frac{kd}{d - kd} \cdot bkd = 0$$

Letting $\frac{kd}{3} = z$, this may be written

$$z^3 \pm az^2 + z \cdot \frac{2\,nA_g\,(d \pm a)}{3\,b} = \frac{2\,nA_gd\,(d \pm a)}{9\,b}$$

If, for simplicity, we let

$$\frac{2\,nA_g\,(d \pm a)}{9\,b} = C$$

then

$$z^3 \pm az^2 + 3\,Cz = Cd \tag{5}$$

Solution of Eq. (5) fixes the width, kd, of the pressure triangle.

z being known, the anchor bolt tension may be found by rearranging Eq. (2), thus

$$T = R \cdot \frac{z \pm a}{d - z} \tag{6}$$

The maximum compression is found from Eq. (1) by expressing Q in terms of f_c, b and z, from which

$$f_c = \frac{R + T}{1.5\,bz} \tag{7}$$

A3. Equations of Art. 59 Respecting Moment Developed by Tension Rivets in Brackets.—As a result of elastic analyses similar to that by Jacob Friedland in *Engineering News*, April 25, 1912, p. 786, it appears that the neutral axis for brackets and beam connections is about one-seventh of the detail up from the bottom.

Referring to Fig. 36, it is evident that the total moment of resistance of the bracket, taken about the neutral axis, is

$$M = M' + M'' \tag{A}$$

where M' = moment of resistance of the tensile forces and M'' = moment of resistance of the toe compression.

Now the total compression equals the total tension, or

$$C = T_1 + T_2 + T_3 + \ldots = \Sigma T_n \tag{B}$$

As the arm of C with respect to the neutral axis is $\frac{2}{21}\,h$, the depth of the whole bracket being h, then

$$M'' = \frac{2\,h}{21} \cdot C = \frac{2\,h}{21} \cdot \Sigma T_n$$

But ΣT_n = sum of the tensile forces in the rivets = $T\Sigma d$, where T = tension in a hypothetical rivet at 1 in. from the neutral axis, and d = distance of any rivet from this axis.

Further, the moment of resistance of any rivet distant d from the neutral

axis is $Td \times d = Td^2$, and the total moment of resistance developed by all the tension rivets is

$$M' = \Sigma Td^2$$

Whence,

$$T = \frac{M'}{\Sigma d^2} \tag{C}$$

Eq. (*A*) may then be written

$$M = M' + \frac{2\,h}{21} \cdot \frac{M'\Sigma d}{\Sigma d^2}$$

or

$$M' = \frac{M}{1 + \dfrac{2\,h\Sigma d}{21\Sigma d^2}} \tag{D}$$

Since tensile stress is proportionate to the distance from the neutral axis,

$$T_n = Td_n = \frac{M'd_n}{\Sigma d^2} \tag{E}$$

Equations (*D*) and (*E*) are Eqs. (1) and (2) of Art. 59.

A4. Eq. (1), Art. 82, Relating to Length of Reinforcing Plates for Beams or Box Girders.—Let the net section moduli of the second plate and the

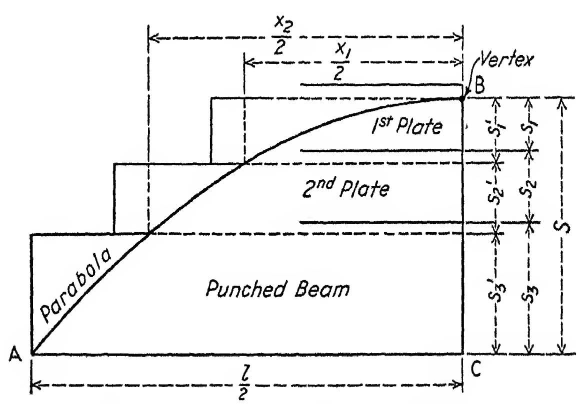


Fig. 135.—Length of Reinforcing Plates.

punched beam at mid-span, with respect to the maximum section, be s_2 and s_3, respectively, and let them be s_2' and s_3' for these elements at the theoretical cut-off of the first and second plates with respect to the net section there existing, as indicated in Fig. 135. Let the required net section modulus of the outer (first) plate at the point where it might be dispensed with be s_1'.

For a uniform load, and for a constant flexural stress along the beam, the

demand for section modulus is represented by the parabola AB, having its vertex at mid-span. Then

$$\frac{\left(\frac{x_1}{2}\right)^2}{\left(\frac{l}{2}\right)^2} = \frac{s_1'}{S}$$

where S = total required section modulus at mid-span. Hence, the theoretical length of the outer plate is

$$x_1 = l\sqrt{\frac{s_1'}{S}} \tag{A}$$

and similarly,

$$x_2 = l\sqrt{\frac{s_1' + s_2'}{S}} \tag{B}$$

$$x_n = l\sqrt{\frac{s_1' + s_2' + \dots . s_n'}{S}} \tag{C}$$

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